Physics 201 Review problems

1. Two objects dropped from a bridge at an interval of 1 s. What happens to the difference in their speeds?

$$v_1(t) = \cancelto{0}{v_{i1}} - gt$$

$$= -gt$$

$$v_2(t) = v_{i2} - gt$$

need $v_2(1) = 0 \quad \Rightarrow \quad 0 = v_{i2} - (10\,m/s^2)(1\,sec)$

$$v_{i2} = 10\,m/s$$

$$v_2(t) = 10\,m/s - gt$$

$$v_1 - v_2 = -gt - (10\,m/s - gt) = -10\,m/s$$

What about position?

$$y_1(t) = -\tfrac{1}{2}gt^2$$

$$y_2(t) = y_{i2} + 10\,m/s \cdot t - \tfrac{1}{2}gt^2$$

need $y_2(1\,sec) = 0 \quad \Rightarrow \quad 0 = y_{i2} + \underbrace{(10\,m/s)(1\,sec)}_{10\,m} - \underbrace{\tfrac{1}{2}(10\,m/s^2)(1\,sec)^2}_{5\,m}$

$$y_{i2} = -5\,m$$

$$y_1 - y_2 = (-\tfrac{1}{2}gt^2) - (-5\,m + 10\,m/s\,t - \tfrac{1}{2}gt^2)$$

$$\boxed{\Delta y = 5\,m - (10\,m/s)\,t}$$

2. Average speed and average velocity between $t=2$ & $t=4$ sec.

knowns: $x(0) = -3$

$x(3) = 3$

$v(3) = 0$

$v(t) = v_0 + at$ $\qquad$ $0 = v_0 + a(3\,\text{sec})$ $\qquad$ $v_0 = -a(3\,\text{sec})$

$x(t) = x_0 + v_0 t + \frac{1}{2}at^2$ $\qquad$ $-3 = x_0 + (-a \cdot 3\,\text{sec})(0) + \frac{1}{2}a(0)^2$

$x_0 = -3\,m$

$v(t) = 4\,m/s - \left(\frac{4}{3}\,m/s^2\right)t$ $\qquad$ $+3m = (-3m) + (-a \cdot 3\,\text{sec})(3\,\text{sec}) + \frac{1}{2}a(3\,\text{sec})^2$

$v(2) = \frac{4}{3}\,m/s$ $\qquad$ $6m = -\frac{1}{2}a(9\,\text{sec}^2) = -\frac{9}{2}\,\text{sec}^2\,a$

$v_0 = 4\,m/s$

$a = -\frac{4}{3}\,\frac{m}{s^2}$

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average velocity = 0

average speed $= \frac{1}{4s-2s}\int_2^4 |v|\,dt = \frac{1}{2s}\int_2^3 (v_0+at)\,dt - \frac{1}{2s}\int_3^4 (v_0+at)\,dt$

$= \frac{1}{2}\left(v_0 t + \frac{1}{2}at^2\right)\Big|_2^3 - \frac{1}{2}\left(v_0 t + \frac{1}{2}at^2\right)\Big|_3^4$

$$\text{avg. speed} = \frac{1}{2s}\left[\left(\left(4\,\tfrac{m}{s}\right)(3\,\text{sec}) + \tfrac{1}{2}\left(-\tfrac{4}{3}\,\tfrac{m}{s^2}\right)(3\,s)^2\right) - \left(\left(4\,\tfrac{m}{s}\right)(2\,\text{sec}) + \tfrac{1}{2}\left(-\tfrac{4}{3}\,\tfrac{m}{s^2}\right)(2\,s)^2\right)\right] \times 2$$

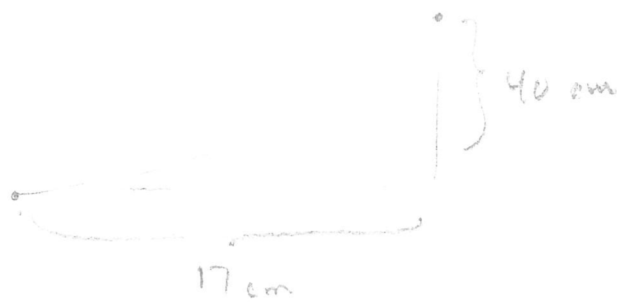
$$= \frac{1}{1s}\left[\underbrace{\left(12\,m - 6\,m\right)}_{6\,m} - \underbrace{\left(8\,m - \tfrac{8}{3}\,m\right)}_{\frac{16}{3}}\right]$$

$$= \frac{1}{s}\left[6\,m - \frac{16}{3}\,m\right]$$

$$= \frac{1}{s}\left[\frac{2}{3}\,m\right]$$

$$\text{avg. speed} = \frac{2}{3}\,\frac{m}{s}$$

3. An electron with $v_x = 3 \cdot 10^6$ m/s



Know.

