

Velocity vectors

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# Introduction

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- We are now using vectors to describe the motion of objects in terms of position, velocity and acceleration. As vectors, these quantities take the following forms:

$$\vec{r} = x\hat{x} + y\hat{y} + z\hat{z}$$

$$\vec{v} = v_x\hat{x} + v_y\hat{y} + v_z\hat{z}$$

$$\vec{a} = a_x\hat{x} + a_y\hat{y} + a_z\hat{z}$$

$\hat{x}, \hat{y}, \hat{z}$  have no units, they just indicate directions

$x, y, z$  have units of length

$v_x, v_y, v_z$  have units of length/time

$a_x, a_y, a_z$  have units of length/time<sup>2</sup>



## Question 1

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- What conditions must be true if the instantaneous velocity is always equal to the average velocity?



# Question 1

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- What conditions must be true if the instantaneous velocity is always equal to the average velocity?

- Answer:

The two vectors must be the same. That means they must have the same magnitude and direction. We can therefore say that when the average and instantaneous speed are equal:

speed is constant

direction is constant



## Question 2

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- Give an example of the following situations:
  - (a) average velocity and instantaneous velocity are the same
  - (b) speed is changing but direction is constant
  - (c) speed is constant but direction is changing



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  - (a) average velocity and instantaneous velocity are the same
  - (b) speed is changing but direction is constant
  - (c) speed is constant but direction is changing
  
- Possible answers:
  - (a) a car traveling on a straight road at constant speed
  - (b) a car accelerating from rest while traveling on a straight road
  - (b) a car rounding a corner without using the brakes



## Question 3

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- If you walk two miles due east in one hour, what is your velocity vector?



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- Answer:

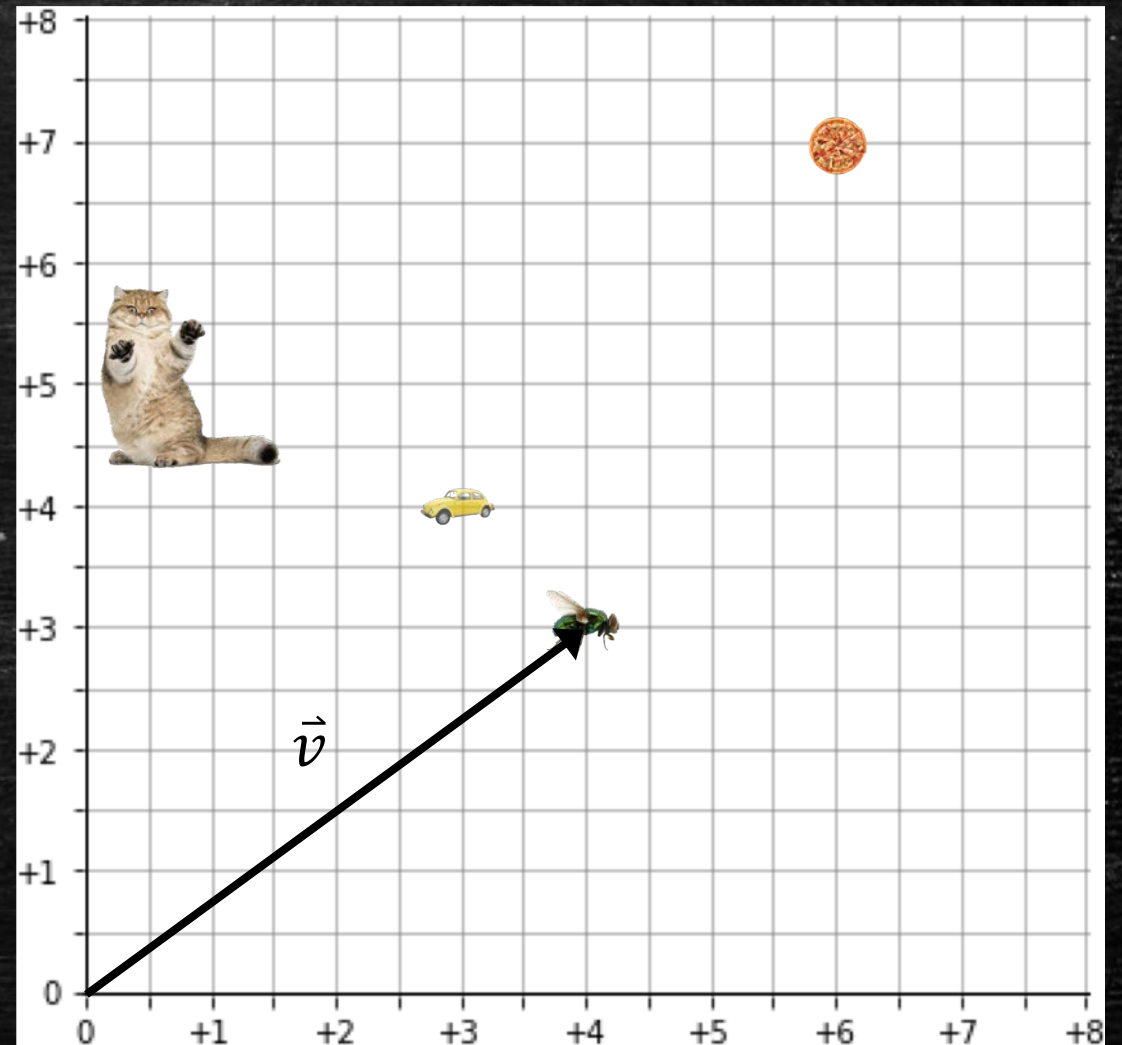
$$\vec{v} = \left( +2.0 \frac{\text{mi}}{\text{hr}} \right) \hat{x} + \left( +0.0 \frac{\text{mi}}{\text{hr}} \right) \hat{y}$$



