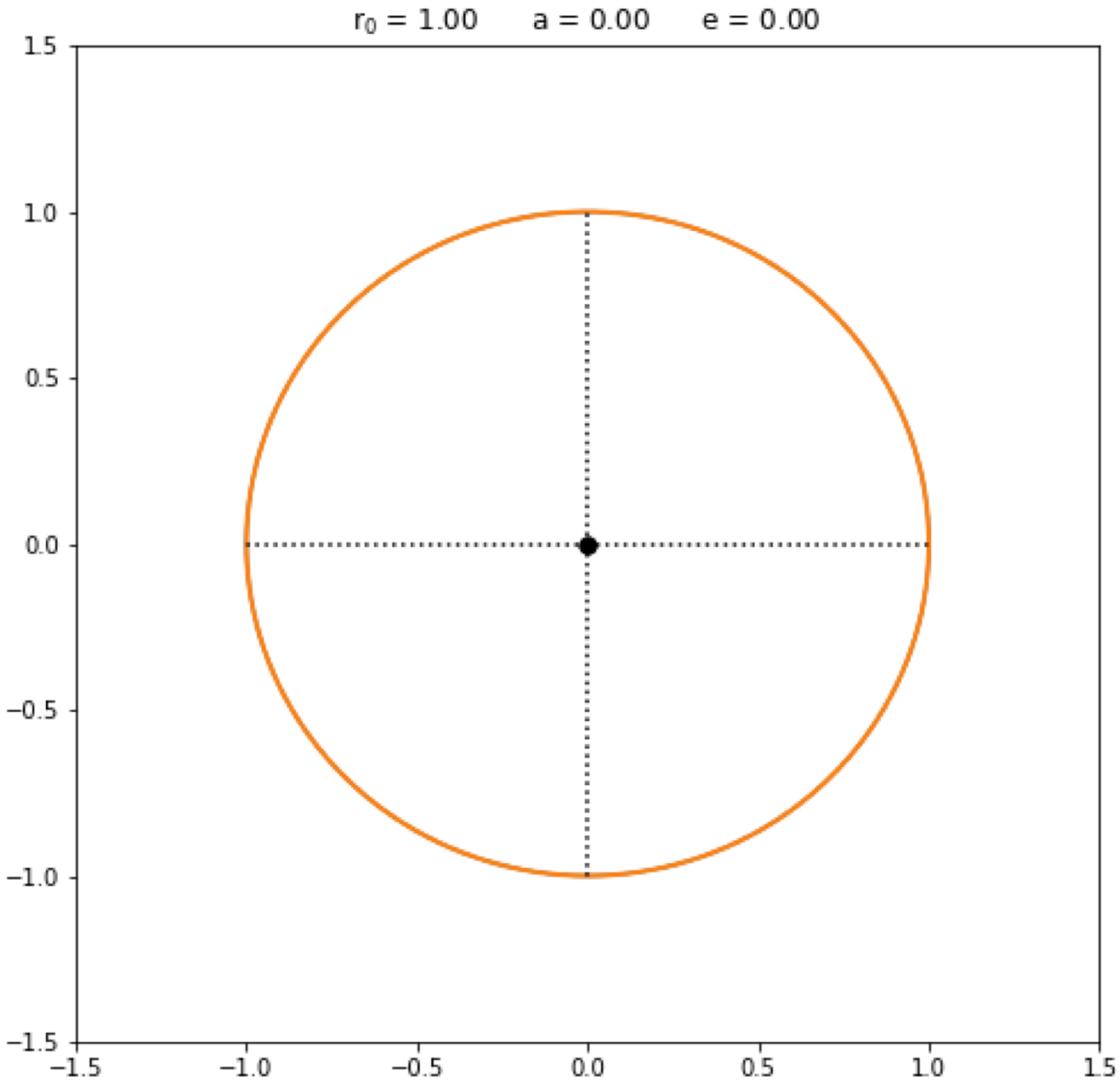
Orbit perturbation analysis

Physics 420

Unperturbed circular orbit

This is the circular orbit to which we will apply a perturbation.

