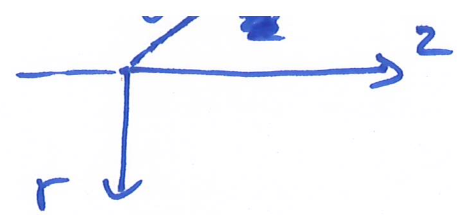
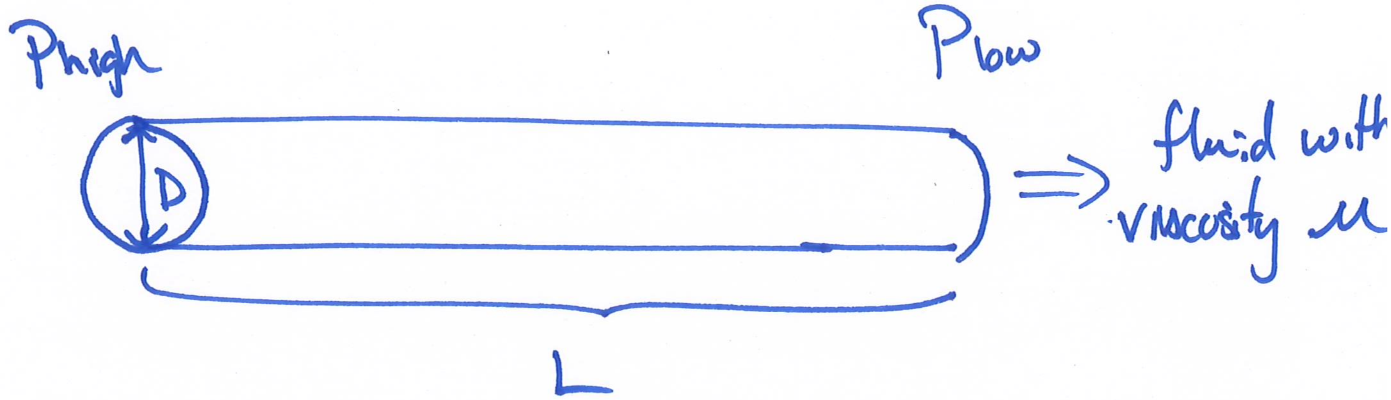
Pipe Flow:

center of pipe:





Q 1: What total flow do we get out of this system?

Q2: What is the <u>shape</u> of the flow inside the pipe?

N.S.

$$\rho \left[\frac{\partial}{\partial t} + (\vec{V} \cdot \vec{\nabla}) \right] \vec{V} = - \nabla \vec{p} + \mu \vec{\nabla} \vec{V}$$

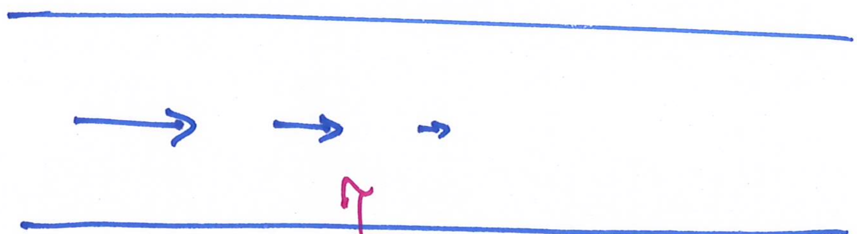
Laminar flow solution: $\frac{\partial}{\partial t} = 0$

$$\vec{v} = v(\hat{z})$$

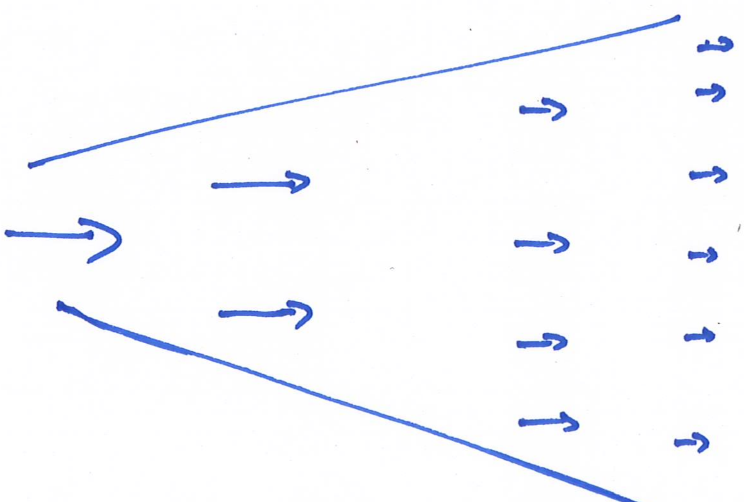
$$(\vec{v}\cdot\vec{\nabla})\vec{v} = \cancel{v_x \frac{\partial}{\partial x} + v_y \frac{\partial}{\partial x}}$$

$$= \left(\cancel{v_r}^{\,0} \frac{\partial}{\partial r} + \cancel{v_\theta}^{\,0} \frac{1}{r}\frac{\partial}{\partial \theta} + v_z \frac{\partial}{\partial z}\right)\vec{v}$$

$$= v_z \boxed{\frac{\partial}{\partial z} v_z}\,\hat{z} = 0$$



not allowed



if pipe is uniform cross section

$$\Rightarrow \frac{\partial}{\partial z} = 0$$

N.S.