travel 17 cm horizontally before striking screen

travel 40 cm vertically in the same time

$v_{yi} = 0$

$$y(t) = \frac{1}{2} a_y t^2 \quad \rightarrow \quad 0.40\,m = \frac{1}{2} a_y t_f^2$$

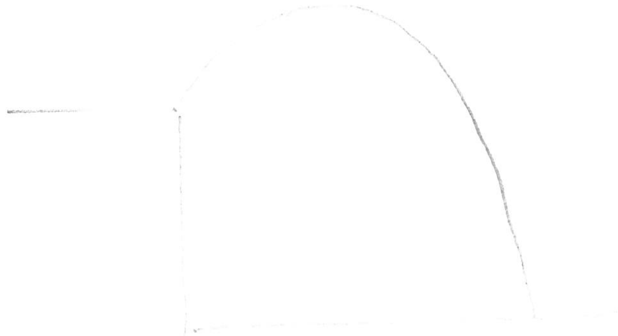
$$x(t) = v_{xi}\, t \qquad 0.17\,m = v_{xi}\, t_f \rightarrow t_f = \frac{0.17\,m}{v_{xi}} = \frac{0.17\,m}{3 \cdot 10^6} = 5.6 \cdot 10^{-8}\,s$$

$$a_y = \frac{2 \cdot 0.40}{t_f^2} = \frac{2 \cdot 0.40\,m}{(5.6 \cdot 10^{-8})^2}$$

$$a_y = 2.5 \cdot 10^{14}\ m/s$$

4 $v_{0x} = 60\ m/s$

$v_{0y} = 175\ m/s$



magnitude of velocity after 21 seconds?

- assume it doesn't hit the ground.

$v_x(t) = v_{0x} = 40\ m/s$

$v_y(t) = v_{0y} - gt$

$v_y(21\text{ sec}) = 175 - (10\ m/s^2)(21\ s)$

$= -35\ m/s$

$|v| = \sqrt{v_x^2 + v_y^2}$

$= \sqrt{(60\ m/s)^2 + (-35\ m/s)^2}$

$= 69.4\ m/s$

5. a) what time does the x-component of the net force on the object reach its maximum magnitude?

$m a_x = F_{Net, x}$

max magnitude at $t = 3.0$ ms $\quad a_x = -20\ m/s^2$

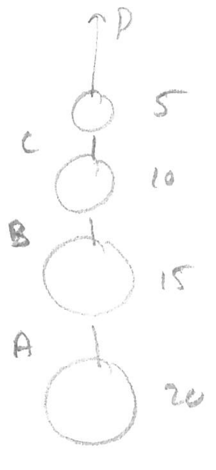
$|F_{net}| = (2.4)(20\ m/s^2) = 48\ N$

b) x component of F_{NET} at $t = 0$ ms & $t = 4$ ms?

$t = 0$ ms $\quad a_x = +5\ m/s^2 \qquad F_{net} = (2.4)(5) = 12\ N$

$t = 4$ ms $\quad a_x = -10\ m/s^2 \qquad F_{NET} = (2.4)(-10) = -24\ N.$

6.



$a_y = +4\ m/s^2$

$m_{tot}\, a_y = -m_{tot} g + P$

$P = m_{tot}(a + g)$

$= (50\text{ kg})(4 + 10)\ m/s^2$

$= 700\ N$

7.