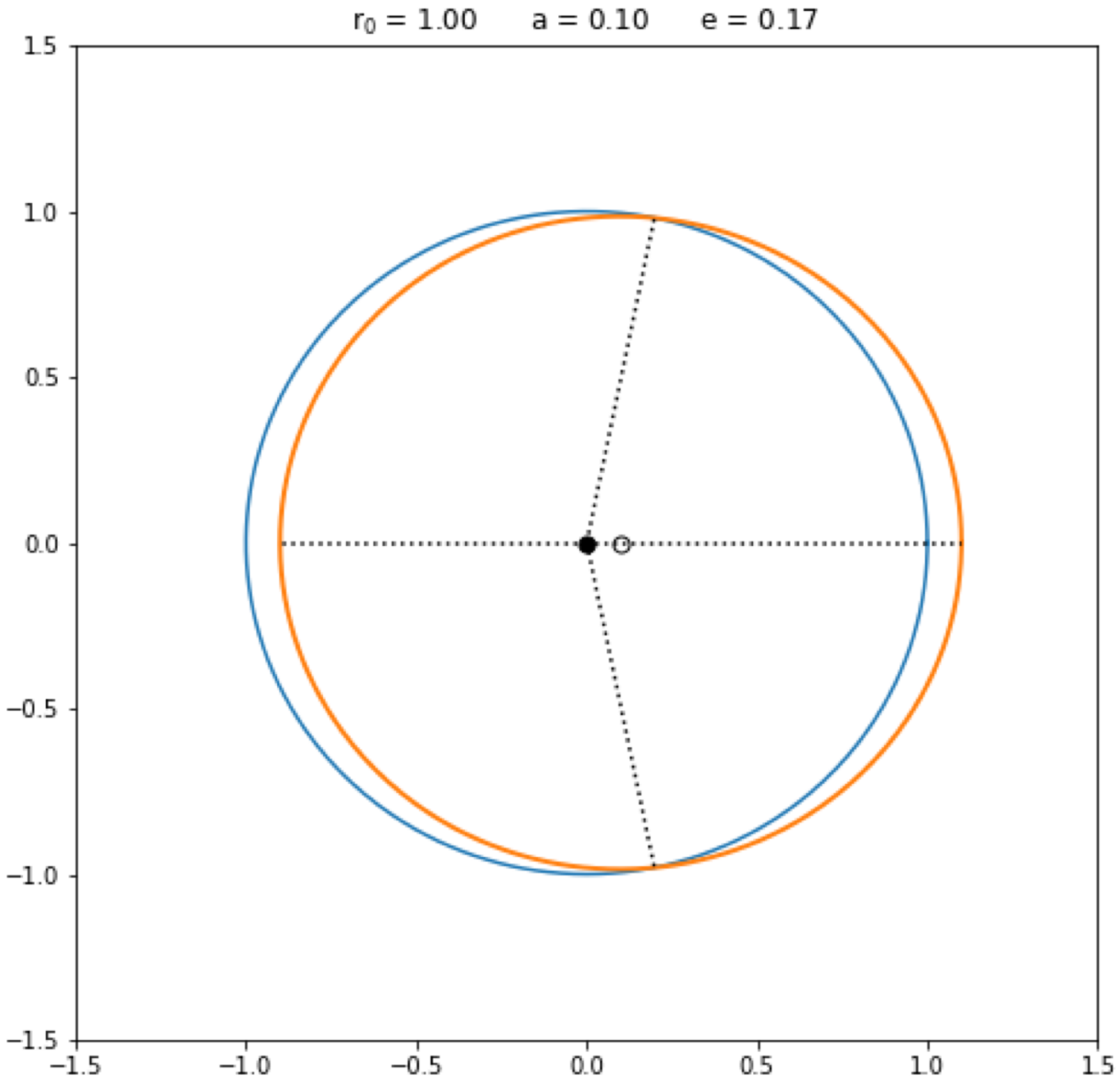


Small perturbation: elliptical-like orbit

The orbit looks very close to a circle.

It is hard to tell if it is actually distorted or just offset.

Careful inspection reveals that while the perturbed orbit appears to have about the same width, the height of the new orbit is a little bit smaller than the circular orbit, and therefor is not just a shifted circle.