# Question 4

All distances measured in miles

- (a) What is the fly's velocity vector (in units of miles/hr) if it left the origin and arrived at the shown position in 30 minutes, assuming that it traveled in a straight line?



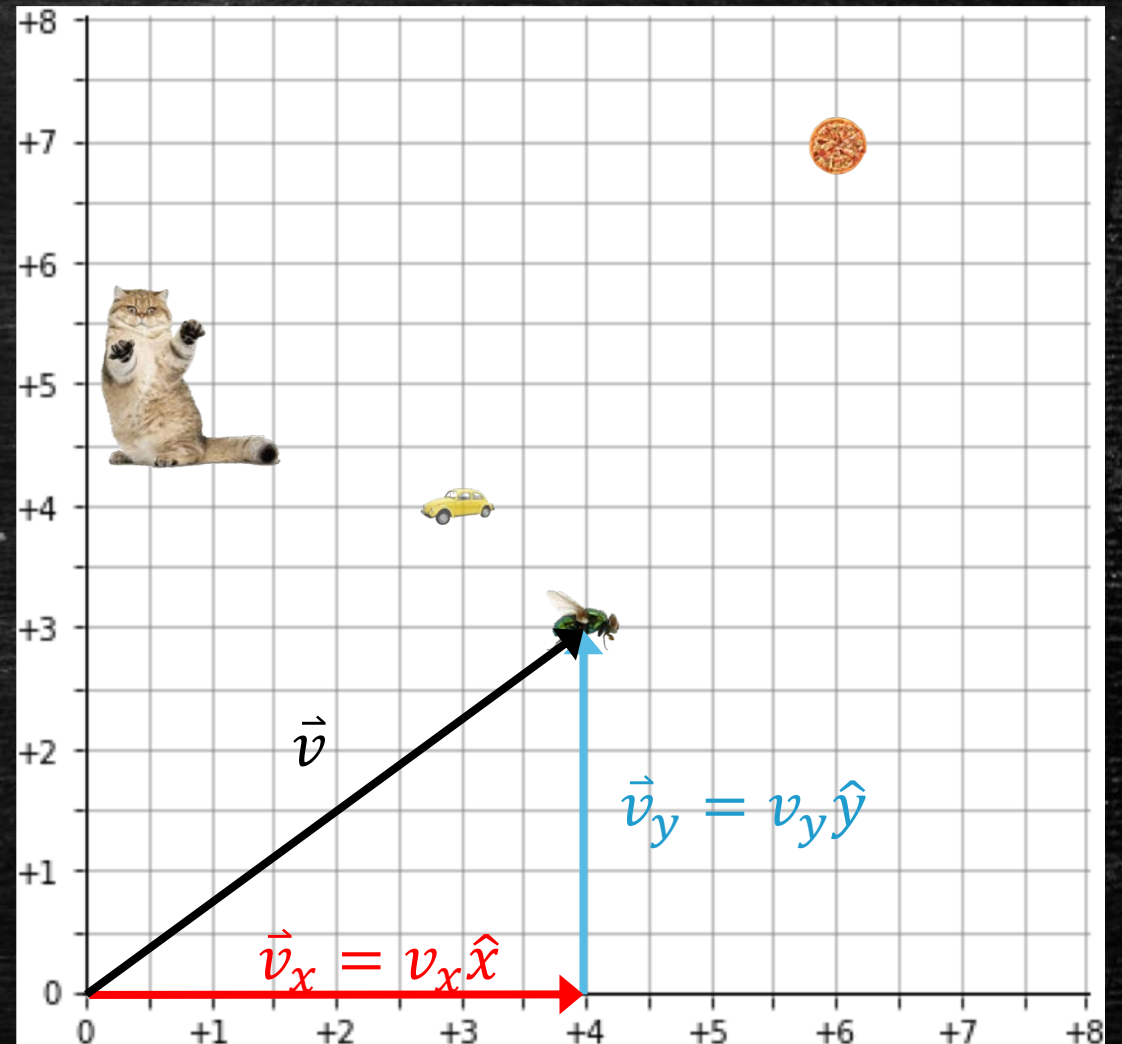


# Question 4

All distances measured in miles

- (a) What is the fly's velocity vector (in units of miles/hr) if it left the origin and arrived at the shown position in 30 minutes, assuming that it traveled in a straight line?

- Answer:
- $\vec{v} = (+8 \text{ mi/hr } \hat{x}) + (+6 \text{ mi/hr } \hat{y})$



