   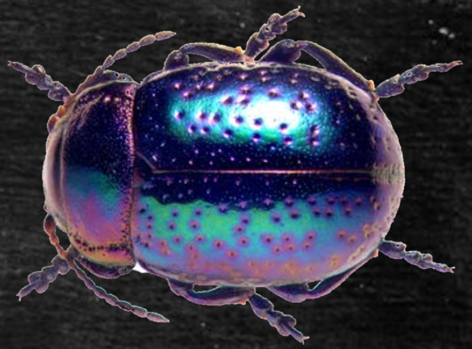

Vector math

Introduction

- We are now using vectors to describe the motion of objects in terms of position, velocity and acceleration. As vectors, these quantities take the following forms:

$\hat{x}, \hat{y}, \hat{z}$ have no units, they just indicate directions

$$\vec{r} = x\hat{x} + y\hat{y} + z\hat{z}$$

x, y, z have units of length

$$\vec{v} = v_x\hat{x} + v_y\hat{y} + v_z\hat{z}$$

v_x, v_y, v_z have units of length/time

$$\vec{a} = a_x\hat{x} + a_y\hat{y} + a_z\hat{z}$$

a_x, a_y, a_z have units of length/time^2

Question 1

- What is the vector dot product of $\hat{x}$ and $\hat{y}$?

(a) +1

(b) -1

(c) 0

(d) cannot be defined for unit vectors

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- What is the vector dot product of $\hat{x}$ and $\hat{y}$?

(a) +1 (c) 0

(b) -1 (d) cannot be defined for unit vectors

- Answer: (b)

- Explanation: $\hat{x}$ and $\hat{y}$ are orthogonal (perpendicular) and the dot product of <u>any</u> two vectors that are orthogonal is always zero.

Question 2

- Calculate the dot product for the following vectors:

$\vec{A} = 3\hat{x} - 4\hat{y}$

$\vec{B} = 8\hat{x} + 5\hat{y}$

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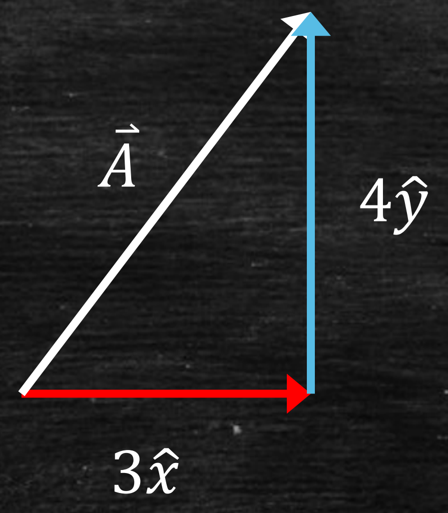
- Answer:

$\vec{A} \cdot \vec{B} = (+3)(+8) + (-4)(+5) = 24 - 20 = 4$

Question 3

- How can you use the dot product to calculate the length of vector $\vec{A}$?

$\vec{A} = 3\hat{x} + 4\hat{y}$



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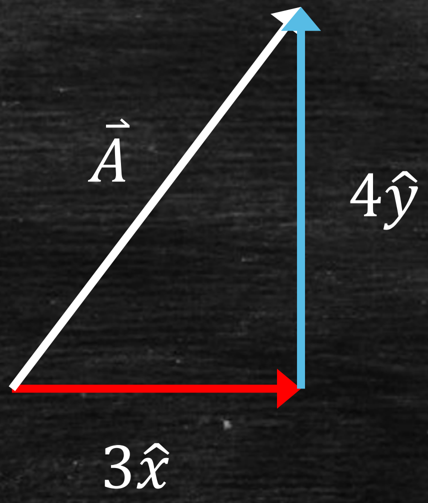


- Answer:

$\vec{A} \cdot \vec{A} = (+3)(+3) + (+4)(+4) = 3^2 + 4^2 = 25$

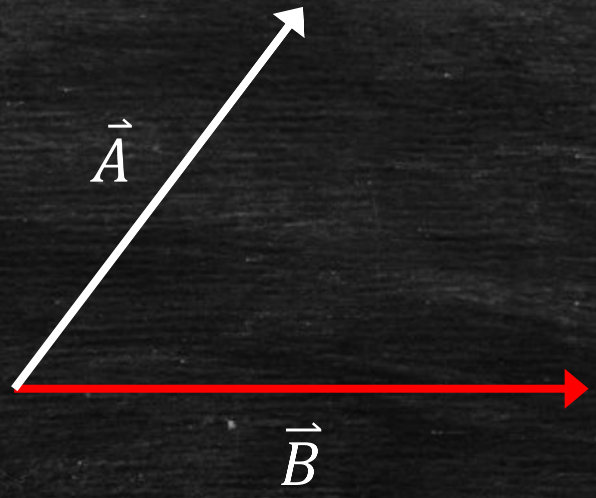
$\vec{A} \cdot \vec{A} = L^2 \quad \Longrightarrow \quad L = \sqrt{\vec{A} \cdot \vec{A}} = 5$

Question 4

- What is the vector dot product of vectors $\vec{A}$ and $\vec{B}$?

$\vec{A} = 3\hat{x} + 4\hat{y}$

$\vec{\mathrm{B}} = 6\hat{x}$



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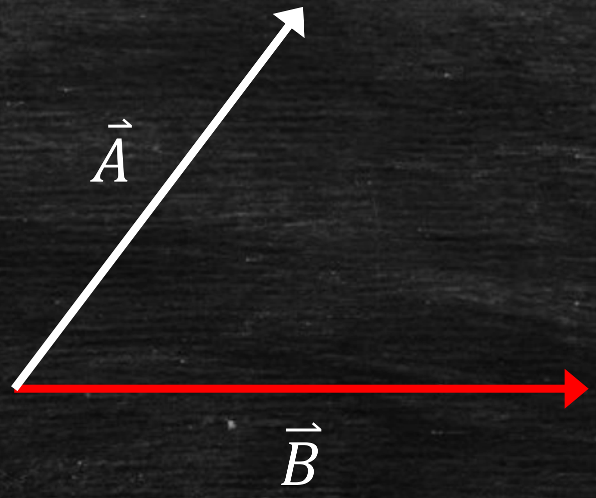


- Answer:

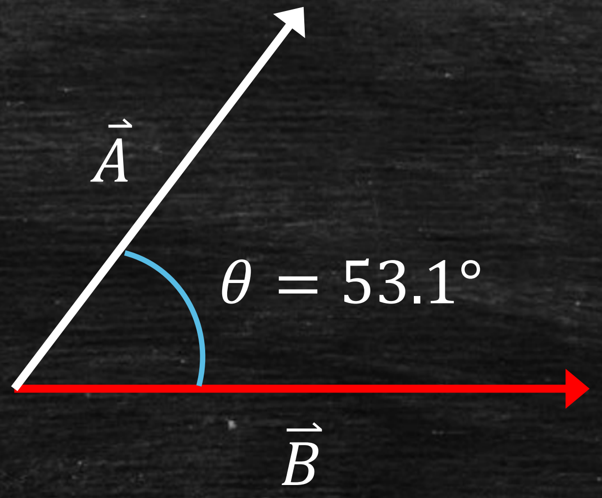
$\vec{A} \cdot \vec{B} = (+3)(+6) + (+4)(0) = 18 + 0 = 18$

Question 5

- What is the vector dot product of vectors $\vec{A}$ and $\vec{B}$?

$|\vec{A}| = 5$

$\vec{B} = 6\hat{x}$



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- What is the vector dot product of vectors $\vec{A}$ and $\vec{B}$?

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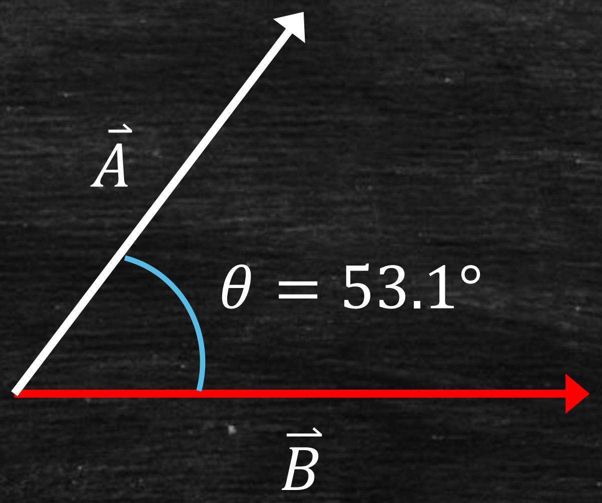