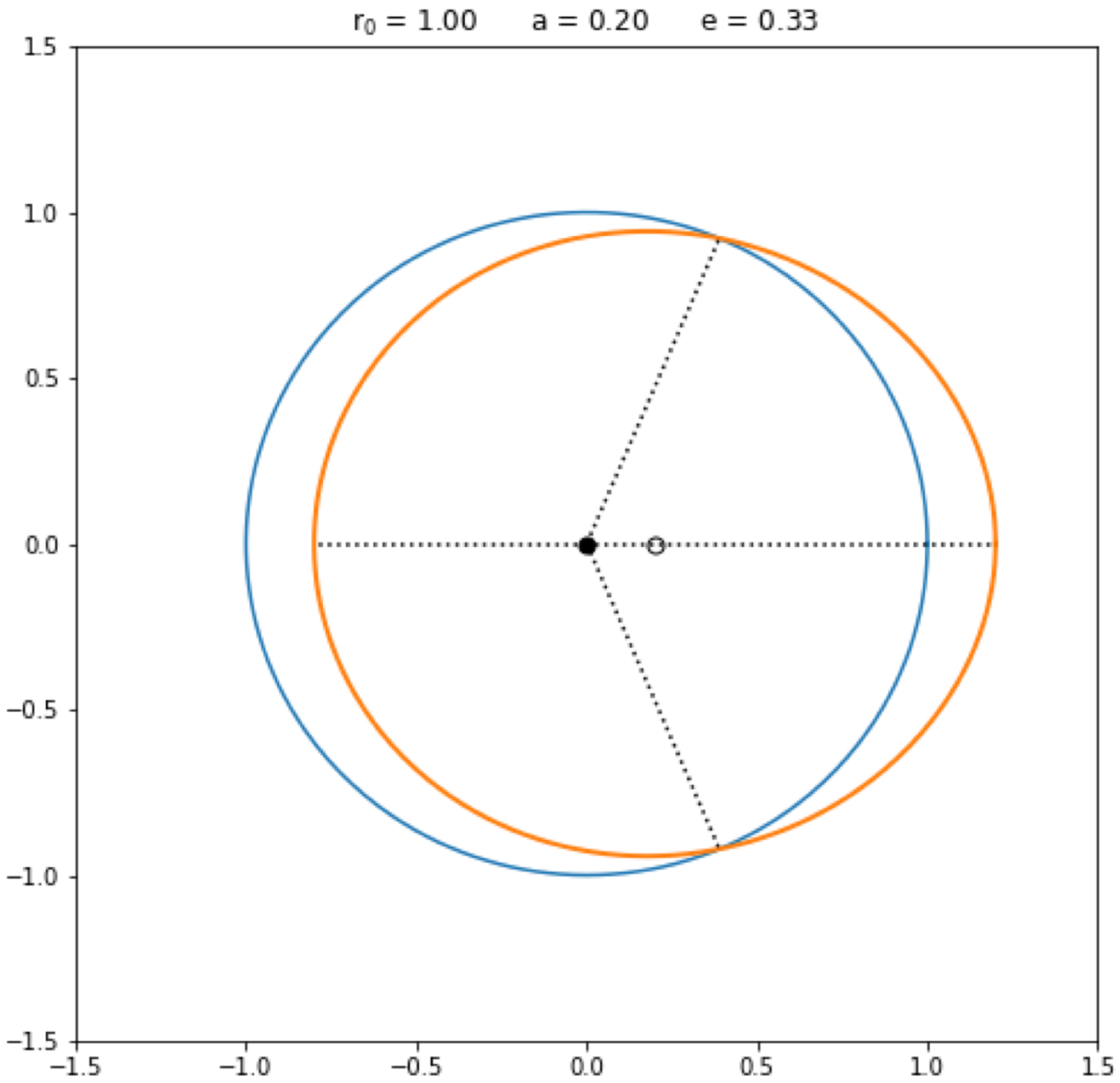
Small perturbation: elliptical-like orbit

Clearly now, this orbit is non-circular.

If we had to wager, it would be entirely reasonable to guess that this orbit is an ellipse.

