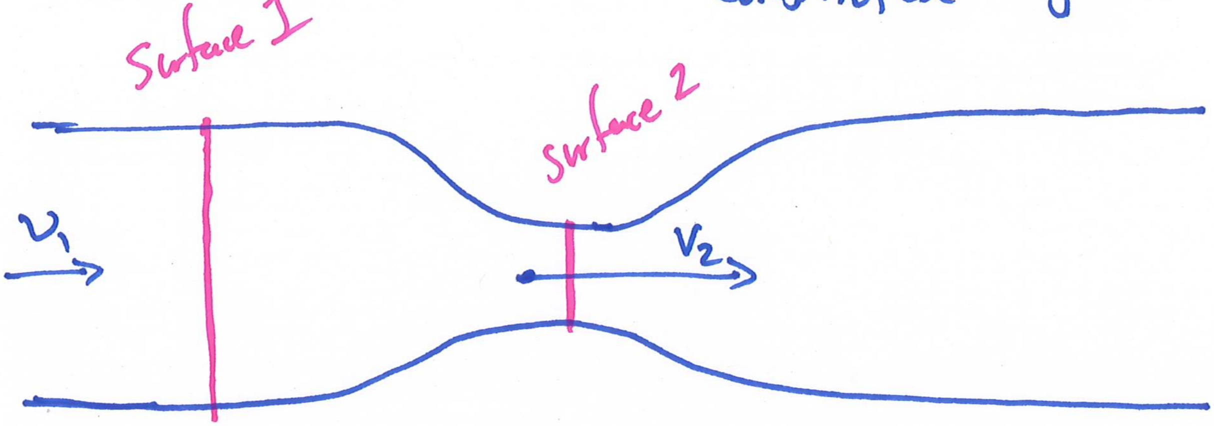
The Venturi tube: a pipe with a constricted region.



Q: How does the velocity change along the center of the pipe?

In the limit of low flow speeds (very subsonic) then the fluid is effectively incompressible.

$$\Delta\rho \sim \rho\, M^2$$

Mach number $\sim \dfrac{V}{V_{sound}}$

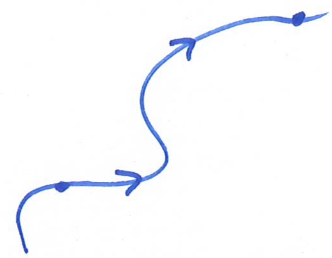
density variation from "normal"

if $M \simeq 0.3$ then $M^2 \simeq 0.1$

$\Delta\rho \simeq (0.1)\,\rho$ $\longleftarrow$ 10% change in density

Phy 201 Review:

$$m\vec{a} = \vec{F}_{NET}$$



↳ choose some path in space
integrate along $d\vec{x}$

$$\int m\vec{a}\cdot d\vec{x} = \int \vec{F}_{NET}\cdot d\vec{x}$$

↓

$$d\vec{x} = \vec{v}\,dt \qquad \vec{a} = \frac{d\vec{v}}{dt}$$

$$\int m\left(\frac{d\vec{v}}{\cancel{dt}}\right)\vec{v}\,\cancel{dt} = W_{NET}$$

$$\int_{v_i}^{v_f} m\,\vec{v}\cdot d\vec{v}$$

$$\frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2 = W_{NET}$$

For low speeds, we can consider constant density along the flow.

Mathematically, this is equivalent to

$$\vec{\nabla} \cdot \vec{V} = 0$$

Conservation of mass told us:

$$\underbrace{\left[\frac{\partial}{\partial t} + \vec{V} \cdot \vec{\nabla} \right]}_{\frac{d}{dt}\rho} \rho = - \rho \left(\vec{\nabla} \cdot \vec{V} \right)$$

$\frac{d}{dt}\rho$ ← as you follow a small volume of fluid

if $\frac{d}{dt}\rho = 0$, the it must be that $\vec{\nabla} \cdot \vec{V} = 0$.

