Note: You are free to use (e.g. copy verbatim) any text in black. All text in red indicates areas where you need to insert your own work.

Experiment #1: Geometric inference of π

Introduction:

A shape, broadly defined, is the pattern of line segments arranged to form a closed boundary. Regular polygons are those shapes that can be constructed from a repeated pattern of an isosceles triangle. The simplest of these is the equilateral triangle (n=3), followed by the square (n=4), pentagon (n=5), and so on, where n counts the number of sides. The limit of n $\rightarrow$ infinity is a vanishingly narrow isosceles triangle repeated an infinite number of times, a shape better known as the circle.

Absent from the definition of a shape is any mention of size. In terms of the geometric quality of shape, a small square and a large square are identical. This size-independent quality of shape is equivalent to the idea that regular polygons can be characterized by a shape parameter. One can define multiple parameters to describe the set of regular polygons. The most obvious choice is the number of sides (n). Another parameter could be the angle of the generating isosceles triangle, which is equal to 360°/n.

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Instructions:

Insert a picture of a polygon with the quantities you measured clearly defined.

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An alternative parameter for polygons is the ratio of the circumference of the shape to twice the "radius" of the polygon, a parameter that will here be referred to as Π. This parameter is interesting because it reaches an asymptotic value in the limit of n $\rightarrow$ infinity, the well-known number 3.14159… which is given the special symbol π. This ratio for the other regular polygons exists but is not known with such fame. The goal of this experiment is to construct regular polygons and directly measure these quantities to determine Π for a set of regular polygons. An approximation for π can be determined by extrapolating this series of measurements to the limit of infinite n.

Experimental setup:

A series of polygons with different n were constructed using a ruler and protractor. The set of polygons explored in this lab had n = <insert list>. Three measurements were made following the construction of each polygon: the total perimeter (circumference), the inner radius (measuring the distance from the center to the midpoint of each edge), and the outer radius (measuring the distance from the center to the vertex). With these measurements, two values of Π can be determined for each polygon: a Π_{inner} and a Π_{outer} formed as the ratio of the circumference divided by the inner our outer diameter, respectively.

Data and Analysis:
In the following tables C represents the circumference, r_{inner} the inner radius, and r_{outer} the outer radius. The angle is the angle of the generating triangle, equal to 360°/n.

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Instructions:

Insert the following table with your data:

n	central angle	C [mm]	r_{inner} [mm]	r_{outer} [mm]	Π_{inner}	Π_{outer}

A few notes about the entries in the data table:
1. The first column, n, is the number of sides of your polygon.
2. The second column, interior angle, is the central angle of each sub-triangle for your polygon. For example, if you were entering your data for the pentagon, you have 5 identical sub-triangles. The central angle of each is equal to 360°/5 = 72°, so you would enter 72° here.
3. The third column, C, is the circumference (or perimeter if you prefer), which is the total distance around the outside of the polygon.
4. The 4th and 5th columns are the radii that you measured.
5. The last two columns are your measurement of the ratio of C/2r.

Once you have your data nicely formatted, you are going to create two separate graphs. Each graph will have two curves on it. We are making two graphs because we want to compare whether there is a better way to present our data. There is no need to use Excel or anything like that for these simple graphs, please make these by hand.

Graph #1 shows Π_{inner} and Π_{outer} versus n (x-axis)

Graph #2 shows Π_{inner} and Π_{outer} versus 360°/n (x-axis).

A few notes about the graphs:
1. To make each graph useful, create them so that they use the full width of your page (don't cram two plots side by side).
2. Examine the data you want to present before defining the scale on your axes, then choose a scale so that you use the full width of the available plot area.
3. Each graph will have two sets of data on it, the data for Π_{inner} and Π_{outer}. Instructors in other classes may have told you not to connect the dots – we don't follow that idea here, and in fact there is good reason to connect the dots for Π_{inner} and, separately, the dots for Π_{outer}. It would look great if you connected them using different colors. We do this because they represent

different things and we want to be able to distinguish them from each other. Without a connecting line we would just see a mash of indistinguishable dots that wouldn't mean much.

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Conclusion:

A series of regular polygons were constructed using a protractor and ruler. For each shape the circumference, inner radius, and outer radius were measured. These measurements provided the necessary information to calculate the two values of Π for each shape. Estimates of π were determined from two plots of this data by considering the limit of Π as n $\rightarrow$ infinity. From the plot of Π versus n we estimate that $\pi \approx$ _______, and from the plot of Π versus 360°/n we estimate that $\pi \approx$ _______.

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Instructions:

Answer the following questions by writing a statement, perhaps a few sentences, that addresses these ideas. That is, do not write the questions explicitly and then answer them:

Which of the plots allowed you to better guess the value of Π for n$\rightarrow$ infinity? Why is this the case?

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