Force between blocks?

$$m_1 a = F_A - F_{21}$$

$$m_2 a = F_{12} \qquad |F_{12}| = |F_{21}|$$

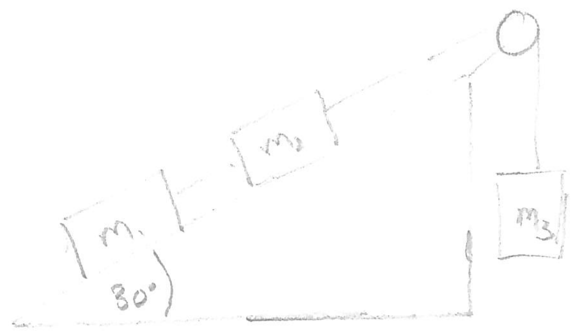
$$m_1 a = F_A - (m_2 a)$$

$$(m_1 + m_2) a = F_A$$

$$a = \frac{F_A}{m_{tot}} = \frac{20\,N}{10\,kg} = 2\,m/s^2$$

$$F_{12} = m_2 a = (4\,kg)(2\,m/s^2) = 8\,N$$

8.



$$(m_1+m_2)\,a = F_{3\text{-}12} - (m_1+m_2)\,g\sin(\theta)$$

$$m_3\,a = m_3 g - F_{12\text{-}3}$$

$$F_{12\text{-}3} = m_3(g-a)$$

$$(m_1+m_2)\,a = m_3(g-a) - (m_1+m_2)\,g\sin\theta$$

$$(m_1+m_2+m_3)\,a = \big(m_3 - (m_1+m_2)\sin\theta\big)\,g$$

$$a = \frac{m_3 - (m_1+m_2)\sin\theta}{m_1+m_2+m_3}\,g$$

$$= \frac{9 - (6+4)\sin(30^\circ)}{6+4+9}\,g = \frac{4}{19}\,g$$

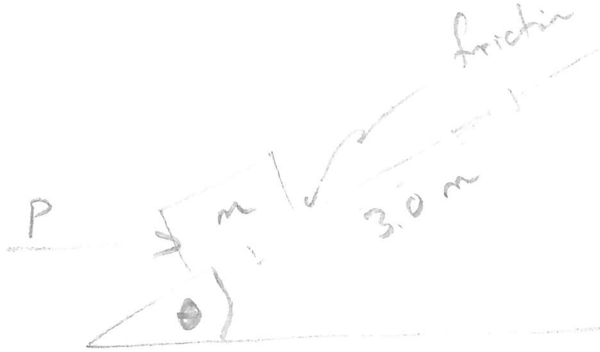
$$m_1 a = F_{21} - m_1 g\sin\theta$$

$$F_{21} = m_1(a + g\sin\theta)$$

$$= m_1\left(\frac{4}{19} + \sin\theta\right)g = 6\text{ kg}\left(\frac{4}{19} + \frac{1}{2}\right)10\text{ m/s}^2$$

$$\approx 42\text{ N}$$

9.



$m = 700$ kg

$P = 5600$ N

$v_i = 1.4$ m/s

$v_f = 2.5$ m/s

Work done by gravity?

$$W_g = \int F \cdot dx = -mg\cos\alpha \,\Delta x$$

$$= -(700)(10)(\tfrac{1}{2})(3\,m)$$

$$= -10\,500\ J$$

Work done by friction?

$$\Delta T = \sum W$$

$$= W_p + W_f + W_g$$

$$W_f = \Delta T - W_p - W_g$$

$$\Delta T = \tfrac{1}{2} m (v_f^2 - v_i^2) = \tfrac{1}{2}(700)(2.5^2 - 1.4^2)$$

$$= 1501.5\ J$$

$$W_p = P \cdot d\cos\theta$$

$$= (5600)(3)(0.87) = 14549.2$$

$$W_f = (1501.5\ J) - (14549.2\ J) - (-10500\ J)$$

$$= -2547.7\ J$$

What is μ for this case?

-> assume constant acceleration

$$W_f = -F_f\, d$$

$$F_f = \mu_k N = \mu_k mg \cos\theta$$

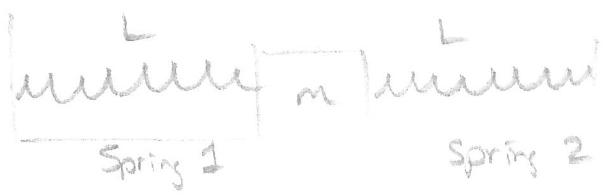
$$W_f = -\mu_k mgd\cos\theta$$

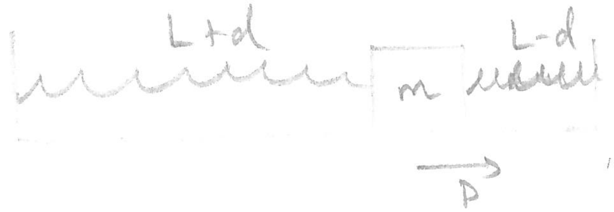
$$\mu_k = \frac{-W_f}{mgd\cos\theta}$$

$$= \frac{-(-2547.7J)}{(700)(10)(3)(0.86)}$$

$$\mu_k = 0.14$$

10.





