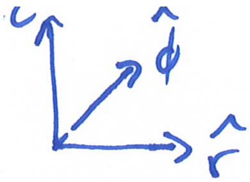
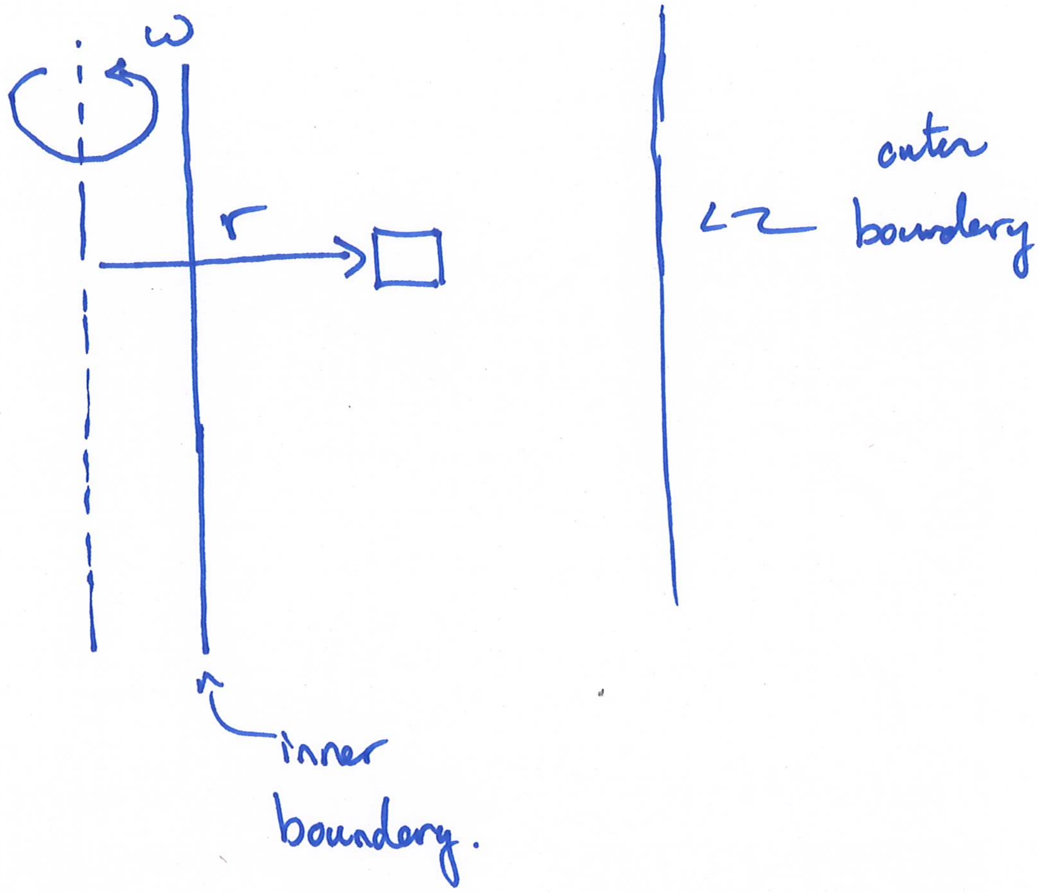
Rotating flows:





<u>case i</u> solid body rotation, fluid everywhere rotates with constant (uniform) angular velocity ω.

$\Rightarrow$ know that the velocity is

$$\vec{V} = \omega r \hat{\phi}$$

Q: What is the pressure in the fluid?

N. S.

$$\rho\left[\cancel{\frac{\partial}{\partial t}} + (\vec{v}\cdot\vec{\nabla})\right]\vec{v} = -\vec{\nabla} p + \mu \nabla^2 \vec{v}$$

$\underbrace{\frac{\partial}{\partial t}}_{0}$

for steady flow

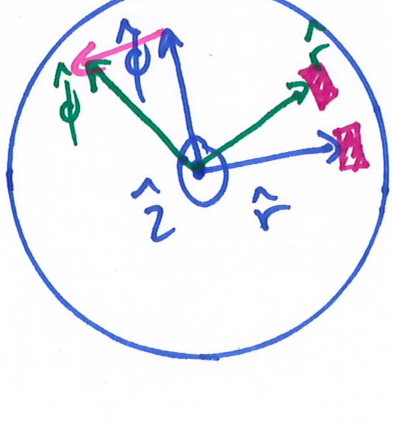
$$\vec{v}\cdot\vec{\nabla} = (\omega r \hat{\phi})\cdot\left(\frac{\partial}{\partial r}\hat{r} + \frac{1}{r}\frac{\partial}{\partial \phi}\hat{\phi} + \frac{\partial}{\partial z}\hat{z}\right)$$

$$= \omega \frac{\partial}{\partial \phi}$$

$$\rho(\vec{v}\cdot\vec{\nabla})\vec{v} = \rho\,\omega\frac{\partial}{\partial\phi}(\omega r\hat{\phi})$$

$$= \rho\,\omega^2\left(\underbrace{\cancel{\left(\frac{\partial}{\partial\phi} r\right)}}_{0}\hat{\phi} + r\underbrace{\frac{\partial}{\partial\phi}\hat{\phi}}_{-\hat{r}}\right)$$

