The viscous force: $\mu \nabla^2 \vec{V}$

↑ viscosity.

$$\eta = \frac{\mu}{\rho}$$ kinematic viscosity.

Why $\nabla^2 \vec{V}$? $\nabla^2 f = \vec{\nabla} \cdot (\vec{\nabla} f)$

$$\nabla^2 f = \left(\frac{d^2}{dx^2} + \frac{d^2}{dy^2} + \frac{d^2}{dz^2} \right) f$$

$$\nabla^2 \vec{v} = \left(\frac{d^2}{dx^2} + \frac{d^2}{dy^2} + \frac{d^2}{dz^2} \right) V_x \hat{x}$$

$$+ \left(\quad \right) V_y \hat{y}$$

$$+ \left(\quad \right) V_z \hat{z}$$

$$\vec{\nabla} f = \left(\frac{d}{dx} \hat{x} + \frac{d}{dy} \hat{y} + \frac{d}{dz} \hat{z} \right) f$$

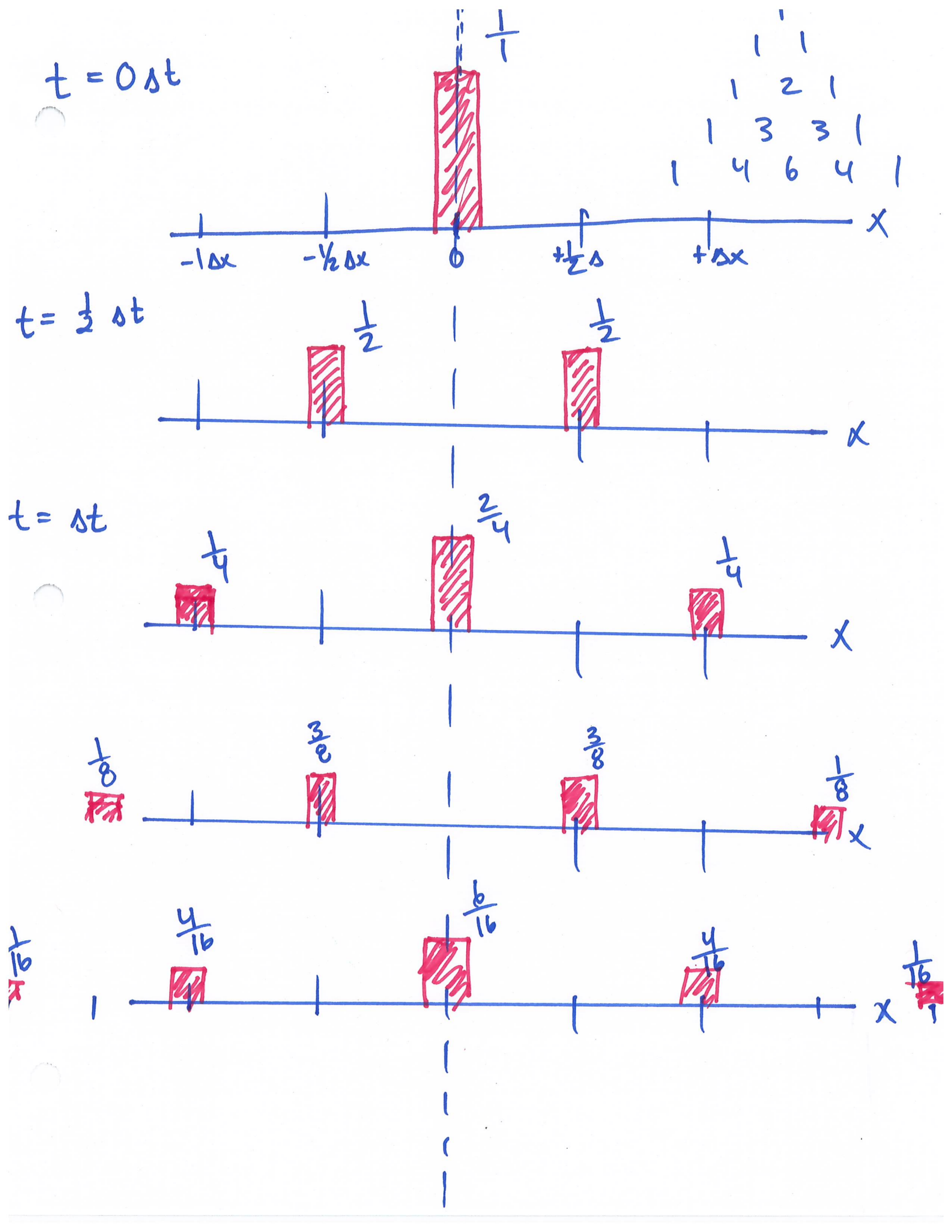
∇^2 should be synonymous with diffusion.

Mathland version of diffusion:

Discrete time steps, discrete spatial locations.

Consider 1-dimensional diffusion

- Have a quantity, that will diffuse. Our quantity can move a distance δx in a time δt.
- Move toward equilibrium by random processes that have equal probability in both directions.


t = 0 Δt
1/1
1
1 1
1 2 1
1 3 3 1
1 4 6 4 1
X
-1Δx
-½Δx
0
+½Δ
+Δx
t = ½ Δt
1/2
1/2
X
t = Δt
2/4
1/4
1/4
X
1/8
3/8
3/8
1/8
X
1/16
4/16
6/16
4/16
1/16
1
X

Why does ∇^2 represent diffusion:

for 1D: $\nabla^2 f = \frac{d^2}{dx^2} f$

$$= \frac{f_{i-1} - 2f_i + f_{i+1}}{\Delta x^2}$$

$$\frac{d}{dx} f = \frac{f_i - f_{i-1}}{\Delta x} \quad \text{or} \quad \frac{f_{i+1} - f_{i-1}}{\Delta x}$$

$$\frac{1}{2}\nabla^2 f = \frac{\frac{1}{2} f_{i-1} - f_i + \frac{1}{2} f_{i+1}}{\Delta x^2}$$

set $\Delta x = 1$ $\quad \frac{1}{2}\nabla^2 f = \frac{1}{2} f_{i-1} - f_i + \frac{1}{2} f_{i+1}$