What value of P is needed to hold the mass in place?

$k = 700 \text{ N/m}$ $\qquad L_0 = 0.25 \text{ m}$

$d = 0.02 \text{ m}$

$L = 0.30 \text{ m}$

$$\vec{F} = \vec{F}_1 + \vec{F}_2$$

case i) $F = -k(L - L_0) + k(L - L_0)$

$= 0$

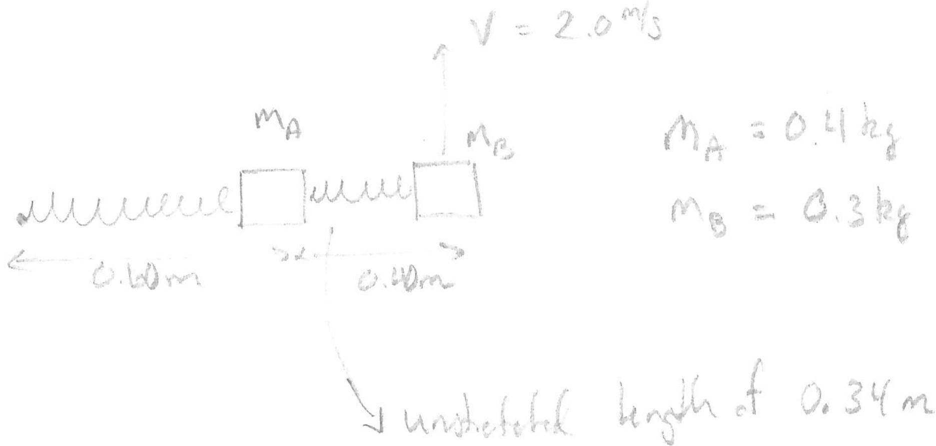
case ii) $F = -k(L + d - L_0) + k(L - d - L_0)$

$= -\left[\cancel{k(L - L_0)} + kd\right] + \left[\cancel{k(L - L_0)} - kd\right]$

$= -2kd$

$= -2(700 \text{ N/m})(0.02 \text{ m}) = 28 \text{ N}$

11.



$m_A = 0.4\ kg$

$m_B = 0.3\ kg$

What is the spring constant of the second spring?

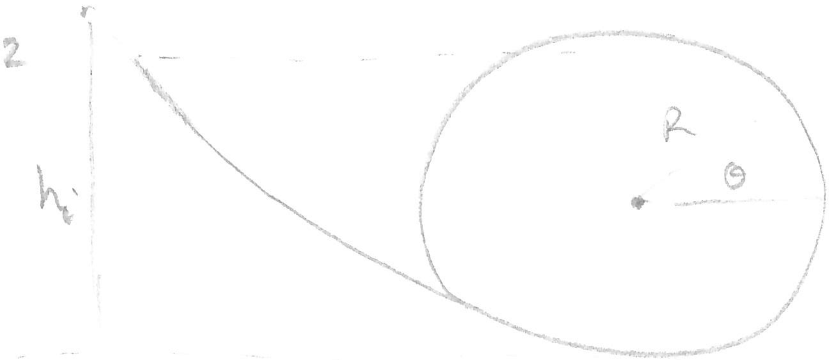
$$m_2 a_2 = -F_s = -k\,\Delta x$$

$$a_2 = -\frac{v^2}{R}$$

$$k = \frac{mv^2}{R\,\Delta x} = \frac{(0.3\ kg)(2\ m/s)^2}{(1.0\ m)(0.06\ m)}$$

$$= 20\ \frac{N}{m}$$

12



What is the max height reached in the loop as a function of the release height?

- assume no friction

→ moving on the circular track, require a centripetal acceleration.

$a_c = m\frac{v^2}{R}$ (toward the center)

→ two forces act toward the center : normal & gravity.

$\angle\theta)$

$$m a_c = N + mg\sin\theta$$

→ when contact ceases, $N \to 0$

critical condition is defined by $N = 0$.

$$\cancel{m} a_c = \cancel{m} g\sin\theta$$

$$\frac{v^2}{R} = g\sin\theta$$

What is v?