$$0 = -\vec{\nabla} p + \mu \nabla^2 \vec{V}$$

$$0 = -\left(\frac{\partial}{\partial r} p \, \hat{r} + \cancelto{0}{\frac{1}{r}\frac{\partial}{\partial \theta} p} \, \hat{\theta} + \frac{\partial}{\partial z} p \, \hat{z}\right)$$

$$+\mu\left(\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial}{\partial r}V_z\right) + \cancelto{0}{\frac{1}{r^2}\frac{\partial^2}{\partial \theta^2}V_z} + \cancelto{0}{\frac{\partial^2}{\partial z^2}V_z}\right)\hat{z}$$

$\cancel{\hat{r}: \quad 0 = -\frac{\partial}{\partial r} p + \mu \frac{1}{r}\frac{\partial}{\partial r}\left(r \frac{\partial}{\partial r} V_z\right)}$

$\cancel{\hat{z}: \quad 0 = -\frac{\partial}{\partial z} p + \mu}$

$$\hat{r}: \quad 0 = -\frac{\partial}{\partial r} p \quad \Rightarrow \quad p = p(z)$$

$$\hat{z}: \quad 0 = -\frac{\partial}{\partial z} p + \mu\left(\frac{1}{r}\frac{\partial}{\partial r}\left(r \frac{\partial}{\partial r} V_z\right)\right)$$

looking for a solution for $v_z(r)$

- know $p = p(z)$

decreases linearly between entrance and exit.

$$\frac{\partial}{\partial z} p = \text{constant.}$$

$$\mu \frac{1}{r} \frac{\partial}{\partial r}\left(r \frac{\partial}{\partial r} v_z \right) = \underset{\left(\frac{\partial p}{\partial z}\right)}{\text{constant}}$$

let $v_z = a r^2 + b r + c$

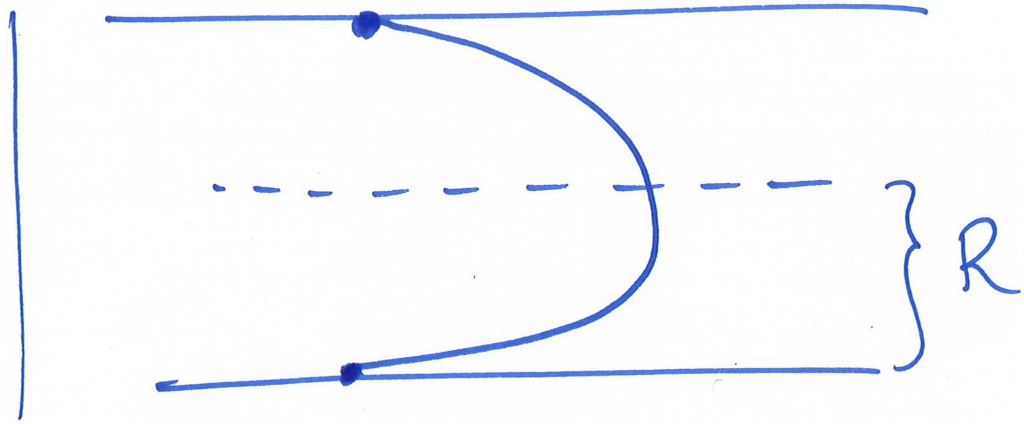
$$r \frac{\partial v_z}{\partial r} = (2ar + b) r = 2ar^2 + br$$

$$\mu \frac{1}{r} \frac{\partial}{\partial r}\left(r \frac{\partial v_z}{\partial r} \right) = \left(\frac{4ar + b}{r} \right) \mu$$

$$= \left(4a + \cancel{\frac{b}{r}}^{\,0} \right) \mu = \text{constant.}$$

$$a = \frac{1}{4\mu} \frac{\partial p}{\partial z}$$

$$\boxed{v(r) = \left(\frac{1}{4\mu}\frac{\partial p}{\partial z}\right) r^2 + c}$$



require $v_z(r_{edge}) = 0$

$$v(R) = \frac{1}{4\mu}\frac{\partial p}{\partial z} R^2 + c = 0$$

$$c = \frac{1}{4\mu}\underbrace{\left(-\frac{\partial p}{\partial z}\right)}_{>0} R^2$$

$$\boxed{v(r) = \frac{1}{4\mu}\left(-\frac{\partial p}{\partial z}\right)\left(R^2 - r^2\right)}$$