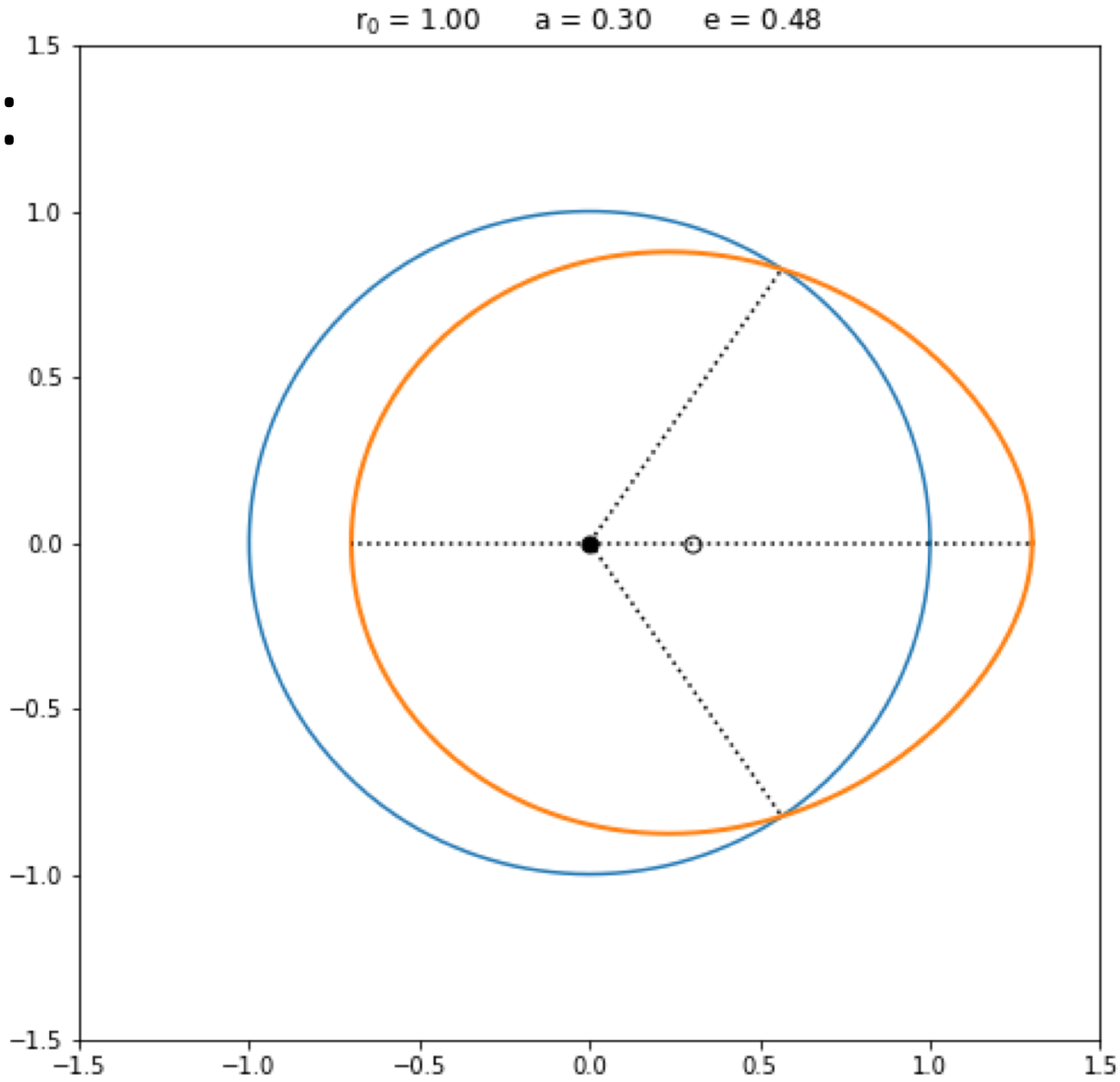
In fact, it is not and ellipse and is just a different mathematical creature that looks fairly similar.

It is impossible to tell the difference by eye.