- Answer:

$$\vec{A} \cdot \vec{B} = |\vec{A}||\vec{B}| \cos(\theta) = (5)(6) \cos(53.1°) = (30)(0.6) = 18$$

Question 6

- What is the vector cross product of $\hat{x} \times \hat{y}$?

(a) $+1$ (c) $+\hat{z}$ (e) 0

(b) -1 (d) $-\hat{z}$ (f) ∞

Question 6

- What is the vector cross product of $\hat{x} \times \hat{y}$?

(a) $+1$ (c) $+\hat{z}$ (e) 0

(b) -1 (d) $-\hat{z}$ (f) ∞

- Answer: (c)

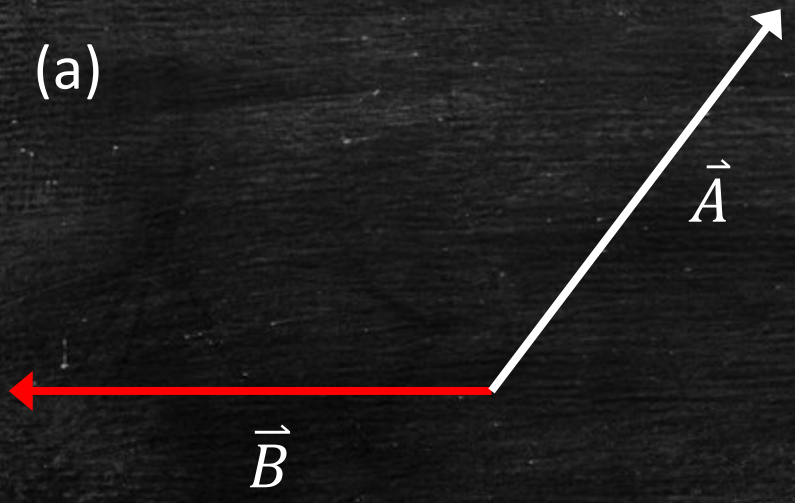
For two distinct vectors, the vector cross product always produces a new vector that is <u>perpendicular to both</u> of the vectors.

The direction can be found using the right hand rule.

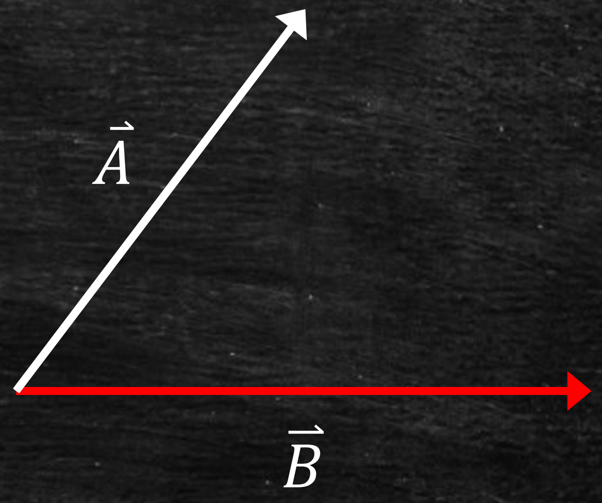
Question 7

- For each of the following, sketch the direction of $\vec{C}$ for $\vec{C} = \vec{A} \times \vec{B}$.