For low-speed flow, density is constant.
If density is constant then the flux of mass is also constant.

mass flux: $\Gamma = \rho A_{surface} V$

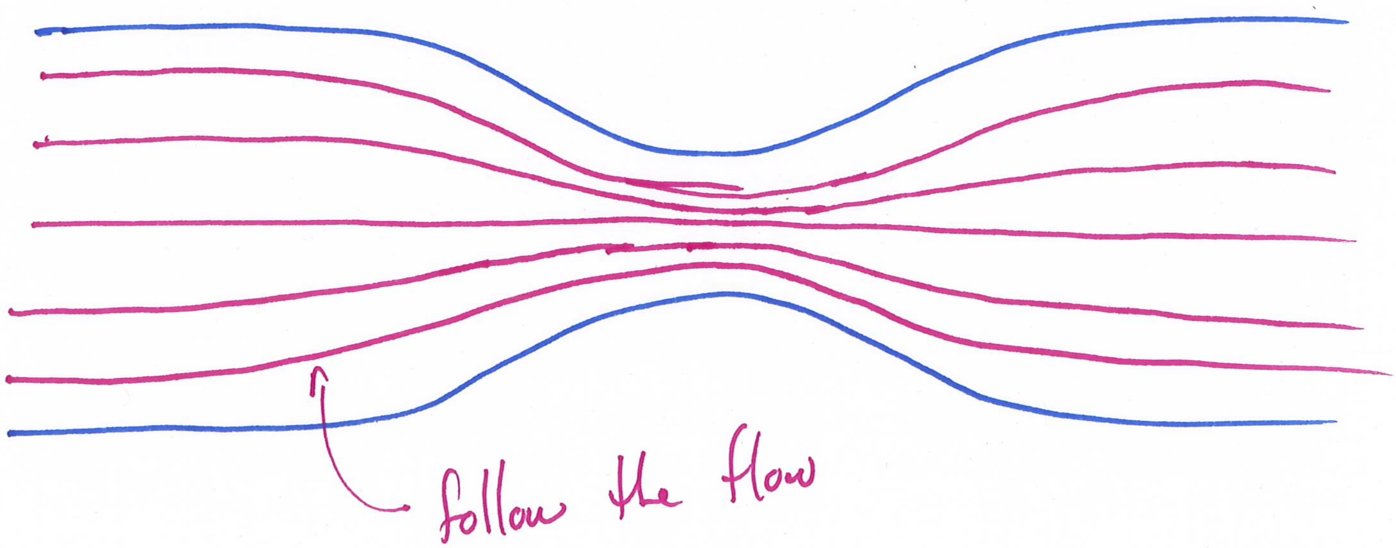
$$\Gamma_{\text{Surface \#1}} = \Gamma_{\text{Surface \#2}}$$

$$\cancel{\rho_1} A_1 V_1 = \cancel{\rho_2} A_2 \boxed{V_2} \qquad \rho_1 = \rho_2$$

$$\boxed{V_2 = \frac{A_1}{A_2} V_1}$$

$\Rightarrow$ flows speed up at points of constriction.

A qualitative derivation of Bernoulli equation.



We now examine N.S. as we follow the flow along a stream line.

N.S.

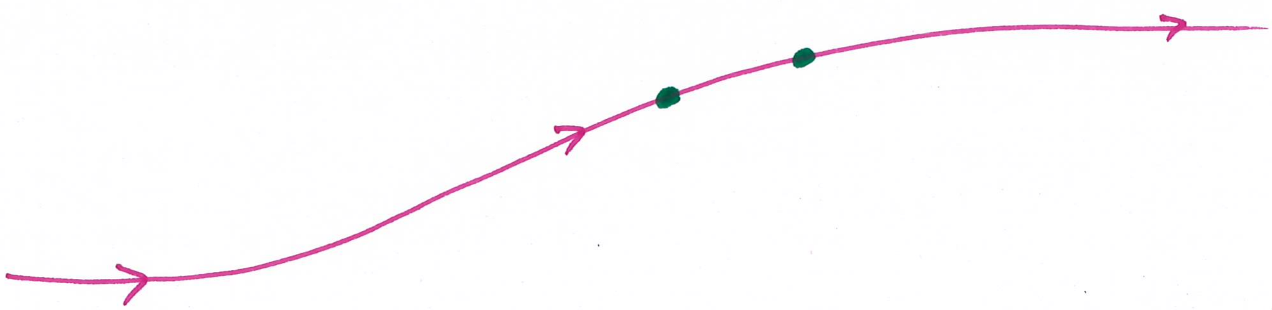
$$\rho\left[\cancel{\frac{\partial}{\partial t}} + (v\cdot\nabla)\right]\vec{v} = -\vec{\nabla}p + \cancel{\mu\nabla^2\vec{v}}$$

- look for time-independent solution "steady-state" flow

- ignore the viscous term

$$\rho\,(\vec{v}\cdot\vec{\nabla})\,\vec{v} = -\vec{\nabla}p$$

$\Rightarrow$ choose to analyze from a coordinate system aligned with the flow.



