Mathematics Study Guide for PHY 201
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This guide provides a survey of a number of mathematical concepts that should be review for you. I understand that not everyone's mathematical preparation was the same, and expect that no one will understand everything here. My expectation is that you will use this guide to help you identify weak spots and ask questions and visit office hours. My goal is to help you, both in terms of core physics content and basic math skills, but I don't read minds so I can only do that if you make an effort to reach out when you need help. There is no shame in admitting you didn't learn something and want to make up for it, but not taking steps to do so when you should is a problem.

Section 1: algebra and graphing

Part 1: solve for x or t, unless otherwise noted

1. $10 = 3x + 1$

2. $10 = t^2 + 1$

3. $10 = -x^2 + 1$

4. $v_f = v_i + a\,t$

5. $m_1 a + m_1 g = m_2 g - m_2 a - \mu m_2 g$ (solve for a)

6. $10 = 3x^3 - 14$

7. $-10 = 3t^3 + 14$

8. $10 = -3x^2 + 2x + 11$

9. $10 = -3x^2 + 2x + \frac{28}{3}$

10. $10x = -3x^3 + 2x^2 + \frac{28}{3}x$

11. $10 = 4 \times 10^{2\pi t/10} - 2$

12. $0 = 4 \times e^x - 2\pi$

13. $0 = 4 \times e^x \times e^{2x^2} - 2\pi$

14. $0 = 6 \times \log_{10}(x) - 2$

15. $0 = 6 \times \log_{10}\left(\frac{x^{1/3}}{2}\right) - 2$

16. $0 = 6 \times \ln(x) - 2$

17. $0 = 6 \times \ln(x^{1/3}) - 2$

Part 2: solve for the unknown coefficients

18. $y = m\,x + b$ given $y(0) = 1$ $y(3) = 10$

19. $y = a\,x + b$ given $y(-1) = 2$ $y(3) = 10$

20. $y = v\,t + y_0$ given $y(t_1) = y_1$ $y(t_2) = y_2$

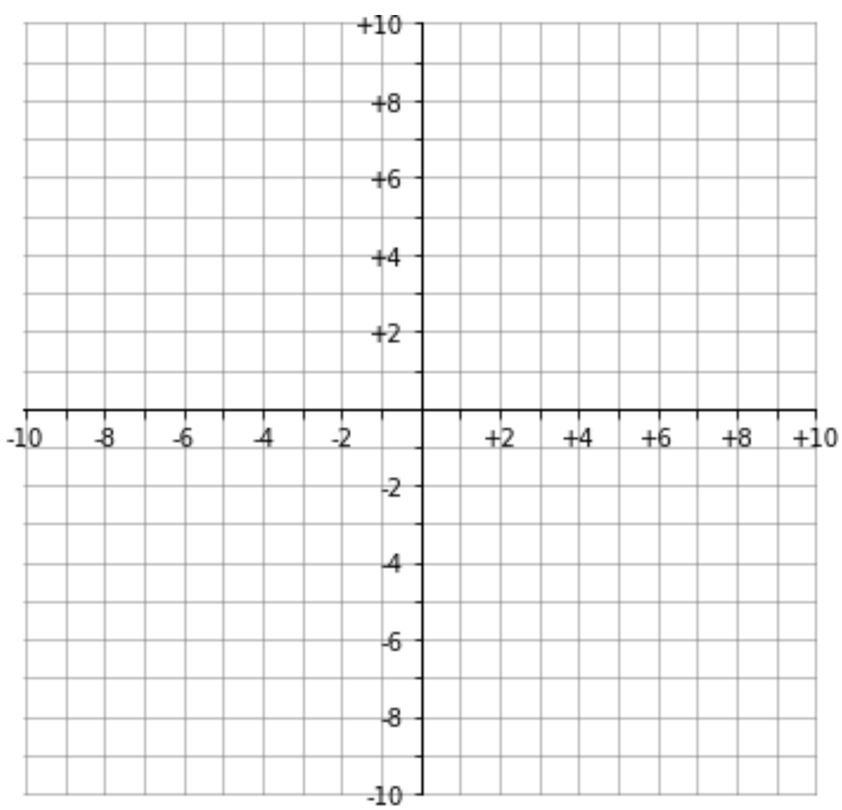
21. $y = a\,x^2 + b\,x + c$ given $y(0) = -1$ $y(1) = 0$ $y(2) = 7$

22. $y = a\,x^2 + b\,x + c$ given $y(-1) = 8$ $y(1) = 0$ min/max at $x = \frac{1}{3}$

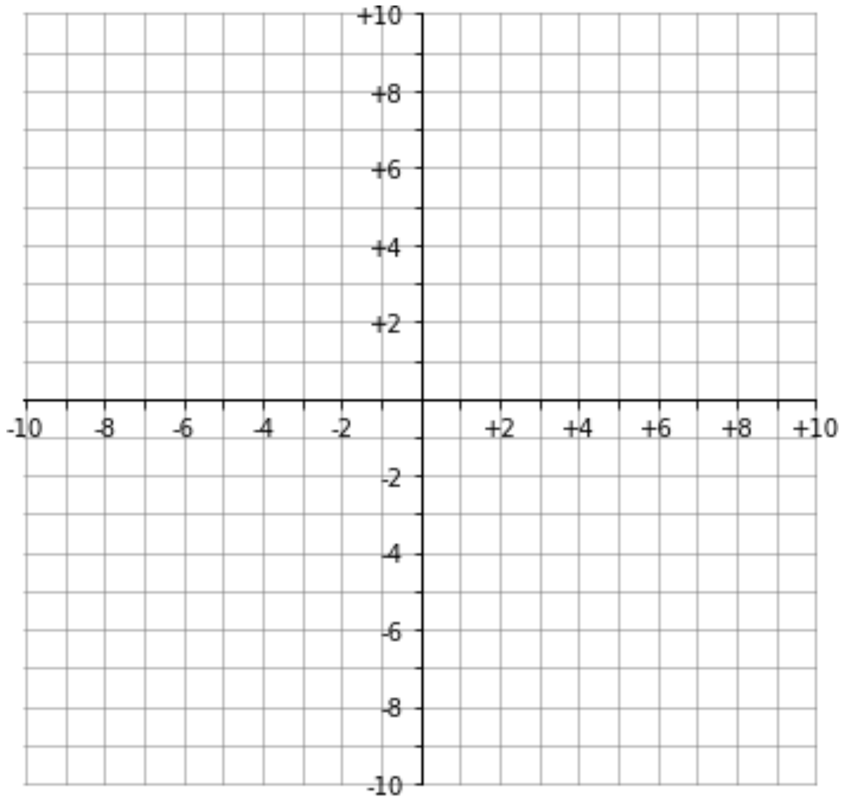
23. $y = a\,x^2 + b\,x + c$ given $y\left(-\frac{1}{3}\right) = 0$ $y(1) = 0$ $y = -\frac{4}{3}$ at min/max