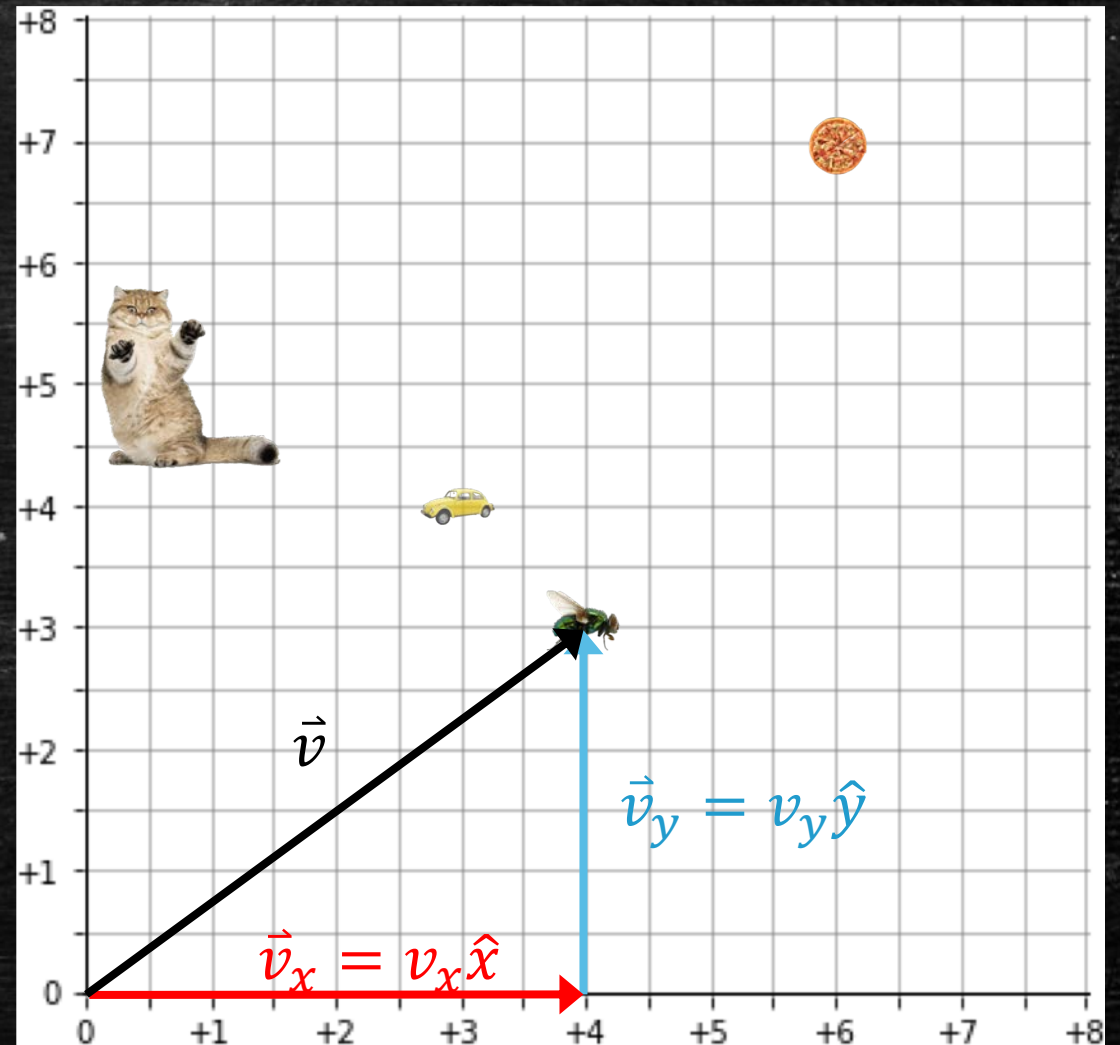


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- (b) What is the the speed of the fly?