top view

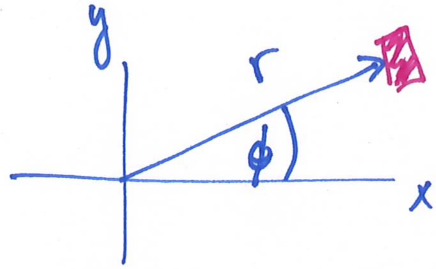


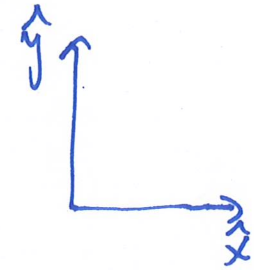
$$= \underbrace{-\rho\,\omega^2 r\,\hat{r}}_{\text{centrifugal force}}$$

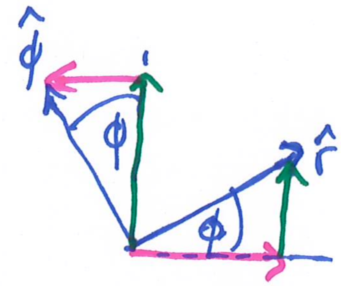
(on RHS)

centripetal acceleration

Consider in cartesian coordinates:







$$x = r\cos(\phi) \qquad y = r\sin(\phi)$$

$$\cancel{\hat{x} = \cos(\phi) \qquad \hat{y} = \sin(\phi)}$$

$$\hat{r} = \cos(\phi)\,\hat{x} + \sin(\phi)\,\hat{y}$$

$$\hat{\phi} = -\sin(\phi)\,\hat{x} + \cos(\phi)\,\hat{y}$$

Consider $\frac{\partial}{\partial\phi}\hat{\phi} = -\cos(\phi)\,\hat{x} - \sin(\phi)\,\hat{y}$

$$= -\underbrace{\left(\cos(\phi)\,\hat{x} + \sin(\phi)\,\hat{y}\right)}_{\hat{r}}$$

$$\frac{\partial}{\partial\phi}\hat{\phi} = -\hat{r}$$

$$\frac{\partial}{\partial\phi}\hat{r} = +\hat{\phi}$$

on the RHS:

$$-\vec{\nabla} p = \cancel{-\left(\frac{\partial}{\partial r}\hat{r} + \frac{1}{r}\frac{\partial}{\partial \phi}\hat{\phi} + \frac{\partial}{\partial z}\hat{z}\right)p}$$

$$-\left(\hat{r}\frac{\partial}{\partial r} + \hat{\phi}\,\frac{1}{r}\frac{\partial}{\partial \phi} + \hat{z}\frac{\partial}{\partial z}\right)p$$

$$\mu \nabla^2 \vec{v} = \mu\left(\frac{1}{r}\frac{\partial}{\partial r} r \frac{\partial}{\partial r} + \frac{1}{r^2}\frac{\partial^2}{\partial \phi^2} + \cancel{\frac{\partial^2}{\partial z^2}}\right)(\omega r \hat{\phi})$$

$$\mu\omega \frac{1}{r}\frac{\partial}{\partial r} r \frac{\partial}{\partial r}(r\hat{\phi}) = \mu\omega\hat{\phi}\underbrace{\frac{1}{r}\frac{\partial}{\partial r} r \frac{\partial}{\partial r} r}_{\frac{1}{r}}$$

$$= \frac{\mu\omega}{r}\hat{\phi}$$

$$\mu\omega \frac{1}{r^2}\frac{\partial^2}{\partial \phi^2} r\hat{\phi} = \frac{\mu\omega}{r^2}\left(r \underbrace{\underbrace{\frac{\partial^2}{\partial \phi^2}\hat{\phi}}_{\frac{\partial}{\partial \phi}(-\hat{r})}}_{-\hat{\phi}} + \underbrace{\cancel{\hat{\phi}\frac{\partial^2}{\partial \phi^2} r}}_{0}\right)$$

$$= -\frac{\mu\omega}{r}\hat{\phi}$$

$$\mu \nabla^2 \vec{V} = \frac{\mu \omega}{r} \hat{\phi} + \left(-\frac{\mu \omega}{r} \hat{\phi}\right)$$

$$= 0$$

$$\boxed{-\rho \omega^2 r \hat{r}} = -\left(\boxed{\hat{r} \frac{\partial}{\partial r}} + \hat{\phi} \frac{1}{r} \frac{\partial}{\partial \phi} + \hat{z} \frac{\partial}{\partial z}\right) p$$

① $-\rho \omega^2 r = -\frac{\partial}{\partial r} p$

② $0 = \frac{1}{r} \frac{\partial}{\partial \phi} p$

③ $0 = \frac{\partial}{\partial z} p$

no variation in pressure in ϕ and z

$\Rightarrow p = p(r)$

Solve for p:

$$\frac{\partial}{\partial r} p = \rho \omega^2 r$$

$$\int_{p_i}^{p(r)} dp = \int_{r_i}^{r} \rho \omega^2 r \, dr$$

$$\boxed{p(r) = p_i + \frac{1}{2} \rho \omega^2 (r^2 - r_i^2)}$$