Part 3: using algebra find the intersection(s) of two curves, then plot the functions and verify.

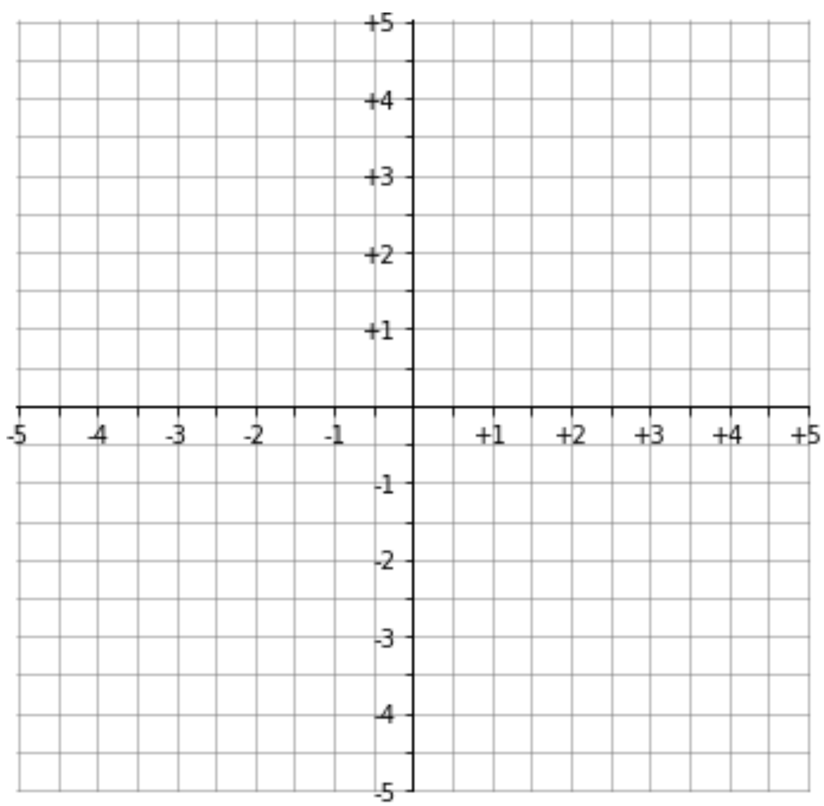
24. $y = 2x + 1$ $y = 4x - 5$



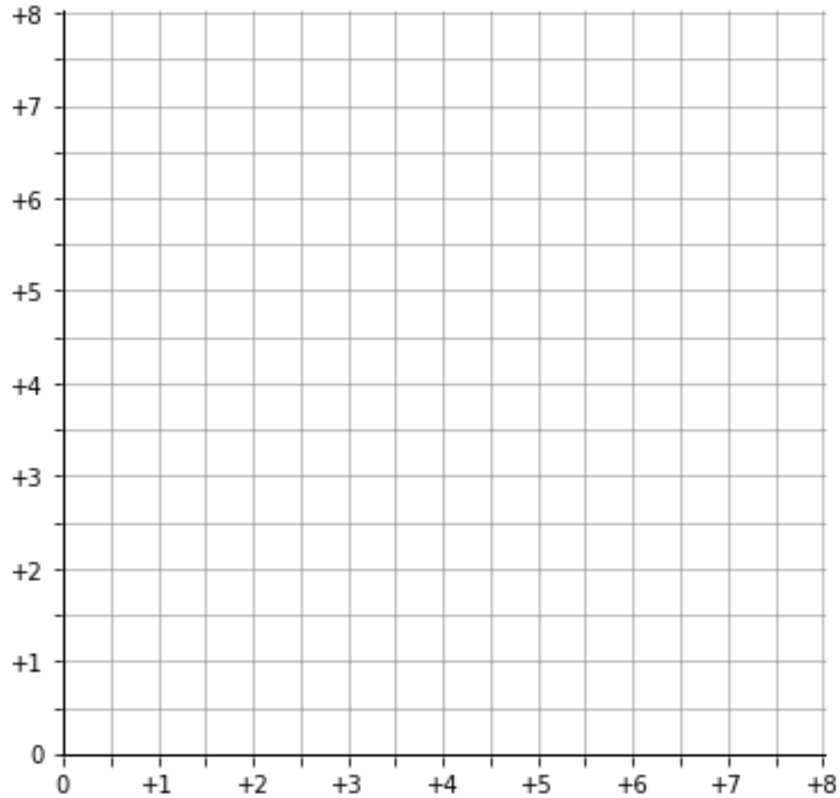
25. $y(x) = 2x + 1$ $y(x) = 2x - 5$



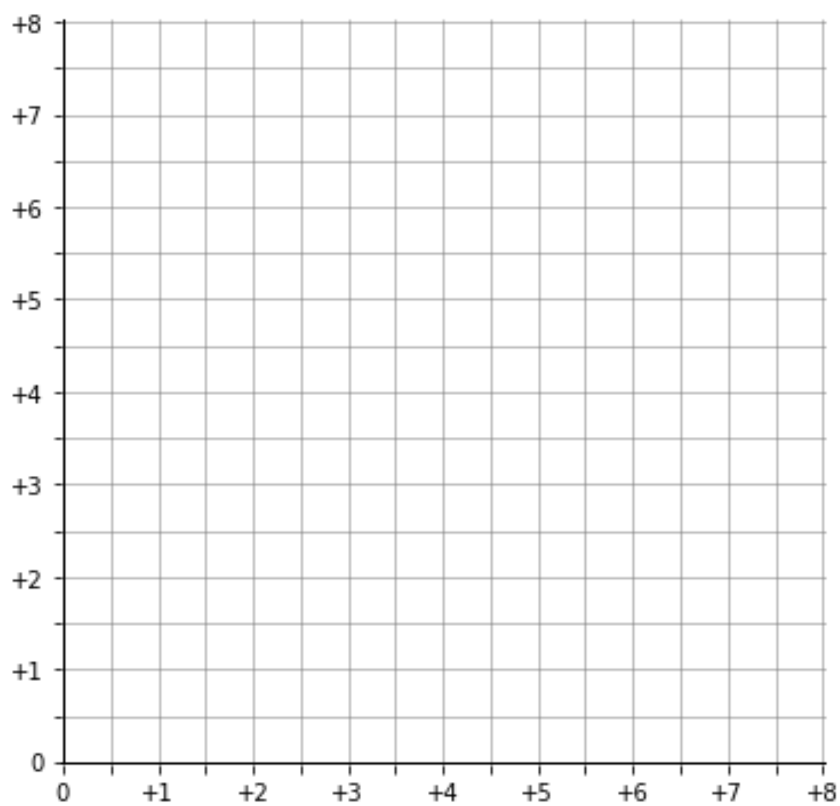
26. $y(x) = 2x + 1$ $y(x) = 3x^2 - 4$



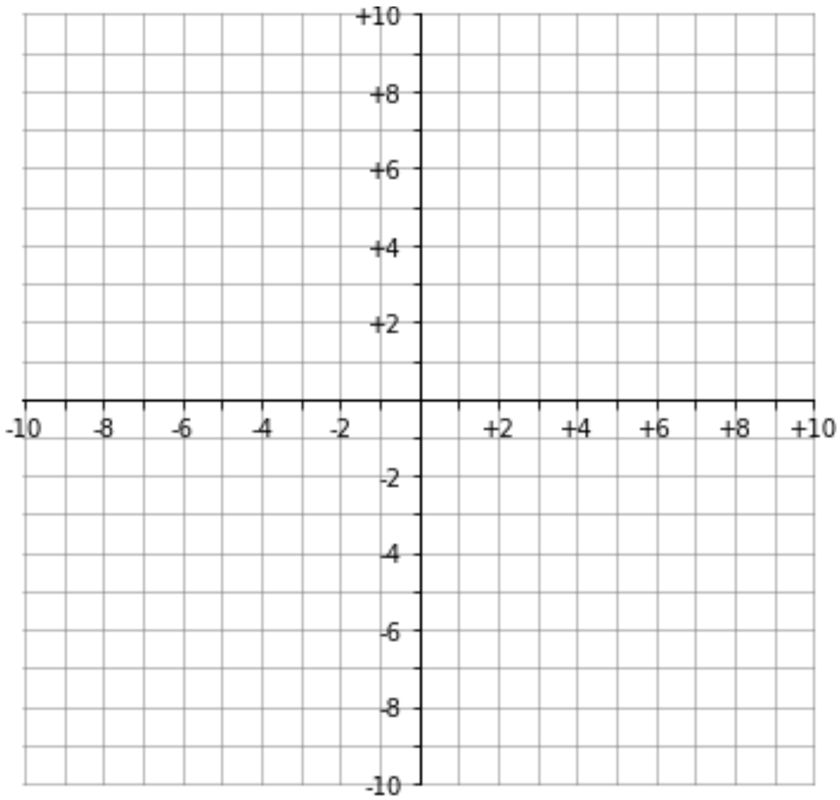
27. $y = -6x + 11$ $y = 3x^2 - 12x + 14$