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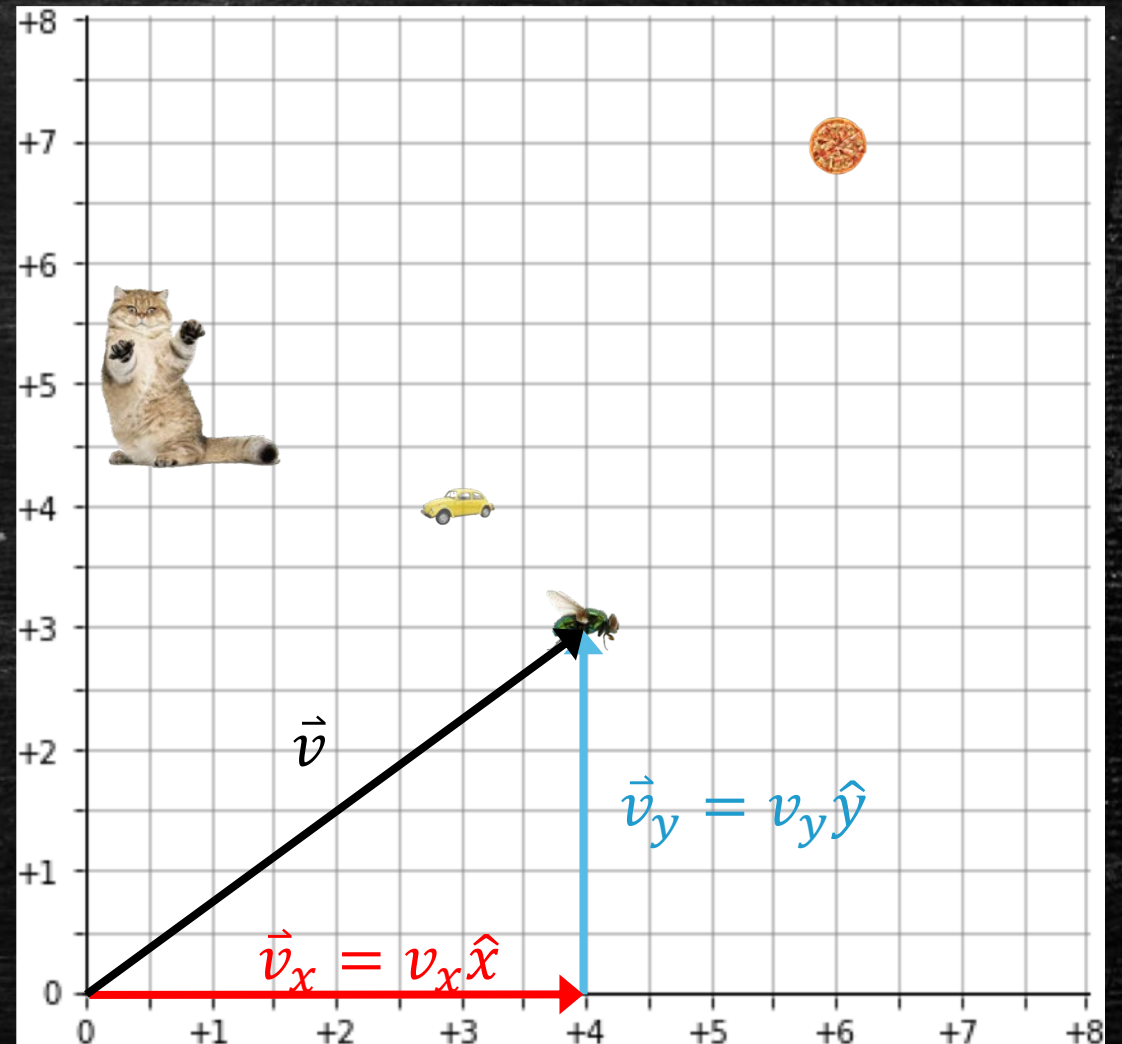
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▪ Answer:

▪  $\vec{v} = (+8 \text{ mi/hr } \hat{x}) + (+6 \text{ mi/hr } \hat{y})$

▪ (b) What is the speed of the fly?

▪ Answer: 10 mi/hr

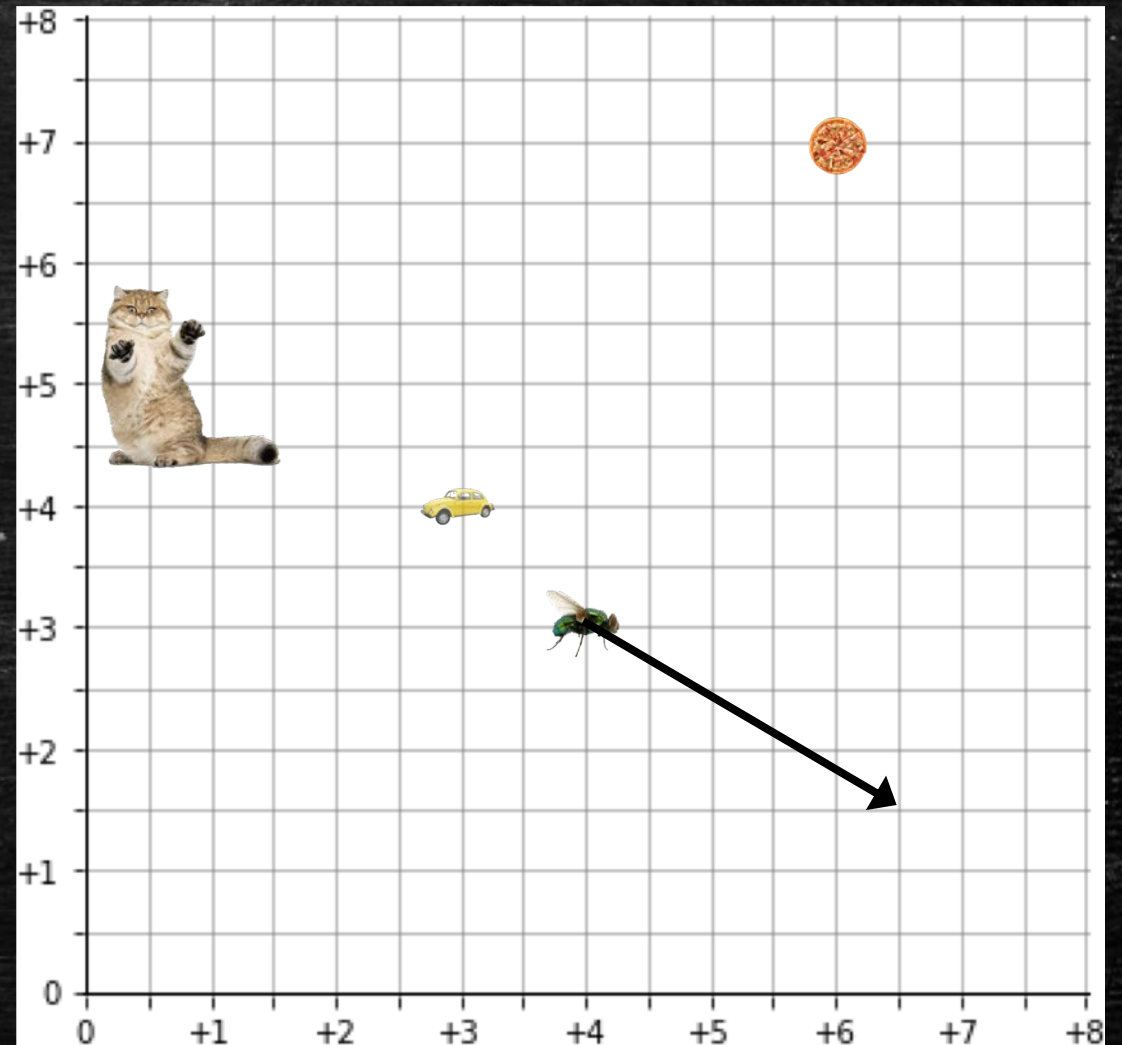




# Question 5

All distances measured in miles

- If the fly continues to move at this same speed and at an