Moderate perturbation: some distortion

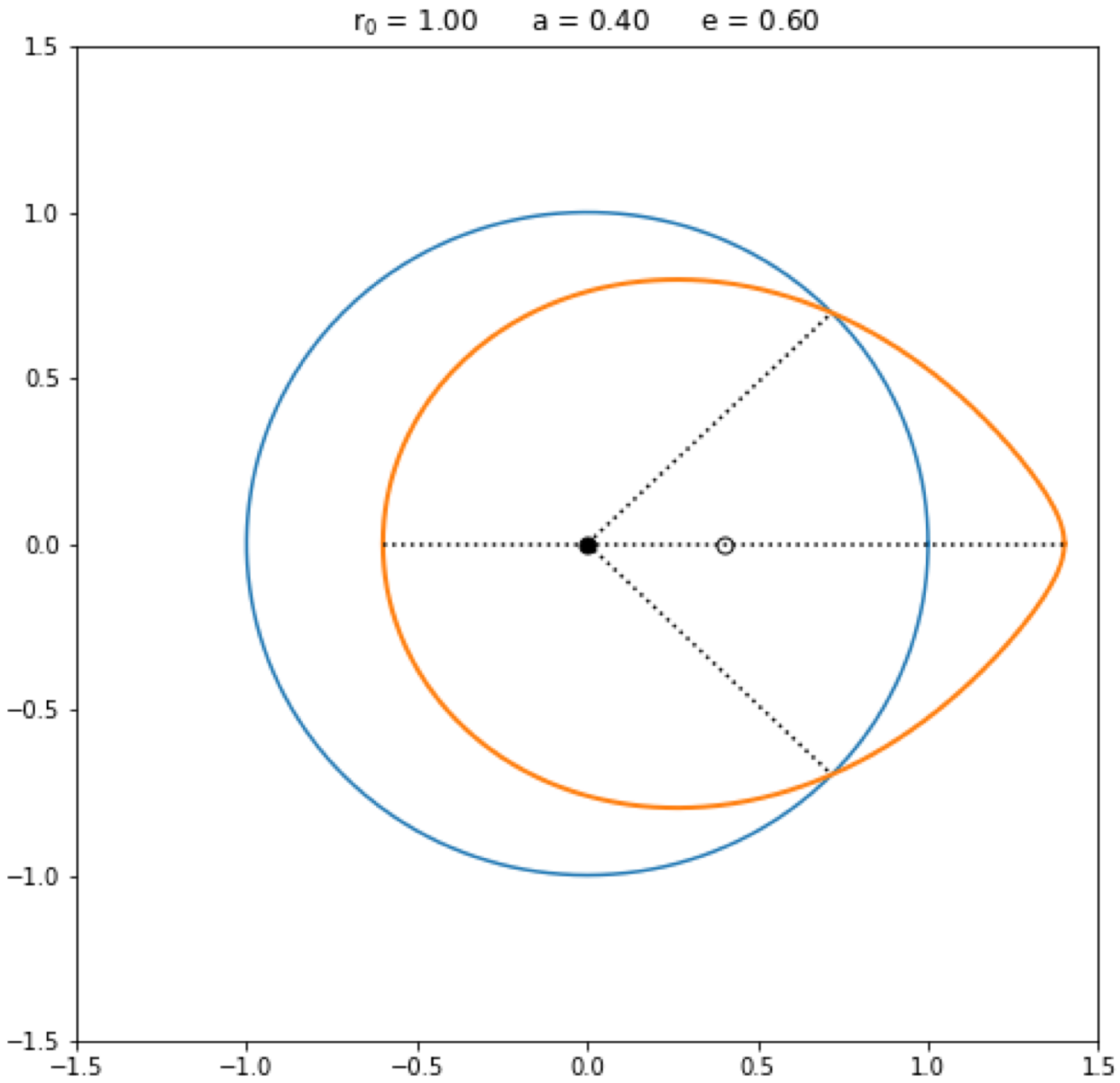
The orbit continues to look more stretched, as expected for increasing ellipticity.



Largish perturbation: pointed orbit

While more stretched than the previous orbit, this one starts to develop a curious feature that it looks fairly egg-shaped and pointed near the aphelion.

We should be highly suspicious of this solution and have concern that the perturbed solution is no longer accurately representing the orbits for this range of perturbations.