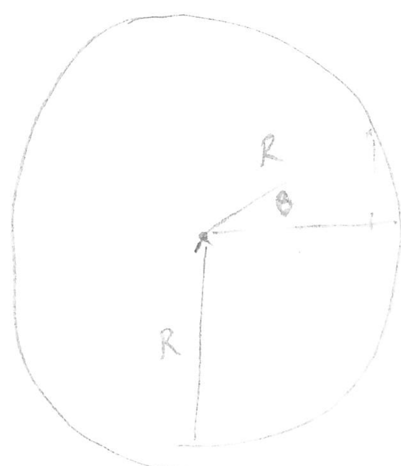
v depends on the angle and release height

without friction, we have conservation of energy.

$$E_i = E_f$$

$$mgh_i = \frac{1}{2}mv^2 + mgh_f$$

what is h_f?



$$h_f = R + R\sin\theta \longrightarrow R\sin\theta = R - h_f$$

$$\cancel{m}gh = \frac{1}{2}\cancel{m}v^2 + \cancel{m}g\,h_f$$

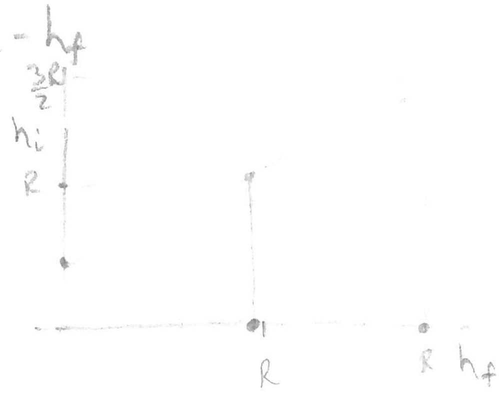
$$v^2 = 2g(h_i - h_f)$$

$$\frac{v^2}{R} = \frac{2g(h_i - h_f)}{R} = g\sin\theta$$

$$2(h_i - h_f) = R\sin\theta = R - h_f$$

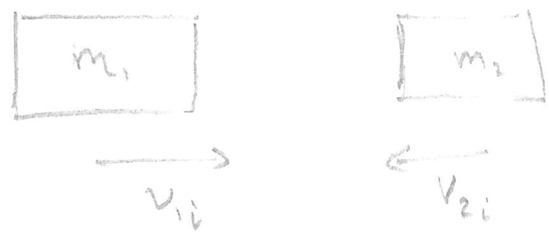
$$2h_i - 2h_f = R - h_f$$

$$h_i = \frac{1}{2}(R + h_f)$$



13.

before collision



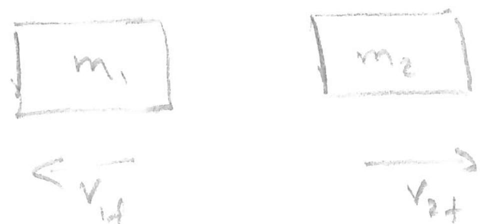
$m_1 = 4\,kg$

$m_2 = 6\,kg$

$v_{1i} = 1.8\ m/s$

$v_{2i} = -0.2\ m/s$

after collision



$v_{1f} = -0.6\ m/s$

$v_{2f} = 1.4\ m/s$

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for elastic collisions

$$v_{1f} = 2v_{cm} - v_{1i}$$

$$v_{2f} = 2v_{cm} - v_{2i}$$

$$v_{cm} = \frac{m_1 v_1 + m_2 v_2}{m_1 + m_2}$$ <— doesn't matter if

$$= \frac{(4\,kg)(1.8\ m/s) + (6\,kg)(-0.2\ m/s)}{4\,kg + 6\,kg} = +0.6\ m/s.$$

$$v_{1f} = 2(0.6\ m/s) - 1.8\ m/s = -0.6\ m/s$$

$$v_{2f} = 2(0.6\ m/s) - (-0.2\ m/s) = 1.4\ m/s$$

Or, compare kinetic energies:

$$E_i = \frac{1}{2} m_1 v_{1_i}^2 + \frac{1}{2} m_2 v_{2_i}^2$$

$$= \frac{1}{2}(4\ kg)(1.8\ m/s)^2 + \frac{1}{2}(6\ kg)(-0.2\ m/s)^2$$

$$= 6.48\ J + 0.12\ J$$

$$= 6.6\ J$$

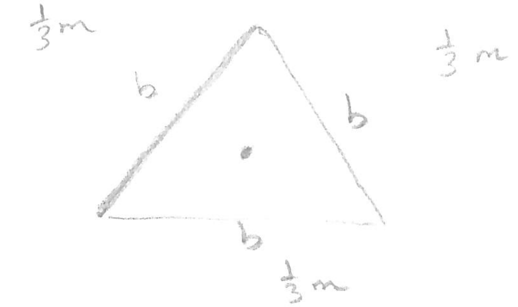
$$E_f = \frac{1}{2} m_1 v_{1_f}^2 + \frac{1}{2} m_2 v_{2_f}^2$$

$$= \frac{1}{2}(4\ kg)(-0.6\ m/s)^2 + \frac{1}{2}(6\ kg)(+1.4\ m/s)^2$$

$$= 0.72\ J + 5.88\ J$$

$$= 6.6\ J$$

14.



moment of inertia

$$I_{1/3} = \int dm\, r^2$$

$$= \int_{-\frac{1}{2}b}^{+\frac{1}{2}b} (\rho\, dx)\,(r_0^2 + x^2)$$

What is r_0?

$$\tan(30°) = \frac{r_0}{\frac{1}{2}b}$$

$$r_0 = \frac{1}{2}b\tan(30°)$$

what is ρ? $\rho = \frac{\frac{1}{3}m}{b} = \frac{m}{3b}$

$$I_{1/3} = \frac{m}{3b}\int_{-\frac{1}{2}b}^{+\frac{1}{2}b} dx\,(r_0^2 + x^2)$$

let $x = bu$
$dx = b\,du$

$$= \frac{m}{3b}\int_{-\frac{1}{2}}^{+\frac{1}{2}} b\,du\; b^2\left(\frac{r_0^2}{b^2} + u^2\right) = \frac{1}{3}mb^2\int_{-\frac{1}{2}}^{+\frac{1}{2}} (a^2+u^2)\,du$$

$$I_{1/3} = \frac{1}{3} m b^2 \left(a^2 u + \frac{1}{3} u^3 \right) \Big|_{-1/2}^{+1/2}$$

$$= \frac{1}{3} m b^2 \left(\underbrace{\underbrace{\left(\frac{\tan(30^\circ)}{2} \right)^2}_{\left(\frac{\frac{1}{3}}{2} \right)^2} \underbrace{\left(\frac{1}{2} - -\frac{1}{2} \right)}_{1}}_{\frac{1}{12}} + \underbrace{\frac{1}{3} \underbrace{\left(\left(\frac{1}{2}\right)^3 - \left(-\frac{1}{2}\right)^3 \right)}_{\frac{1}{4}}}_{\frac{1}{12}} \right)$$

$$= \frac{1}{3} m b^2 \left(\frac{1}{6} \right)$$

$$I_{tri} = 3 \, I_{1/3}$$

$$= \frac{1}{6} m b^2$$

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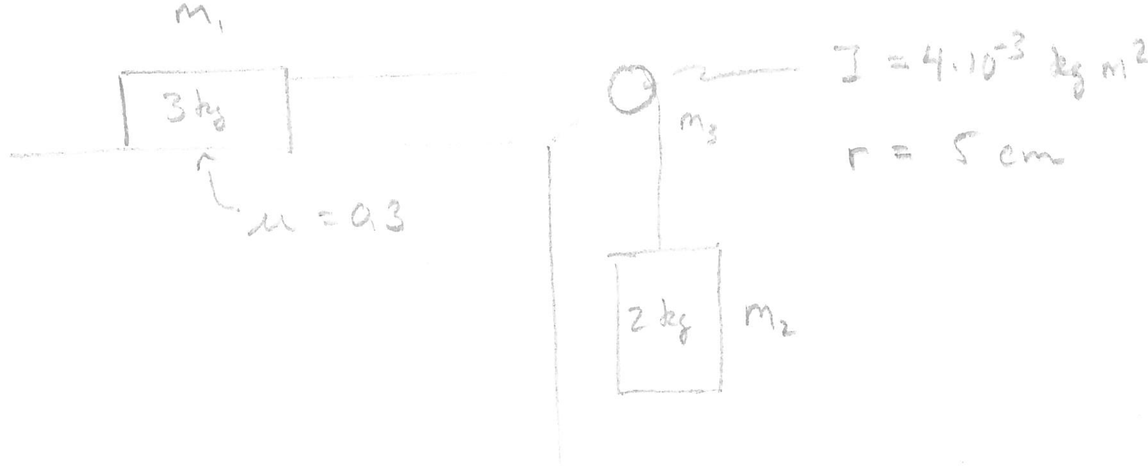