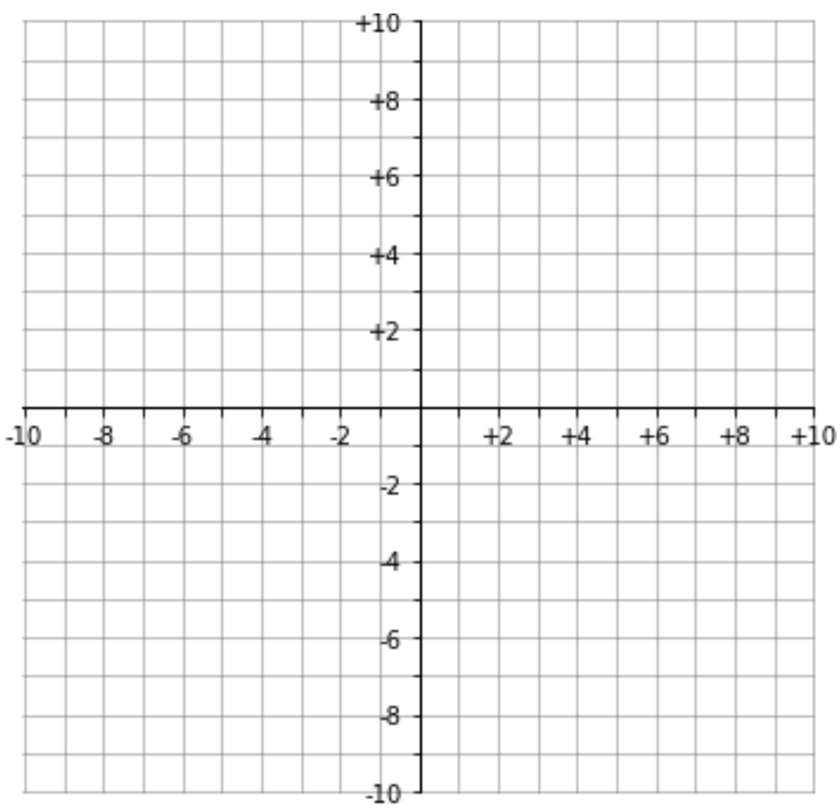
28. $y = -6x + 5$ $y = 3x^2 - 12x + 14$



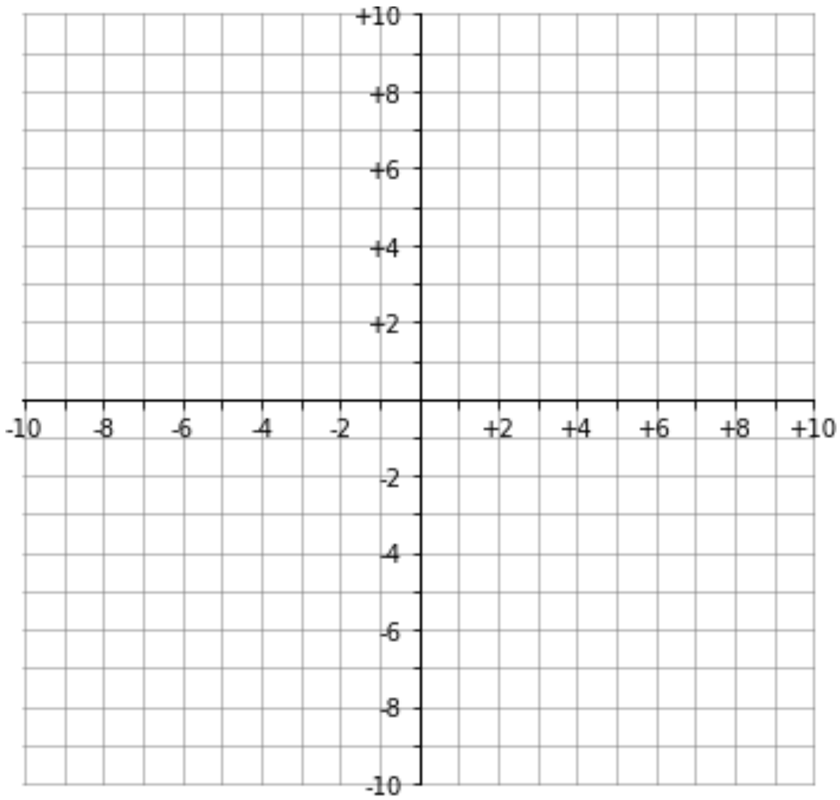
29. $y = 1x^2 - 2x - 6$ $y = 3x^2 - 12x + 2$



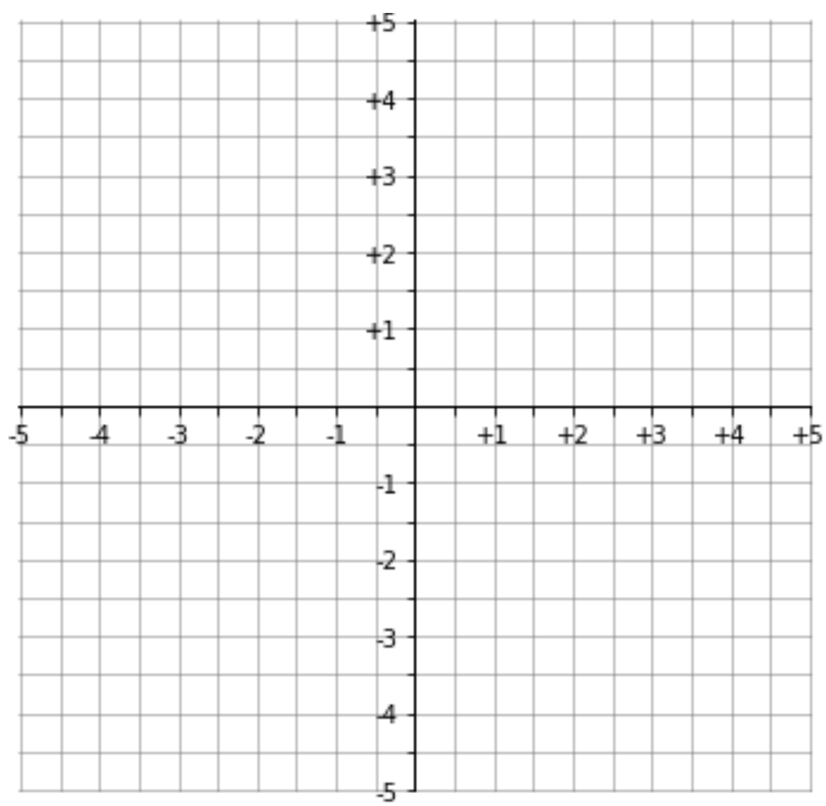
30. $y = -1x^2 - 2x + 9$ $y = 3x^2 - 12x + 13$