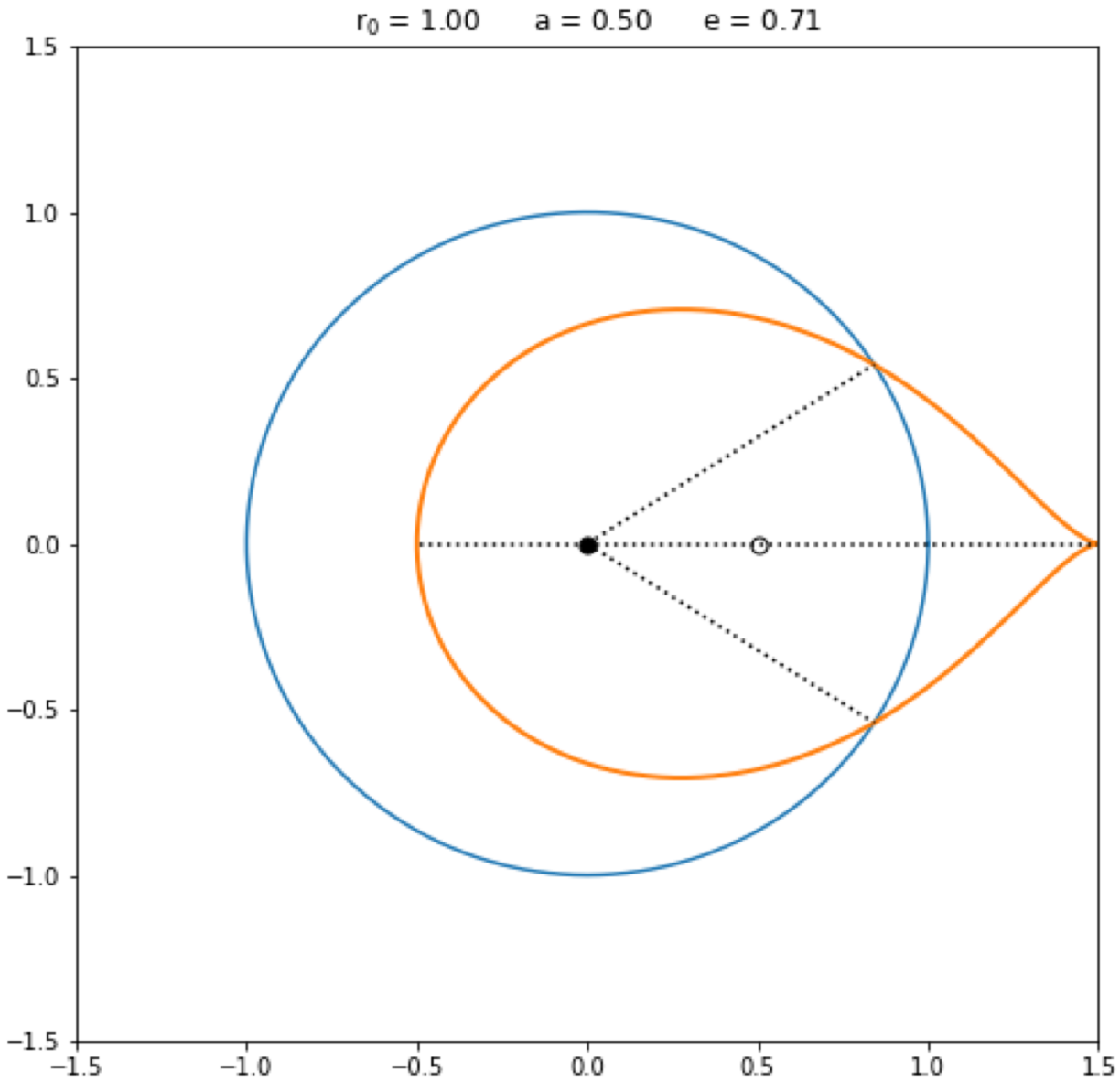


Large perturbation: unphysical orbit

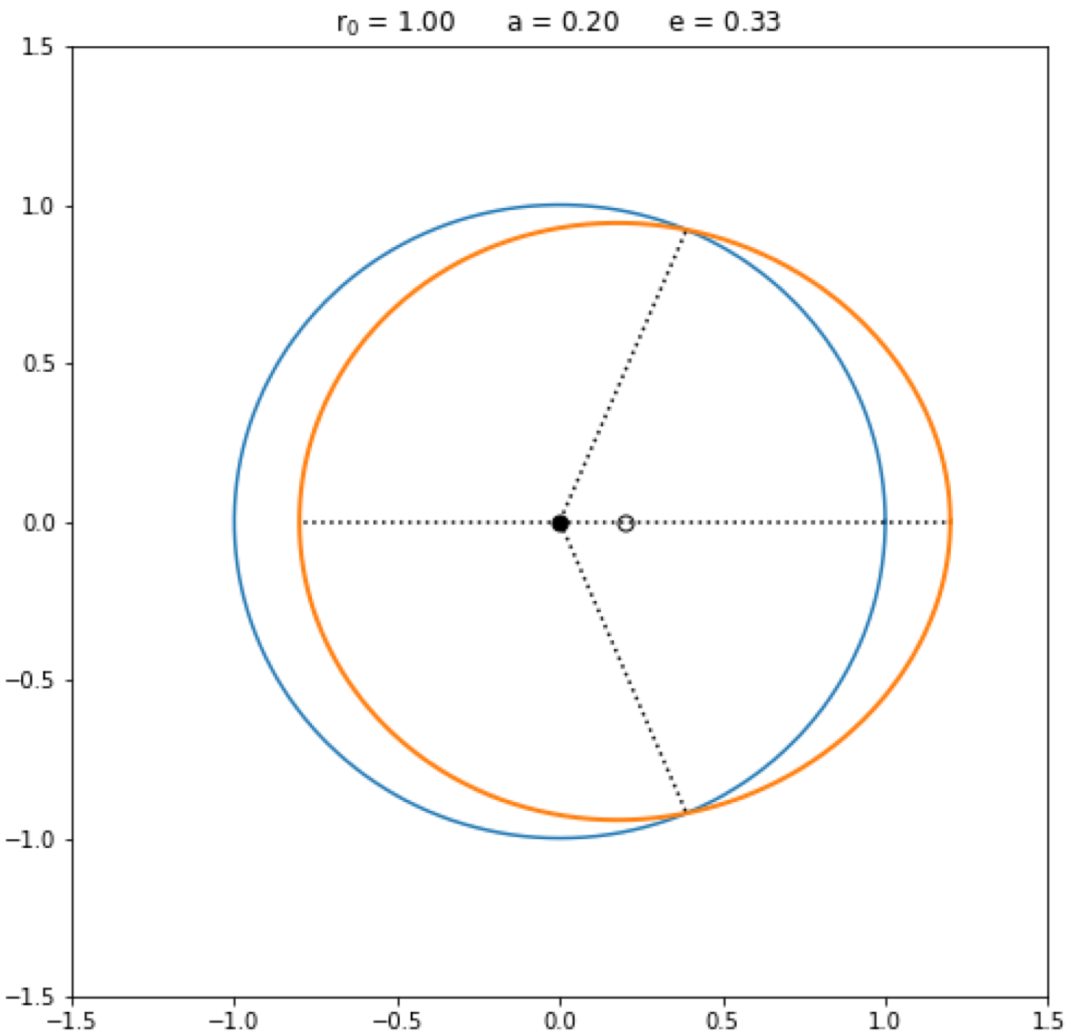
The orbit now develops a distinctive point.