Use energy analysis to find the speed of the upper block for $\Delta x = 0.6$ m.

$$V_1 = V_2 = V_3 \text{ (edge)}$$

$$\Delta T = \sum W$$

$$= W_f + W_g$$

$$W_f = \vec{F}_f \cdot \Delta \vec{x} = -F_f \, \Delta x = -\mu N_1 \, \Delta x$$

$$= -(0.3)(30\,N)(0.6\,m) = -5.4\,J$$

$$W_g = \vec{F}_g \cdot \Delta \vec{x} = F_g \, \Delta x = m_2 g \, \Delta x$$

$$= (2\,kg)(10\,m/s^2)(0.6\,m)$$

$$= +12\,J$$

Starting at rest: $\Delta T = T_f = \frac{1}{2} m_1 V_1^2 + \frac{1}{2} m_2 V_2^2 + \frac{1}{2} I_3 \omega^2$

$$v = \omega r \qquad \omega = \frac{V}{R}$$

$$T = \frac{1}{2}\left(m_1 + m_2 + \frac{I_o}{r^2}\right) v^2$$

$$v^2 = \frac{2\,(W_g + W_f)}{m_1 + m_2 + \frac{I_o}{r^2}}$$

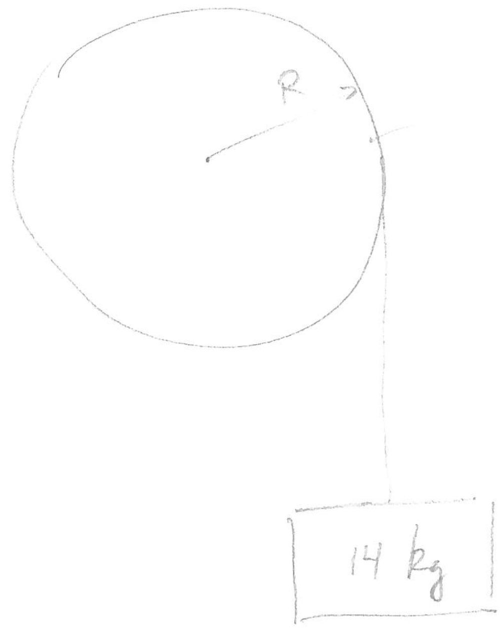
$$= \frac{2\,(12\,J - 5.4\,J)}{3\,kg + 2\,kg + \frac{4 \cdot 10^{-3}}{(5 \cdot 10^{-2})^2}} = \frac{2 \cdot 6.6\,J}{(5 + 1.6)\,kg}$$

$$= \frac{2 \cdot 6.6\,J}{6.6\,kg}$$

$$= 2\,(m/s)^2$$

$$v \simeq 1.4\ m/s$$

16.



$I = \frac{1}{2}MR^2$

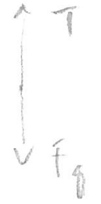
what is a?

$\Delta y = \frac{1}{2} a \Delta t^2$

$a = 2\frac{\Delta y}{\Delta t^2} = \frac{2(10\,m)}{(2\,sec)^2}$

$= 5\ m/s^2$

When released from rest, it is observed that the block descends a distance of $\Delta y = 10$ m in $\Delta t = 2$ sec. What is the moment of inertia of the wheel?



$ma = mg - T$

$I\alpha = RT \qquad T = \frac{I\alpha}{R}$

$\alpha = \frac{a}{R} \qquad T = \frac{Ia}{R^2}$

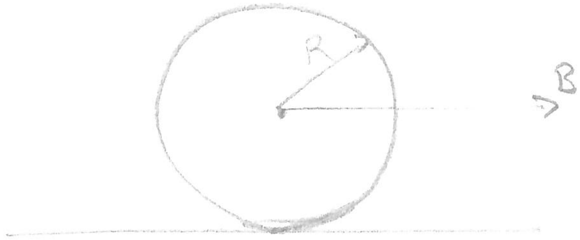
$ma = mg - \frac{I}{R^2}a$

$I = \frac{m(g-a)R^2}{a} = \left(\frac{g}{a} - 1\right) mR^2$

$= \left(\frac{10}{5} - 1\right)(14\,kg)(2.0\,m)^2 = 56\ kgm^2$

$m = \frac{2I}{R^2}$

17.



