METEOR IMPACT LAB

PHYSICS 201

Artist's impression of the Chicxulub impact event

BACKGROUND & THEORY

- To model something as complex as a meteor colliding with a planet we need to have handle on the situation, some place to start. We look to our laws and theories of physics to see which one(s) we can apply.
- We begin by looking to conservation of energy, and try to convince ourselves that we should express this statement as the kinetic energy of the meteorite being transferred into various forms of energy at impact:

$$K_{meteor} = K_{ejecta} + E_{friction} + E_{phase\ changes} + E_{seismic\ waves} + \ \ldots$$

- This is a very complex equation, and figuring out how to solve it would take a lot of hard work and time that we don’t have - you could likely make a whole career out of trying to solve this problem!

Meteor Crater
Arizona

SIMPLIFYING THE SYSTEM

- We can’t actually solve the whole thing, so what do we do? We simplify it and aim to keep the most important parts. We always do this. When we solve projectile motion problems (in this class) we typically ignore air resistance and a bunch of other small effects, like the gravitational influence of the moon. We know these influences are there and certainly affect the actual motion, but they’re also small compared to the other effects and we can therefore get the problem mostly right by ignoring them.
- **Our central assumption:** the kinetic energy of the meteorite becomes kinetic energy of the ejecta, and we ignore everything else.

$$K_{meteor} \approx K_{ejecta}$$

- We can now begin to model this system. The motion of the meteor is fairly simple, it’s mass and collision velocity are well-defined, but the ejecta is very complex, a bunch of loose material moving with different angles and speeds. However, let us assume that all of the ejected mass from the crater moves with a single representative velocity so that we can express the kinetic energies as follows:

$$K_{meteor} \approx \frac{1}{2} M_{meteor} v_{meteor}^2 \qquad K_{ejecta} \approx \frac{1}{2} M_{ejecta} v_{ejecta}^2$$

MODELING THE MASS AND VELOCITY OF THE EJECTA

- We now assume that the crater is half of a sphere and that that entire mass of material is ejected, that is, the ejected mass is equal to the density times the volume of the crater (half a sphere).

$$M_{ejecta} = \rho_{soil} V = \rho_{soil}\left[\frac{1}{2}\frac{4}{3}\pi r^3\right] = \rho_{soil}\left[\frac{1}{2}\frac{4}{3}\pi\left(\frac{D}{2}\right)^3\right] = \rho_{soil}\frac{\pi}{12}D^3$$

- The velocity term is harder to pin down, and in fact, we don't really know much about it. The best we can do at this point is to make a really simple model. Knowing nothing else, but guessing that perhaps the velocity is somehow related to the size of the crater, we consider the following models:

  - Model 1: ejecta velocity increases with increasing crater size

$$v_{ejecta} = \alpha D = \alpha D^1$$

  - Model 2: ejecta velocity is constant (independent of the crater size)

$$v_{ejecta} = \alpha = \alpha D^0$$

  - Model 3: ejecta velocity decreases with increasing crater size

  (this seems like a bad model, but we can include it for completeness)

$$v_{ejecta} = \frac{\alpha}{D} = \alpha D^{-1}$$

MAKING A GENERALIZED MODEL FOR VELOCITY

- The fact is that we don’t really know which of those models is more correct than the others, and there are an infinite number of other models we could consider.
- Without having any other information, we leave the specific form of the velocity dependence on crater size unknown and we will let the data tell us how it should be.
- We use a 2-parameter model for velocity, meaning that there are two unknown numbers: **α** and p.

$$v_{ejecta} = \alpha D^p$$

PUTTING IT TOGETHER

- Combining our models for the mass and velocity of the ejecta we have:

$$K_{ejecta} = \frac{1}{2} M_{ejecta} v_{ejecta}^2 = \frac{1}{2}\left[\frac{\pi}{12}\rho_{soil} D^3\right][\alpha D^p]^2$$

- Collecting all of the constants together, and all of the D's together we have:

$$K_{ejecta} = \left[\frac{\pi}{24}\rho_{soil}\alpha^2\right] D^{3+2p}$$

- Let's simplify this expression a bit more by defining a constant c equal to all the stuff out front:

$$K_{ejecta} = cD^{3+2p}$$

$$c = \frac{\pi}{24}\rho_{soil}\alpha^2$$

SETTING UP THE EQUATION FOR PLOTTING

- We will separately measure the kinetic energy and the diameter of the crater. But it is not so easy to plot the data and determine what the values of p and **α** should be.

- To make plotting easier, we take the logarithm of both sides of the equation:

$$\log\left(K_{ejecta}\right) = \log(cD^{3+2p}) = \log(c) + \log(D^{3+2p}) = \log(c) + (3+2p)\log(D)$$

$$\log\left(K_{ejecta}\right) = \log(c) + (3+2p)\log(D)$$

- We can think of this as an expression like y = mx + b when we make the following definitions:

$$\mathrm{y} = \log\left(K_{ejecta}\right)$$

$$m = 3 + 2p$$

$$x = \log(D)$$

$$b = \log(c)$$

SOLVING FOR THE MODEL PARAMETERS

- We can make either two graphs (one for the 2nd floor drop and one for the 3rd floor drop) or combine all the data into one graph.

- Plot the log(kinetic energy) on the y-axis, and the log(crater diameter) on the x-axis.

- Once we have the data plotted, we can fit a line to the data and find slope (m) and y-intercept (b). Once we have these numbers we can solve for α and p.

- The density of the soil is about 1600 kg/m^3.

- USE SI UNITS FOR ALL MEASUREMENTS – that means convert mass to kg, distances to meters.

WHAT YOU SHOULD CALCULATE

Once you have your model parameters, you should answer the following questions:

1. Look back at our model that we developed and make an estimate of the mass of the ejected material from the crater.
2. Does the ejecta travel high enough into the atmosphere to get trapped in the jet stream so that the dust can circulate around the world? Use our model for the velocity of the ejecta, and assume that at least some of it travels straight upward.
3. Do a little reading and find the crater diameter of the Chixulub crater (estimates vary). Use this information to calculate the mass and the diameter of the meteor that made that crater, assuming that the impact speed is about 17000 miles/hour and that the meteor was made of pure iron. You might need to consider some more unit conversions. You can get this info by first finding the kinetic energy of the meteor given the info on the crater size.