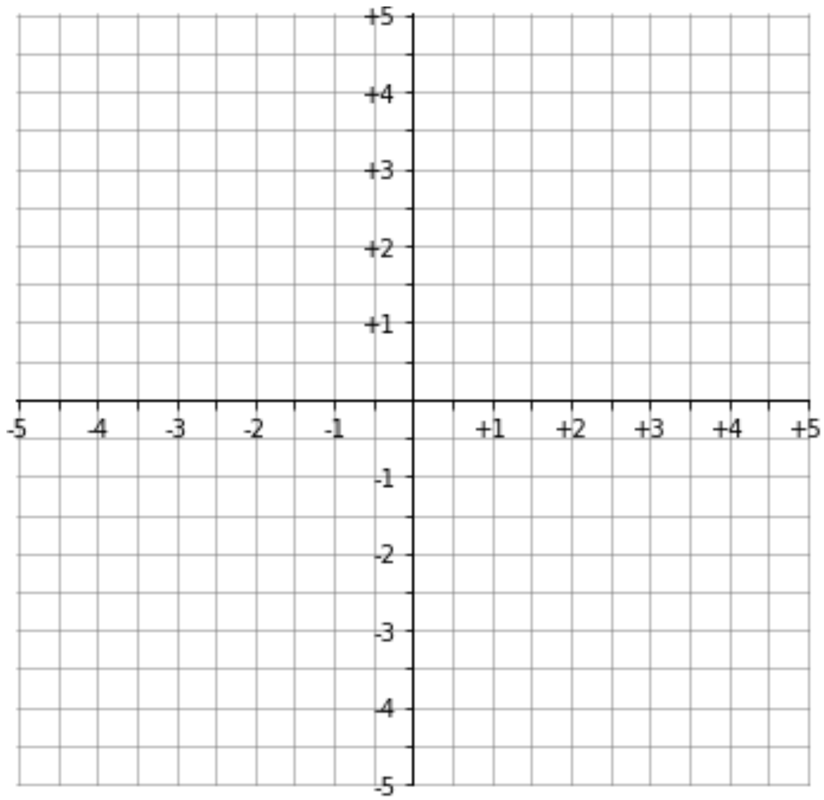
31. $y = -3x^2 - 2x + 5$ $y = 3x^2 - 12x + 13$