angle of  $329^\circ$  relative to the horizontal for 17.5 minutes, what are ...  
(a) the x & y components of the velocity,





# Question 5

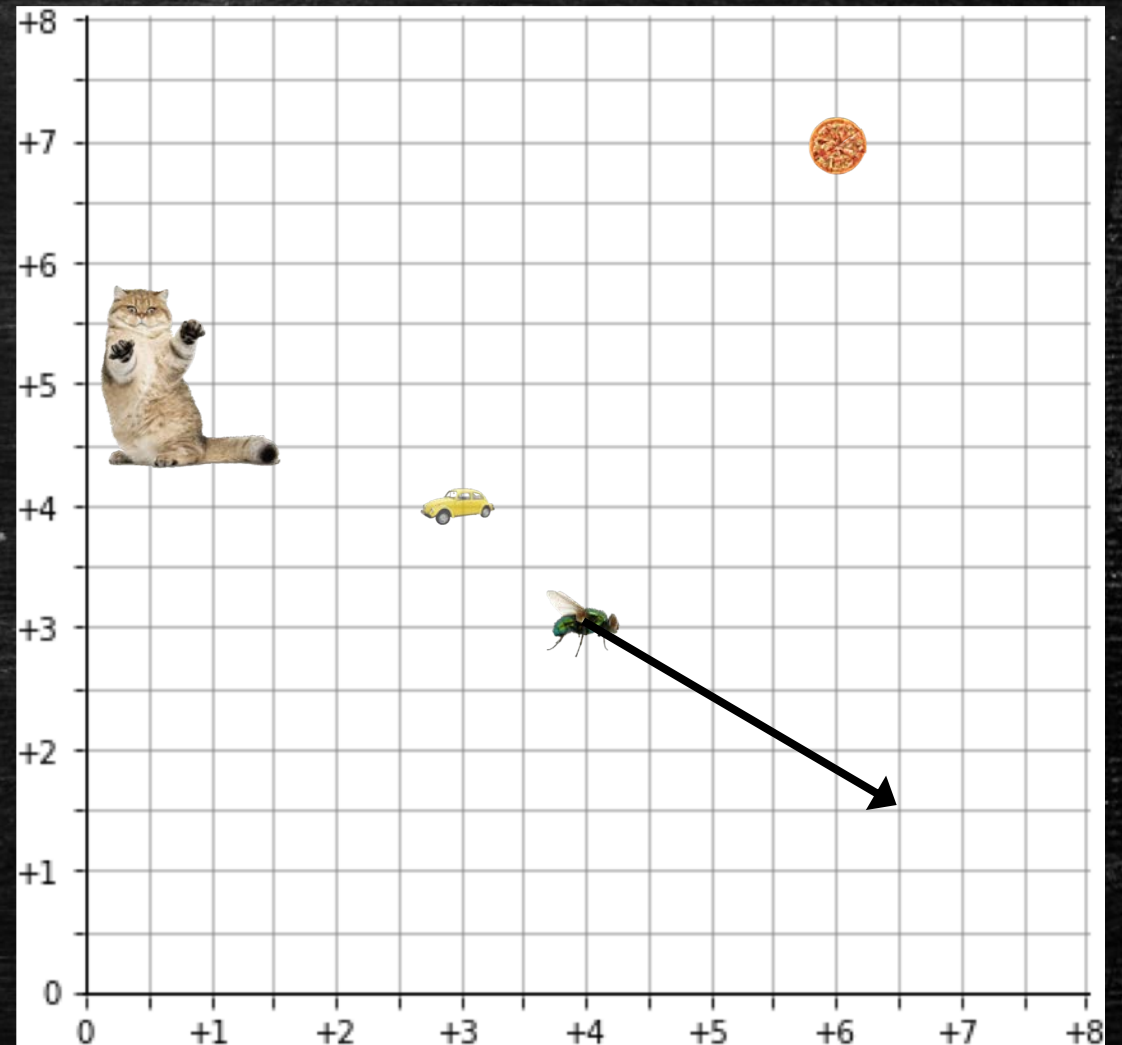
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(a) the x & y components of the velocity,