Clearly, this feature is not physical and means that our approximation is broken for this level of perturbation.

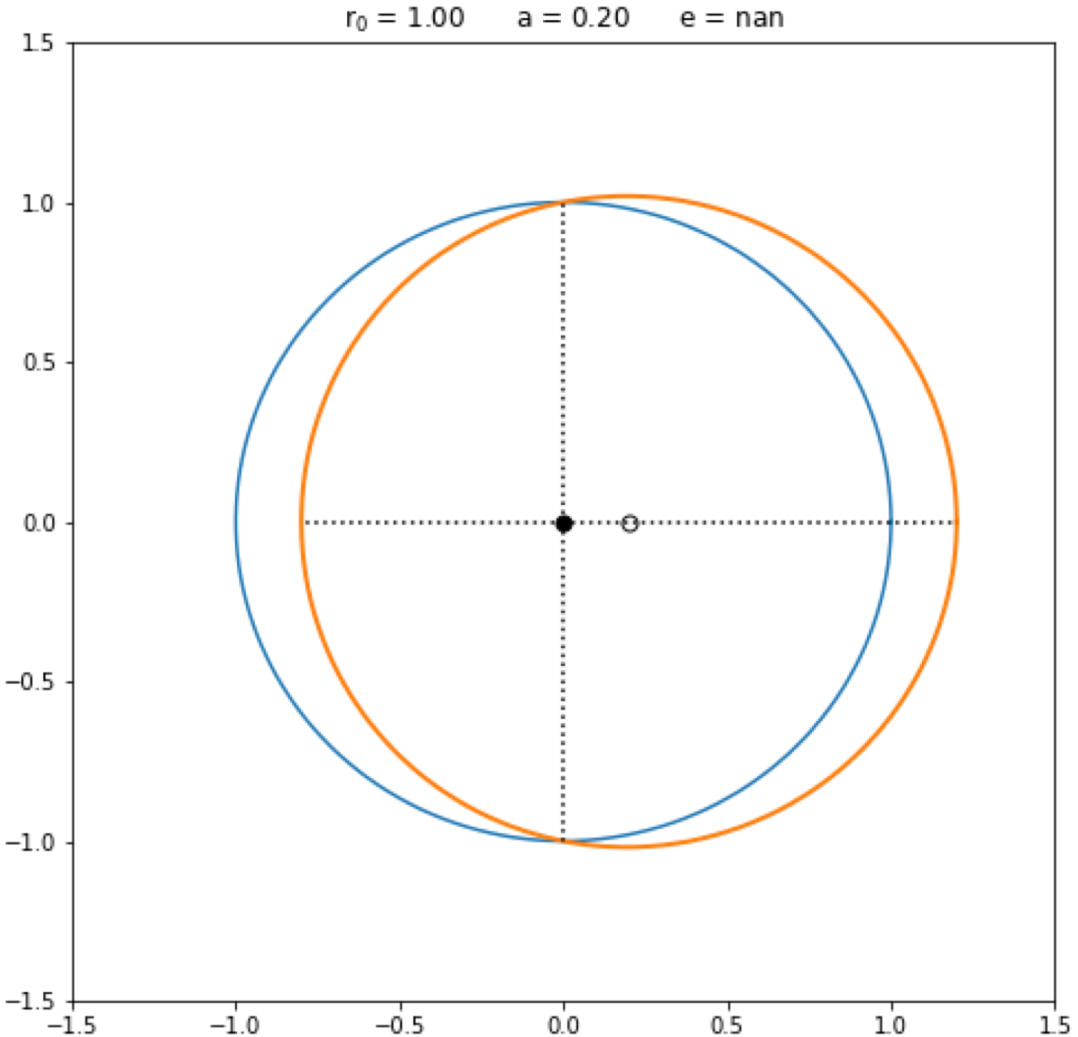
If we were doing a careful study of orbits, we might want to define a range of usefulness that is considerably smaller than a = 0.50 for the perturbed model.