(a)



(b)



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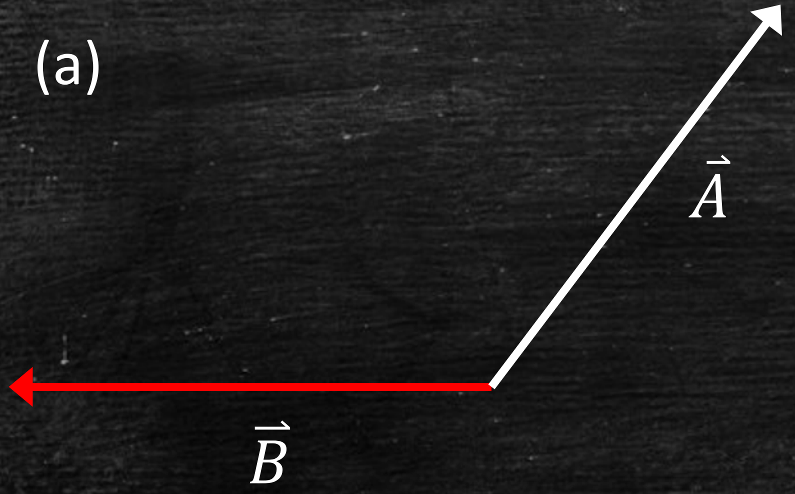
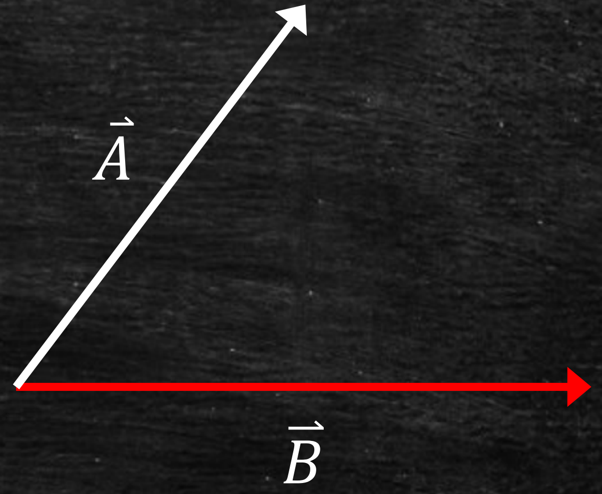


(b)



- Answers:

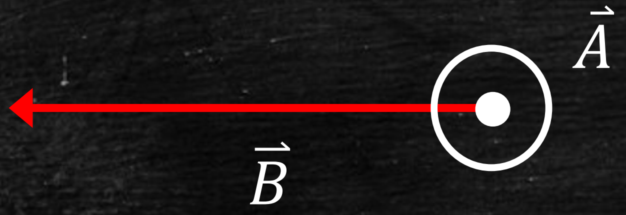
(a)

(b)

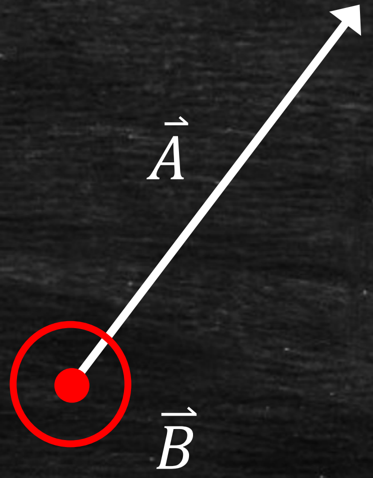
Question 8

- For each of the following, sketch the direction of $\vec{C}$ for $\vec{C} = \vec{A}\times\vec{B}$.

(a)



(b)



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- For each of the following, sketch the direction of $\vec{C}$ for $\vec{C} = \vec{A} \times \vec{B}$.

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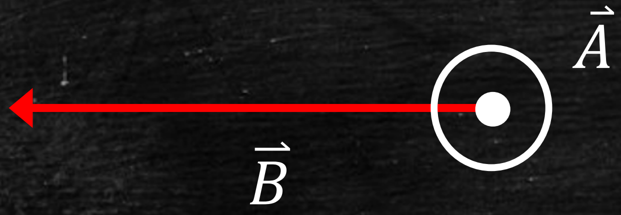
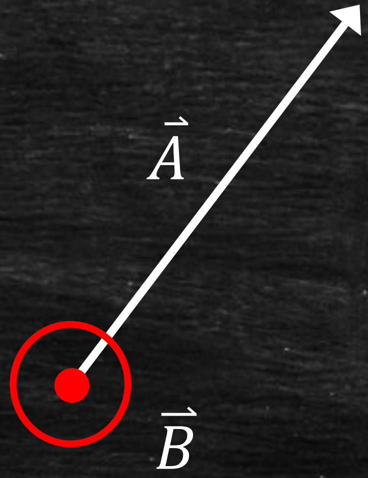