along a streamline

$$\rho\,(\vec{v}\cdot\vec{\nabla})\vec{v} \Rightarrow \vec{\nabla}\left(\tfrac{1}{2}\rho v^2\right)$$

N.S.

$$\vec{\nabla}\left(\tfrac{1}{2}\rho v^2\right) = -\vec{\nabla} p$$

$$\vec{\nabla}\left(\tfrac{1}{2}\rho v^2 + p\right) = 0$$

$$\boxed{\tfrac{1}{2}\rho v^2 + p = \text{constant}}$$

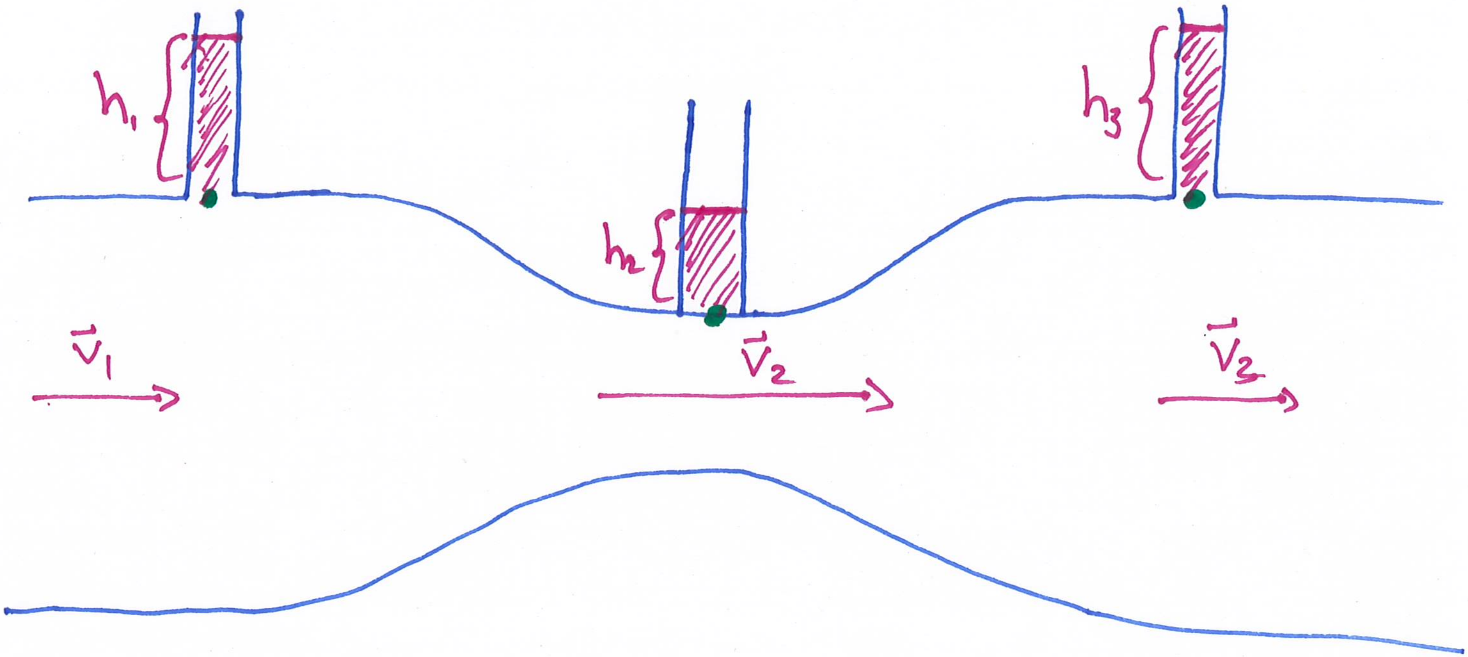
Bernoulli: Equation (simplified)

To include gravity: add $-\rho g \hat{z}$

$$-\vec{\nabla}(\rho g z)$$

$$\Rightarrow \tfrac{1}{2}\rho v^2 + p + \rho g z = \text{constant}.$$

Pressure measurements in a Venturi tube:



At each point along the tube, we have:
(at bottom of tubes at edge of pipe)