- Answers:

(a)  $v_x = +8.6$  mi/hr,  $v_y = -5.2$  mi/hr





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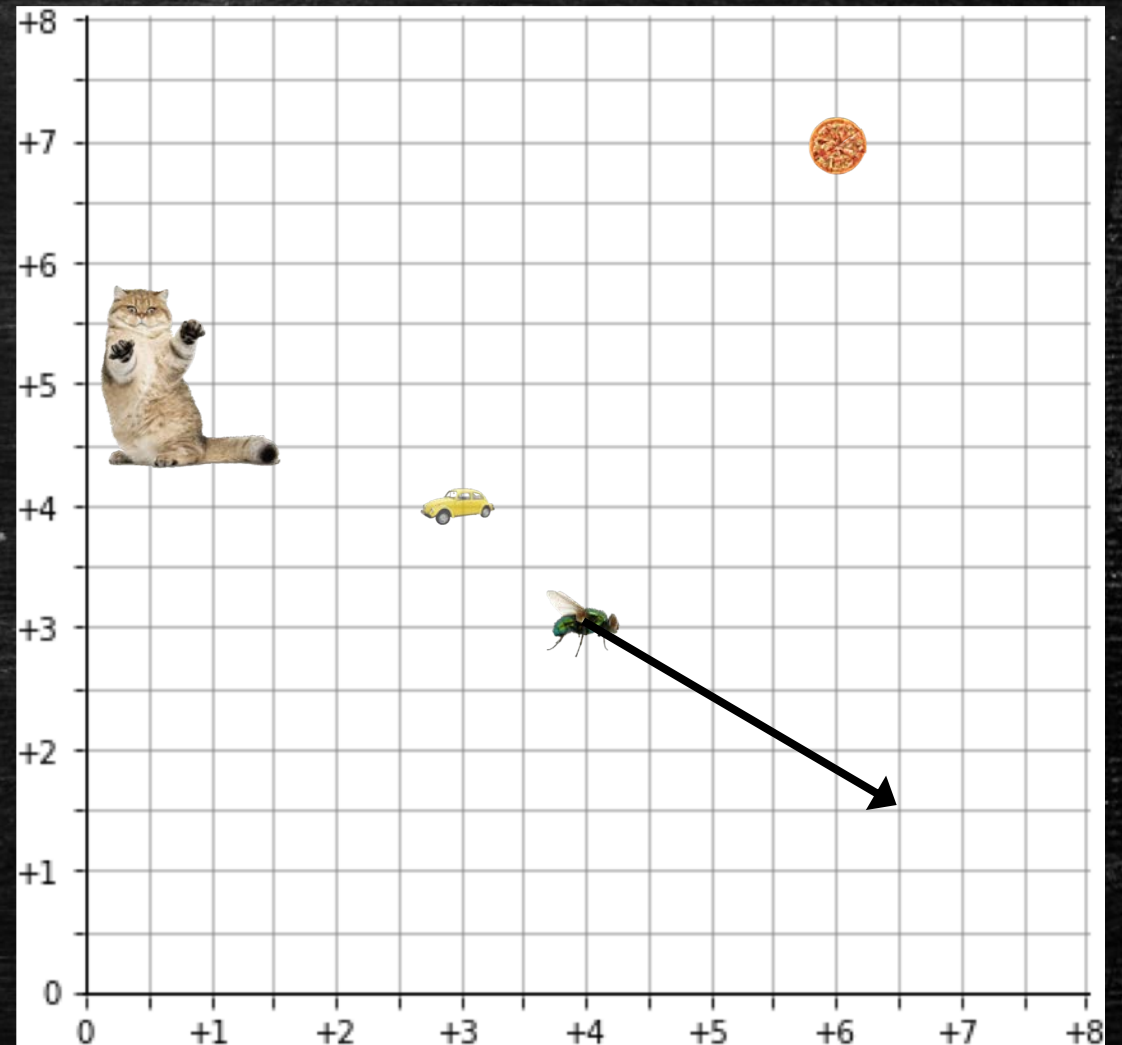
All distances measured in miles

- If the fly continues to move at this same speed and at an angle of  $329^\circ$  relative to the horizontal for 17.5 minutes, what are ...

