Note: You are free to use (e.g. copy verbatim) any text in black. All text in red indicates areas where you need to insert your own work.

Experiment #2: Emergency landing

Introduction:

A pilot of a malfunctioning airliner with inoperable landing gear and a malfunctioning navigation computer needs to make an emergency landing at the Denver airport located 100 miles due east of its current position. Two things must happen to maximize the possibility of survival in this unfortunate situation. The first, a necessary condition, is that the airplane must land at the airport. A landing anywhere else in the Rockies will end in total disaster with almost absolute certainty. A second condition, that the aircraft land with almost completely empty fuel tanks, is not an absolute requirement but is a condition that should be satisfied so that the pilot minimizes the chances of an explosion on landing.

The flight controls in the aircraft are malfunctioning such that the pilot must program the remaining flight path to the airport in exactly two straight-line segments. The computer will only accept a distance of 50 miles for the first leg. The current fuel on board gives the pilot a total range of 180 miles.

The following analysis presents a set of possible paths between the origin and destination, calculating the total distance traveled along those paths. An optimal path is suggested based on the due conditions of arriving at the airport and doing so with minimal remaining fuel.

Data and Analysis:

The only constraint used in this analysis is that the total eastward displacement across the 2 legs of any path must be d_0 = 100 miles due east, and that the distance traveled on the first leg of the path is exactly 50 miles. The pilot effectively has a single decision to make, which is the direction of the first leg, θ_1.

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Instructions: Insert a drawing showing the origin, destination and the two legs. Show on the drawing $\vec{d}_0$, $\vec{d}_1$, $\vec{d}_2$, θ_1, and θ_2.

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The unknown quantities in this analysis are therefore the two remaining parameters that described the second leg. These are d_2, the length of the second leg and θ_2, the angle of the second leg with respect to the eastward vector. The equations used to determine these parameters are as follows.

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Instructions: Insert a derivation of the equations you used to solve for d_2 and θ_2, starting from your knowledge of the total path between origin and destination as shown in your figure.

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Possible solutions to this system were considered by scanning a range of values for θ_1 and calculating the total distance, $d_{tot} = d_1 + d_2$, for each path. For this analysis, a total of <insert number> paths were chosen. These calculations were then compared to a set of "experimental" values based on sketches of the paths.

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Instructions: Fill out the following table for your paths.

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			EXPERIMENT		THEORY
Path number	θ_1	θ_2	d_2 [units]	d_{tot} [units]	d_{tot} [units]
1					
2					
…					

Conclusion:

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Instructions:

Answer the following questions in your concluding statement. Do not simply write these questions and provide an answer to them. Incorporate your answers in a written paragraph of your own composition.

1. What did you find for the parameters of the best path? Is your number exact, or was there some rounding involved? What range of paths would leave with the aircraft with 0-5 miles of fuel?

2. How did your theoretical analysis compare against your measurements? Was there good agreement? If not, what went wrong experimentally, or can you suggest a modification to the theory to make it better?

BONUS:
In the above work, we either drew a set of triangles or made a series of calculations for different angles θ_1. Instead of scanning these various trials to find the best one, is it possible to derive a single equation that provides the optimal angle θ_1? If so, show the derivation and give an example of how it works.

for other distances. Are there limitations on what ranges of distances allow you to succeed in satisfying both conditions?s

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