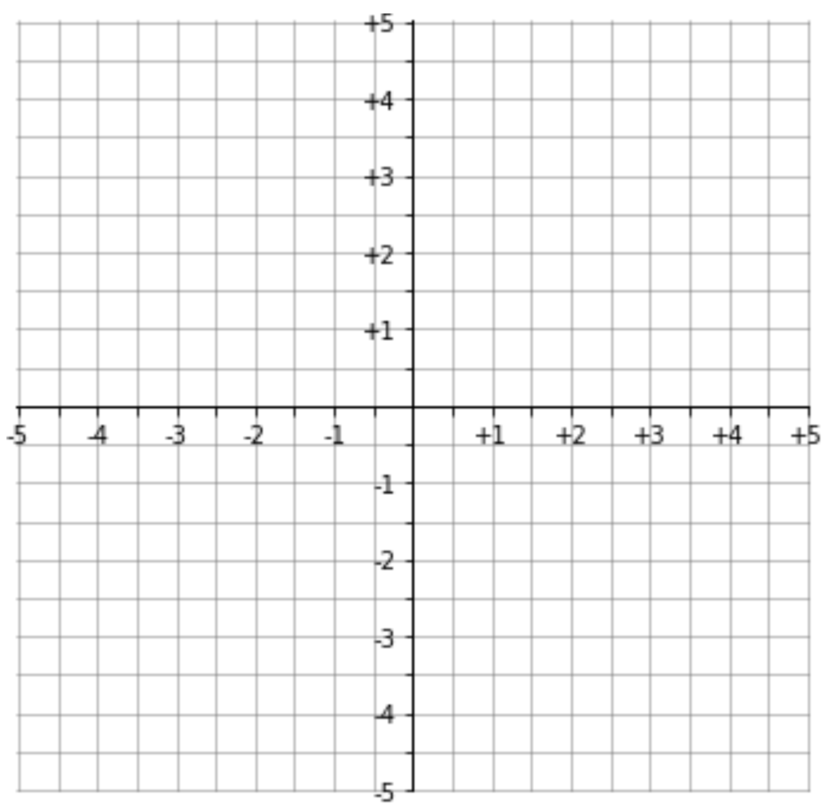
32. $x^2 + y^2 = 16$ $x + y = 4$



33. $x^2 + y^2 = 16$ $x^2 + y = 4$



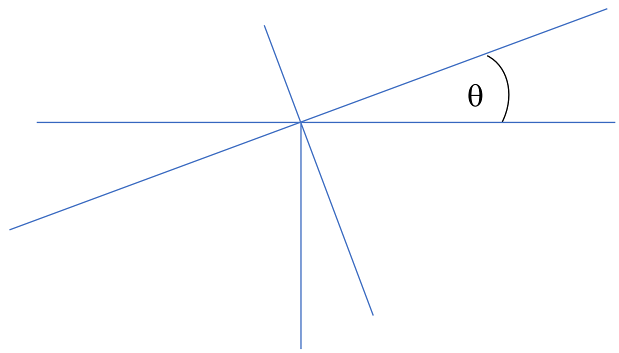
34. $x^2 + y^2 = 16$ $x^2 + y = 5$



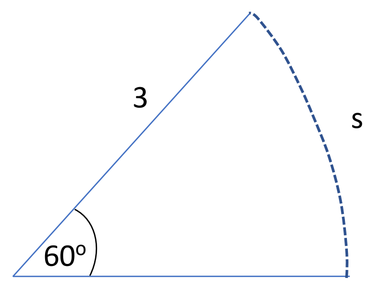
Section 2: geometry and trigonometry

Part 1: basic geometry

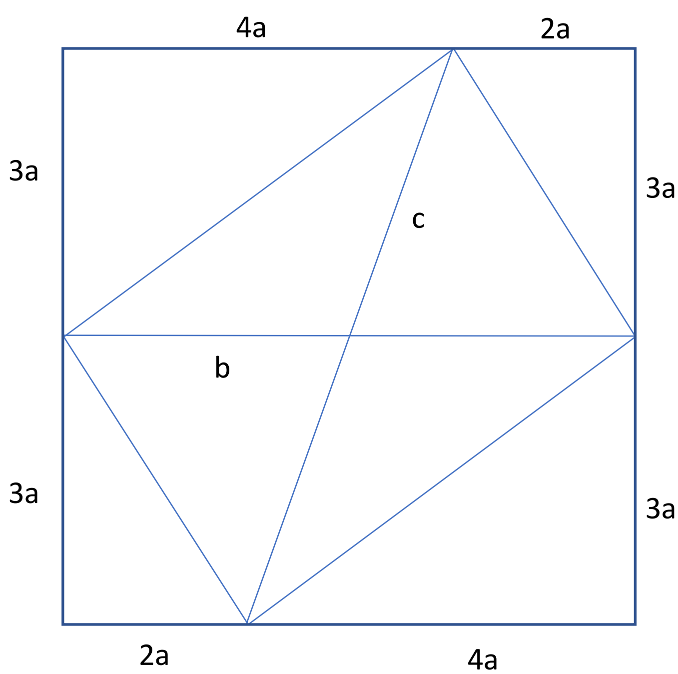
1. Identify all the other angles in the diagram below. You may assume that angles that look like right angles are indeed so.



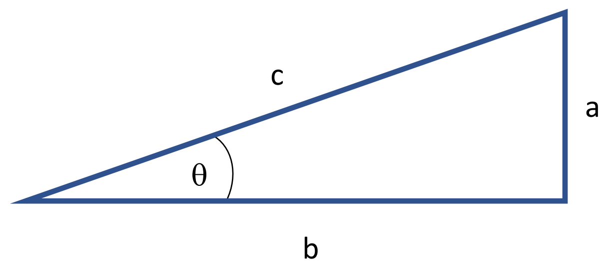
2. What is the length of circular arc length defined as s?