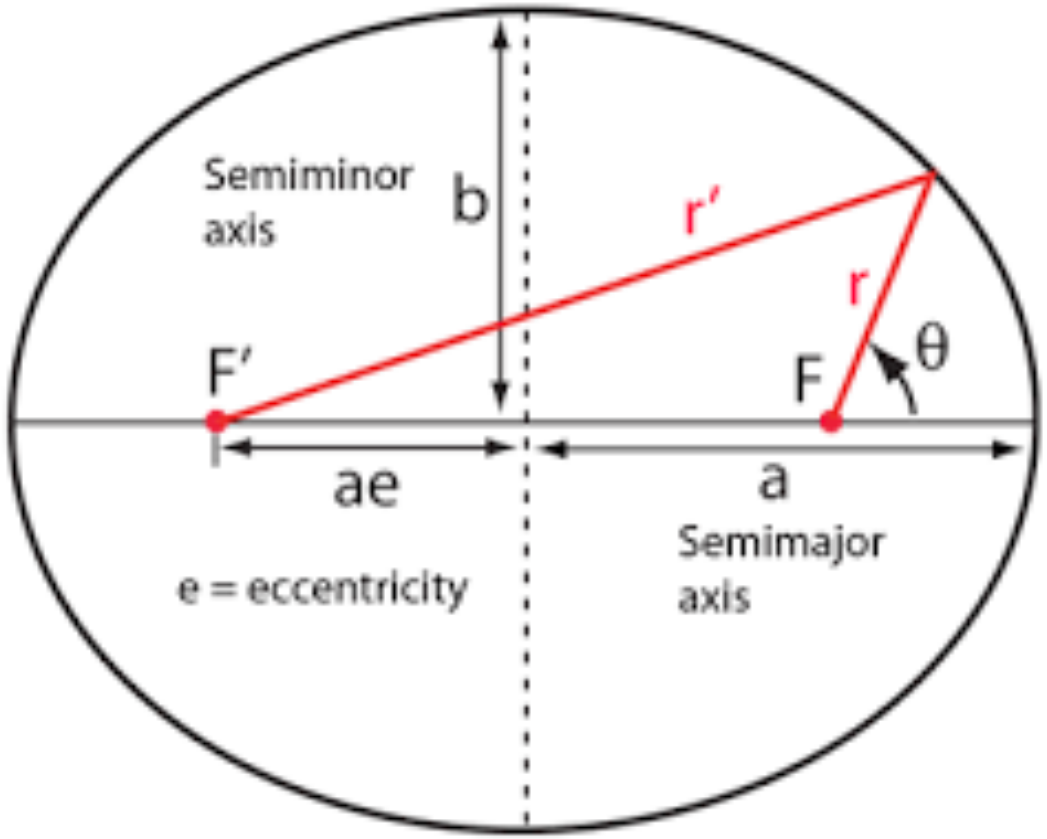
with perturbations to radius and angle



with perturbations to radius only



Definitions



semi-major axis: a

semi-minor axis: b

eccentricity: e

distance between
center and foci: ae

$$e = \sqrt{1 - \left(\frac{b}{a}\right)^2}$$