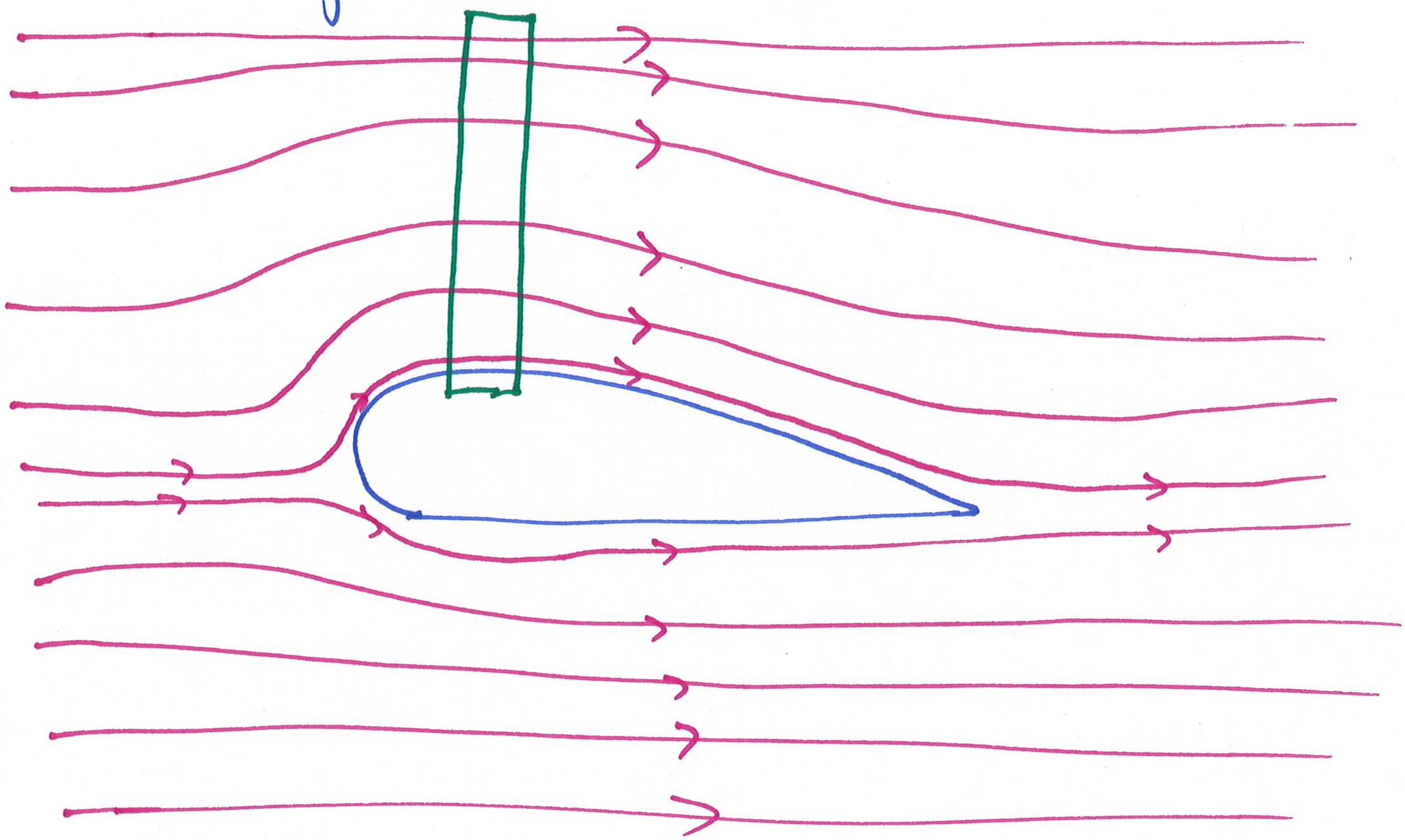
$$\frac{1}{2}\rho v_1^2 + P_1 = \frac{1}{2}\rho v_2^2 + \boxed{P_2}$$

$$P_2 = P_1 + \frac{1}{2}\rho\left(v_1^2 - v_2^2\right)$$

$$= P_1 + \frac{1}{2}\rho v_1^2\left(1 - \underbrace{\left(\frac{A_1}{A_2}\right)^2}_{>1}\right)$$

$$\boxed{P_2 = P_1 - \frac{1}{2}\rho v_1^2\left(\frac{A_1^2}{A_2^2} - 1\right)}$$

Lift generation in wings:



Note: far from the wing the flow is undisturbed.

In the region near the wing (green box) the flow looks like that in a Venturi tube.

This compression of the flow means that the flow must accelerate, and by Bernoulli's equation, this flow has less pressure $\Rightarrow$ lift.