3. Solve for the length of the unknown side defined as b and c and the area of the inner parallelogram.



Part 2: Provide definitions for the following trigonometric functions given the triangle below



4. $\sin(\theta) =$

5. $\cos(\theta) =$

6. $\tan(\theta) =$

7. $\csc(\theta) =$

8. $\sec(\theta) =$

9. $\cot(\theta) =$

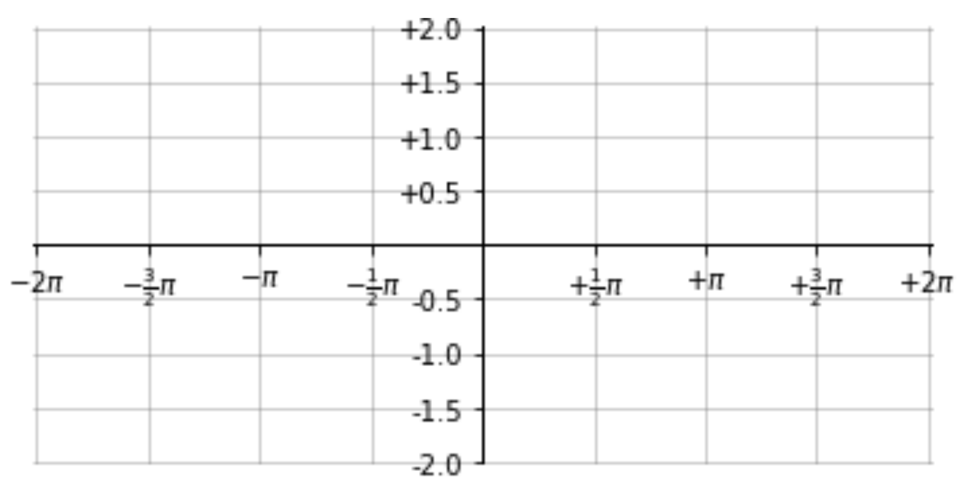
10. $\sin^{-1}\left(\frac{a}{c}\right) =$

11. $\cos^{-1}\left(\frac{b}{c}\right) =$

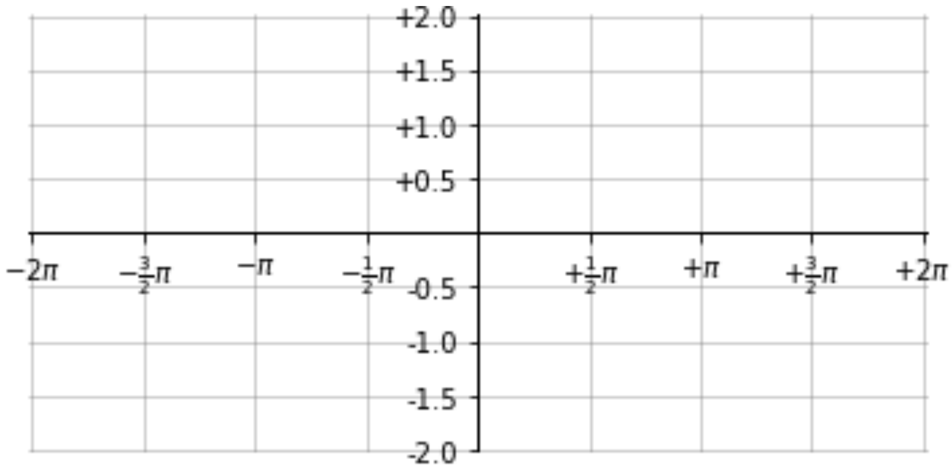
12. $\tan^{-1}\left(\frac{a}{b}\right) =$

Part 3: Plot the following functions.