$R = 0.65\ m$

$m = 51\ kg$

$a = 2.8\ m/s^2$

what B is needed?

method 1: about the center

$ma = B - F_f$

$I\alpha = R F_f \rightarrow F_f = \frac{I\alpha}{R} = \frac{I a}{R^2}$

$B = ma + \frac{I}{R^2} a = \left(m + \frac{I}{R^2}\right) a$

$B = \left(m + \frac{1}{2}m\right) a$

$= \frac{3}{2} ma = \frac{3}{2}(50\ kg)(2.8\ m/s^2)$

$= 210\ N$

method 2 about the edge

$I\alpha = RB$

$B = \frac{I\alpha}{R} = \frac{I}{R^2} a$

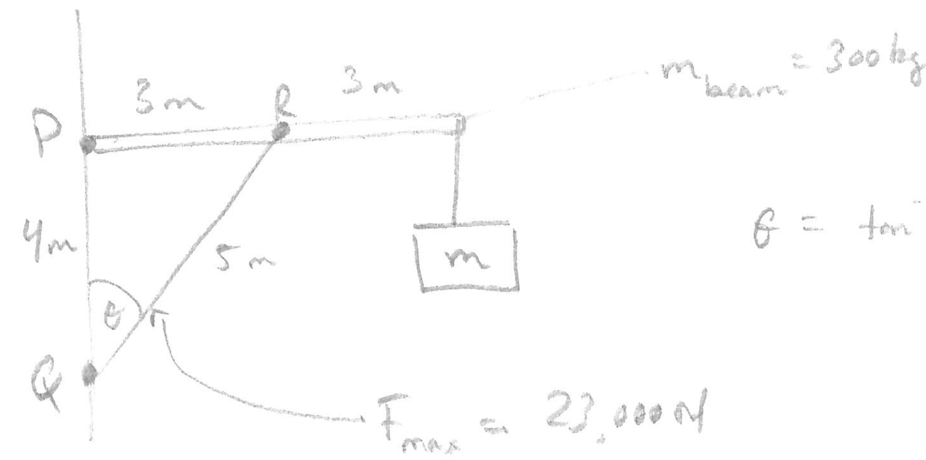
$I = \frac{1}{2}MR^2 + mR^2$ (parallel axis theorem)

$B = \frac{3}{2}\frac{MR^2}{R^2} a$

$B = \frac{3}{2} Ma$

$l = 6\,m$

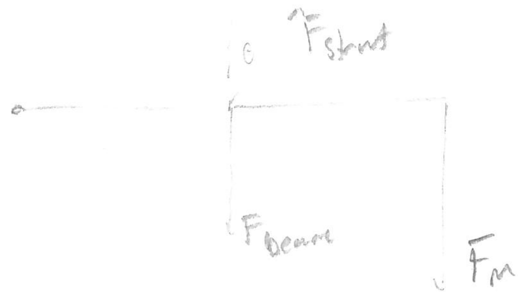
18.



$\theta = \tan^{-1}\left(\frac{3}{4}\right) = 37°$

at maximum load, find horizontal component of force at P.

Consider rotation about P.



$$0 = \sum \tau = F_{strut}\left(\frac{l}{2}\right)\underbrace{\cos\theta}_{\frac{4}{5}} - \underbrace{F_{beam}}_{m_{beam}g}\frac{l}{2} - \underbrace{F_m}_{Mg} l$$

$$M = \frac{1}{gl}\left(\frac{1}{2}F_{strut}\, l\left(\frac{4}{5}\right) - \frac{1}{2}m_{beam}\, g l\right)$$

$$= \frac{2}{5}\frac{F_{strut}}{g} - \frac{1}{2}m_{beam}$$

$$= \frac{2}{5}\frac{(23{,}000)}{10} - \frac{1}{2}(300) = 770\,kg.$$

$$F_{horizontal} = F_{strut} \sin(\theta)$$

$$= (23{,}000)\left(\frac{3}{5}\right)$$

$$= 13\,800 \text{ N}$$