A Macroscopic Model of Quantum Mechanical Systems

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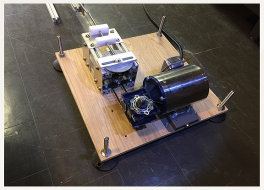
Abstract

This research uses a computer simulation in Python to model a network of coupled pendula that has been constructed as a macroscopic model for quantum mechanical systems. Our system consists of 22 actual masses and 2 dummy masses representing specified boundary conditions. A Runge-Kutta integrator was used for the simulation because of its ability to "cancel" the error associated with each timestep. The damping parameter used in the simulation was experimentally determined and could change over time to achieve good convergence of the simulation. Simulation results for a range of experimental conditions will be presented to show resonance and wave tunneling phenomena.

The Experimental Apparatus

The machine:



Drive Mechanism

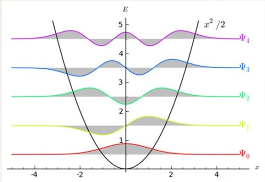
Coupled Pendulum

The mechanism driving the system is a .5 horsepower electric motor. This motor is capable of providing various frequencies to the system, necessary to observe the system with different inputs. The coupled pendulum is composed of steel masses held up by fishing line. We are also using plastic blocks, used to provide different effective lengths of the pendulum.

Experiments/measurements:
Measurements were taken of the system at various frequencies and mass displacements were recorded. In order to make these calculations and understand what is happening, a relationship between quantum tunneling and coupled-pendula dispersion need to be understood. Calculations were made using a Runge-Kutta integrator with a time dependent damping constant.

Quantum/Coupled-pendula Dispersion Relation

Particles that travel through space can be treated as a probability distribution. In quantum mechanics the probability of particle position is modeled as a wave.



Source: Physics Stack Exchange

Schrödinger Equation in one dimension:

$$i\hbar\frac{\partial}{\partial t}\Psi = -\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2}\Psi + V\Psi$$

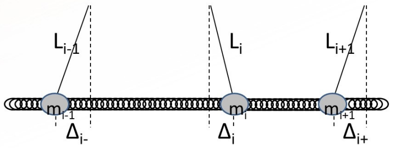
After considering ``local'' solutions of the form:

$$\Psi(x,t) = \Psi_0\cos(kx - \omega t + \phi)$$

And transforming this Fourier representation into differential equations, a dispersion relation can be found, and wavenumber can be solved for:

$$\mathrm{k} = \left(\frac{2m}{\hbar}\right)^{1/2}(\omega - \omega_0)^{1/2}$$

Now we consider our system of coupled pendula



The net force on each mass of our system is as follows:

$$m_i\frac{\partial^2}{\partial t^2}\Delta_i = m_i\frac{g}{L_i}\Delta_i + k_{sp}(\Delta_{i+1} - 2\Delta_i + \Delta_{i-1})$$

When we take the limit of a continuous distribution of mass, and Fourier analysis, we can solve similarly for the wavenumber and obtain:

$$k = \left[\frac{1}{v}(\omega + \omega_0)^{1/2}\right](\omega - \omega_0)^{1/2}$$

Damping Coefficients/Results

Three different damping coefficients were considered. In Fig.1 we investigated a high, constant damping coefficient.

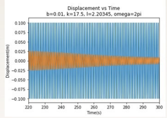


Fig. 1: High, Constant Damping

Amplitudes become too low to study over time.
In Fig.2 we considered a low, constant damping coefficient.

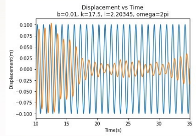


Fig.2: Low, constant damping

This type of chaotic motion does not settle down quick enough.
In Fig.3 we consider an initially high damping constant that settles down over time.

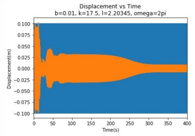
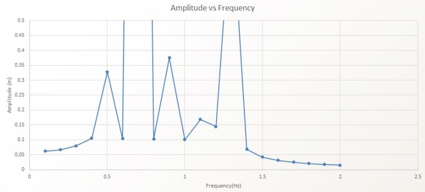


Fig.3: Time dependent damping

A time dependent damping constant settles chaotic motion and provides large enough amplitudes to study.
Results investigating amplitudes vs frequencies with a decreasing damping coefficient is provided below.



We are looking for resonant frequency. In our simulation, it appears as if there is an interesting resonance triplet in which there is a build-up in amplitude, a drop-off, and then large amplitude resonance. These resonance patterns appear in chemistry. It does appear that our system does reach resonant frequency at 0.7 Hz for this simulation.
Future Work:
Further simulations at more frequencies to study this resonance frequency triplet as well as observe quantum tunneling behaviors at resonant frequencies.

With respect to quantum tunneling, we plan to install a very large potential well to the system and try to observe tunneling phenomena.