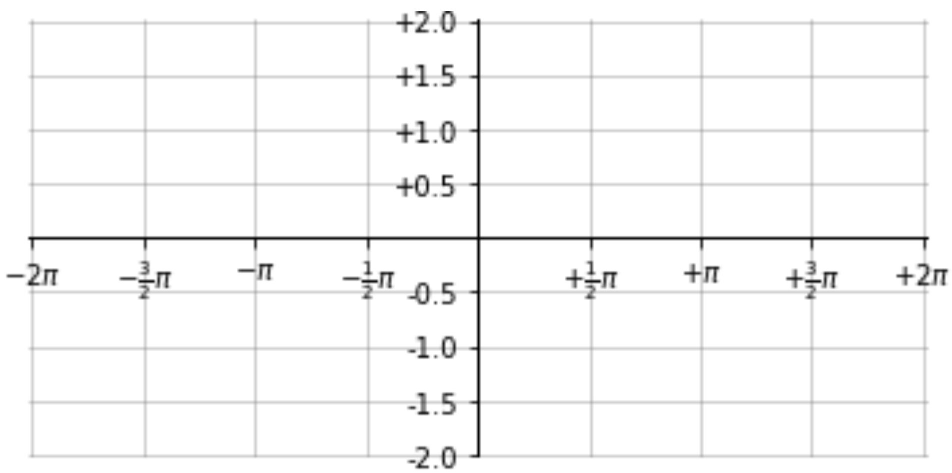
13. $y(x) = \sin(x)$



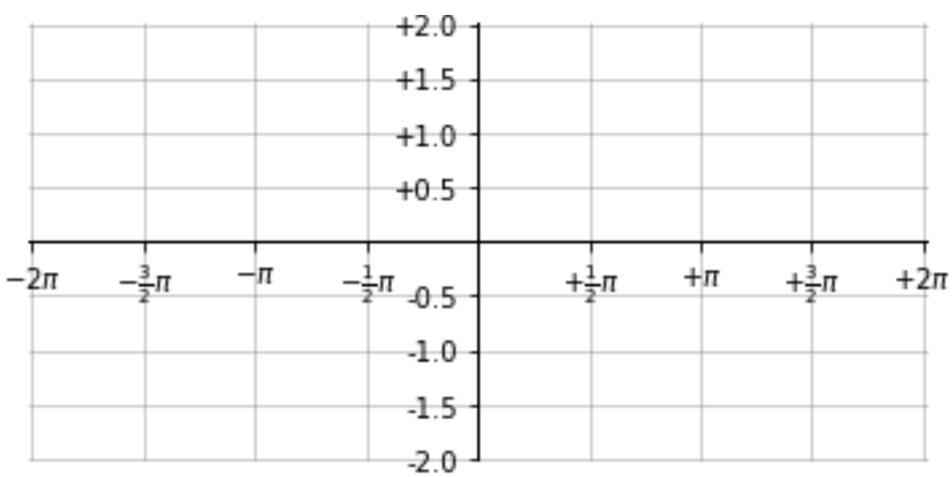
14. $y(x) = 2\cos(x)$



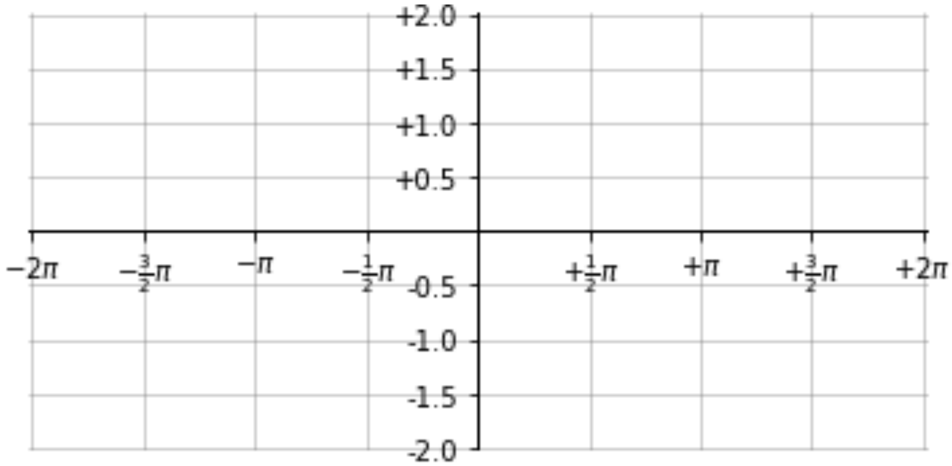
15. $y(x) = \cos(2\,x)$



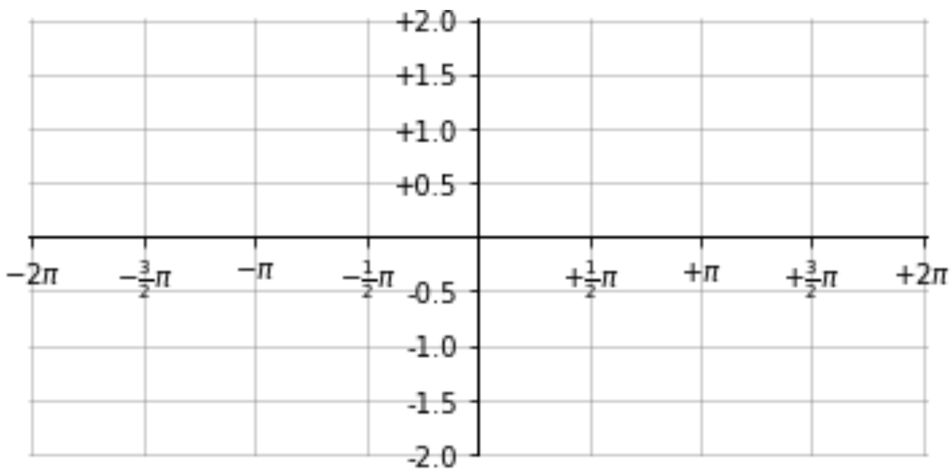
16. $y(x) = 2\cos\left(\frac{x}{2}\right)$