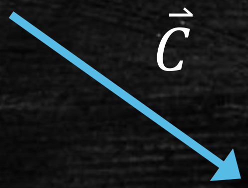
(b)



- Answers:

(a)

(b)

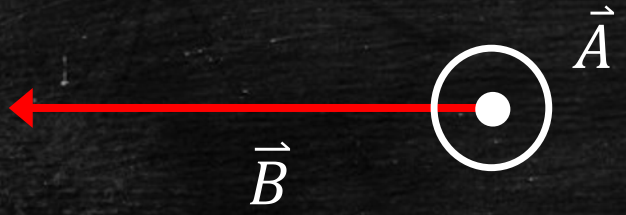


Question 9

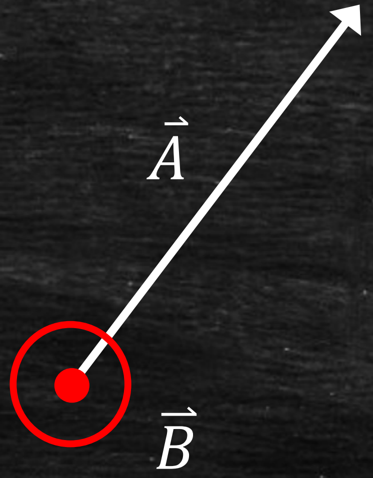
$|\vec{A}| = 5 \quad |\vec{B}| = 4$

- For each of the following, calculate the magnitude of $\vec{C}$ for $\vec{C} = \vec{A} \times \vec{B}$.

(a)



(b)



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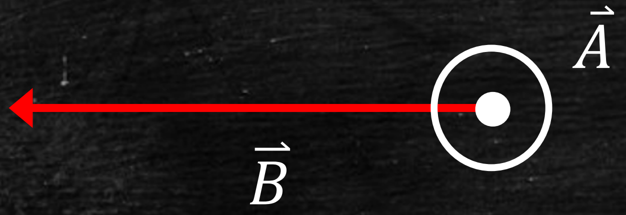
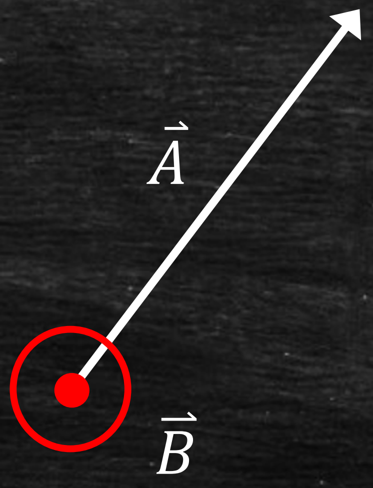
(a)



(b)



- Answer:

(a) $|\vec{C}| = |\vec{A}||\vec{B}|\sin(\theta)$
$= (5)(4)\sin(90°)$
$= 20$

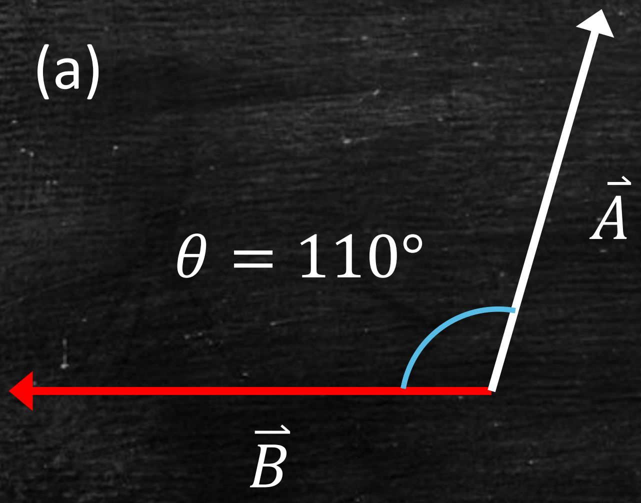
(b) $|\vec{C}| = 20$

Question 10

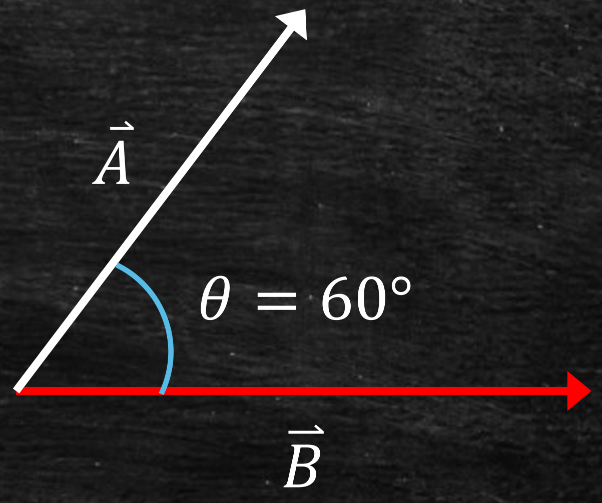
$|\vec{A}| = 5 \qquad |\vec{B}| = 4$

- For each of the following, calculate the magnitude of $\vec{C}$ for $\vec{C} = \vec{A} \times \vec{B}$.

(a)



(b)



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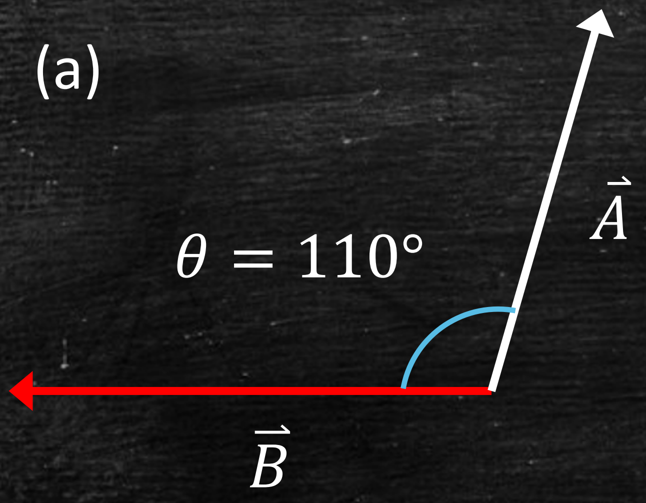
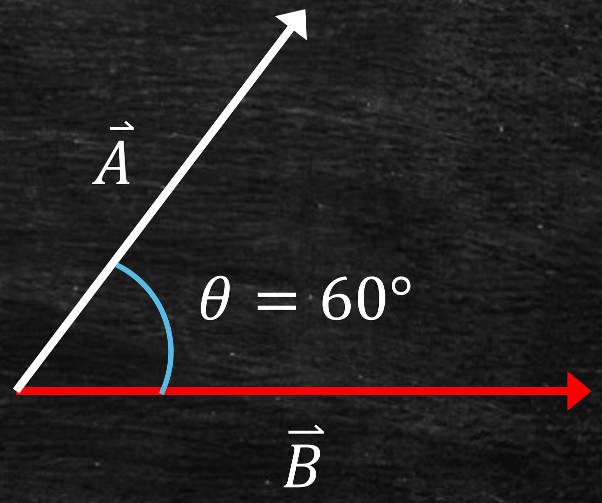
- For each of the following, calculate the magnitude of $\vec{C}$ for $\vec{C} = \vec{A} \times \vec{B}$.

(a)



(b)



- Answer:

(a)
$$|\vec{C}| = |\vec{A}||\vec{B}|\sin(\theta) = (5)(4)\sin(110°) = (20)(0.939) = 18.8$$

(b)
$$|\vec{C}| = |\vec{A}||\vec{B}|\sin(\theta) = (5)(4)\sin(60°) = (20)(0.866) = 17.3$$

Group Work

- Solve for the components of vector $\vec{B}$ given the constraints:
  (a) $\vec{A} \cdot \vec{B} = 0$
  (b) $\vec{A} = 3\hat{x} - 4\hat{y} + 2\hat{z}$
  (c) $\vec{B}$ must be in the x-y plane
  (d) $|\vec{B}| = 10$