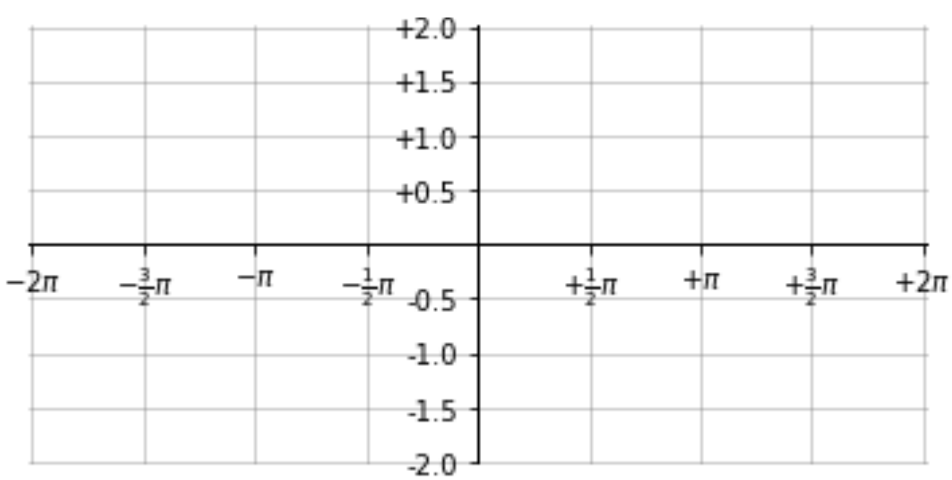
17. $y(x) = 2\cos\left(x - \frac{\pi}{2}\right)$



18. $y(x) = 2\sin\left(x + \frac{\pi}{2}\right)$



19. $y(x) = \tan(x)$



Section 3: calculus

Part 1: calculate the derivative of each function

1. $y(x) = 3x + 2$

2. $y(x) = 3x^2 + 2x + 2$

3. $y(x) = 2\, x^{1.5} + 3$

4. $y(x) = \frac{2}{x} + 3$

5. $y(x) = \frac{2}{x^{1.5}} + 3$

6. $y(t) = 2t + 3$

7. $f(t) = 4\, t^{1.5} - 1$

8. $f(t) = 3 - \frac{2}{t^{1.5}}$

Part 2: find the coordinates of the minima or maxima of the function

9. $f(x) = 2x + 3$

10. $f(x) = 3x^2 + 2x + 2$

11. $f(x) = x^3 - x^2 - x + 2$

12. $f(x) = x^3 - x^2 + x + 2$

13. $f(x) = \frac{2}{x^2} - \frac{3}{x}$

Part 3: calculate the velocity given the position

14. $y(t) = (2\text{ m/s})\, t + 3\text{ m}$

15. $y(t) = (2\text{ m/}s^2)\, t^2 + (3\text{ m/s})\, t + 2\text{ m}$

16. $x(t) = (1\text{ m/}s^3)\, t^3 - (1\text{ m/}s^2)\, t^2 + (1\text{ m/s})\, t + 2\text{ m}$

17. $x(t) = (2\text{ m }s^2)\, t^{-2} - (3\text{ m s})\, t^{-1}$

Part 4: calculate the acceleration given the position

18. $y(t) = (2\ \mathrm{m/s})\ t + 3\ \mathrm{m}$

19. $y(t) = (2\ \mathrm{m}/s^2)\ t^2 + (3\ \mathrm{m/s})\ t + 2\ \mathrm{m}$

20. $x(t) = (1\ \mathrm{m}/s^3)\ t^3 - (1\ \mathrm{m}/s^2)\ t^2 + (1\ \mathrm{m/s})\ t + 2\ \mathrm{m}$

21. $x(t) = (2\ \mathrm{m}\ s^2)\ t^{-2} - (3\ \mathrm{m\ s})\ t^{-1}$

Part 5: calculate the area under the curve

22. $y(x) = x$ from $x = 0$ to $x = 1$

23. $y(x) = x$ from $x = -1$ to $x = 1$

24. $y(x) = 2x + 3$ from $x = 0$ to $x = 2$

25. $y(x) = 2x + 3$ from $x = -2$ to $x = 2$

26. $y(x) = 2x^2 + 3x + 2$ from $x = 0$ to $x = 2$

27. $y(x) = 2x^2 + 3x + 2$ from $x = -2$ to $x = 2$

Part 6: calculate the velocity given the acceleration

28. $a_x(t) = 2\ \mathrm{m}/s^2$ given $v_x(0\ s) = -3\ \mathrm{m}/s$

29. $a_x(t) = 2\ \mathrm{m}/s^2$ given $v_x(2\ s) = 1\ \mathrm{m}/s$

30. $a_y(t) = (4\ \mathrm{m}/s^3)\ t + 3\ \mathrm{m}/s^2$ given $v_y(0\ s) = 2\ \mathrm{m}/s$

Part 7: calculate the position given the acceleration

31. $a_x(t) = 2\ \mathrm{m}/s^2$ given $v_x(0\ s) = -3\ \mathrm{m}/s$ and $x(0\ s) = 1\ \mathrm{m}$

32. $a_x(t) = 2\ \mathrm{m}/s^2$ given $v_x(2\ s) = +1\ \mathrm{m}/s$ and $x(0\ s) = 1\ \mathrm{m}$

33. $a_y(t) = (2\ \mathrm{m}/s^3)\ t$ given $v_y(0\ s) = -3\ \mathrm{m}/s$ and $y(0\ s) = 1\ \mathrm{m}$

34. $a_y(t) = (2\ \mathrm{m}/s^3)\ t$ given $v_y(2\ s) = +3\ \mathrm{m}/s$ and $y(0\ s) = 1\ \mathrm{m}$

35. $a_y(t) = (2\ \mathrm{m}/s^3)\ t$ given $y(0\ s) = 1\ \mathrm{m}$ and $y(1\ s) = 2\ \mathrm{m}$