- (a) the x & y components of the velocity,
- (b) the relative displacement of the fly, and

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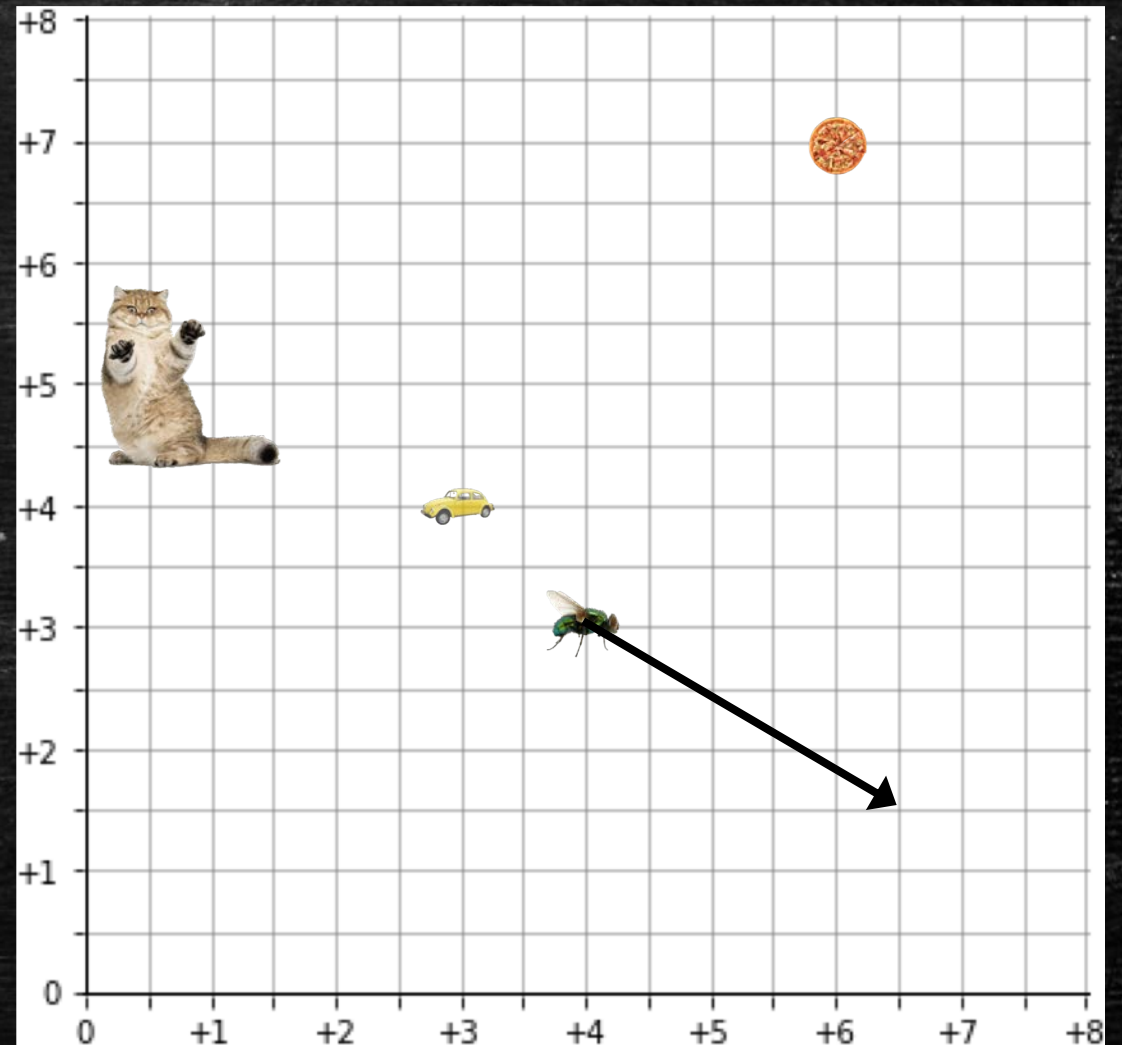
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- Answers:

(a)  $v_x = +8.6$  mi/hr,  $v_y = -5.2$  mi/hr

(b)  $\Delta x = +2.5$  mi,  $\Delta y = -1.5$  mi

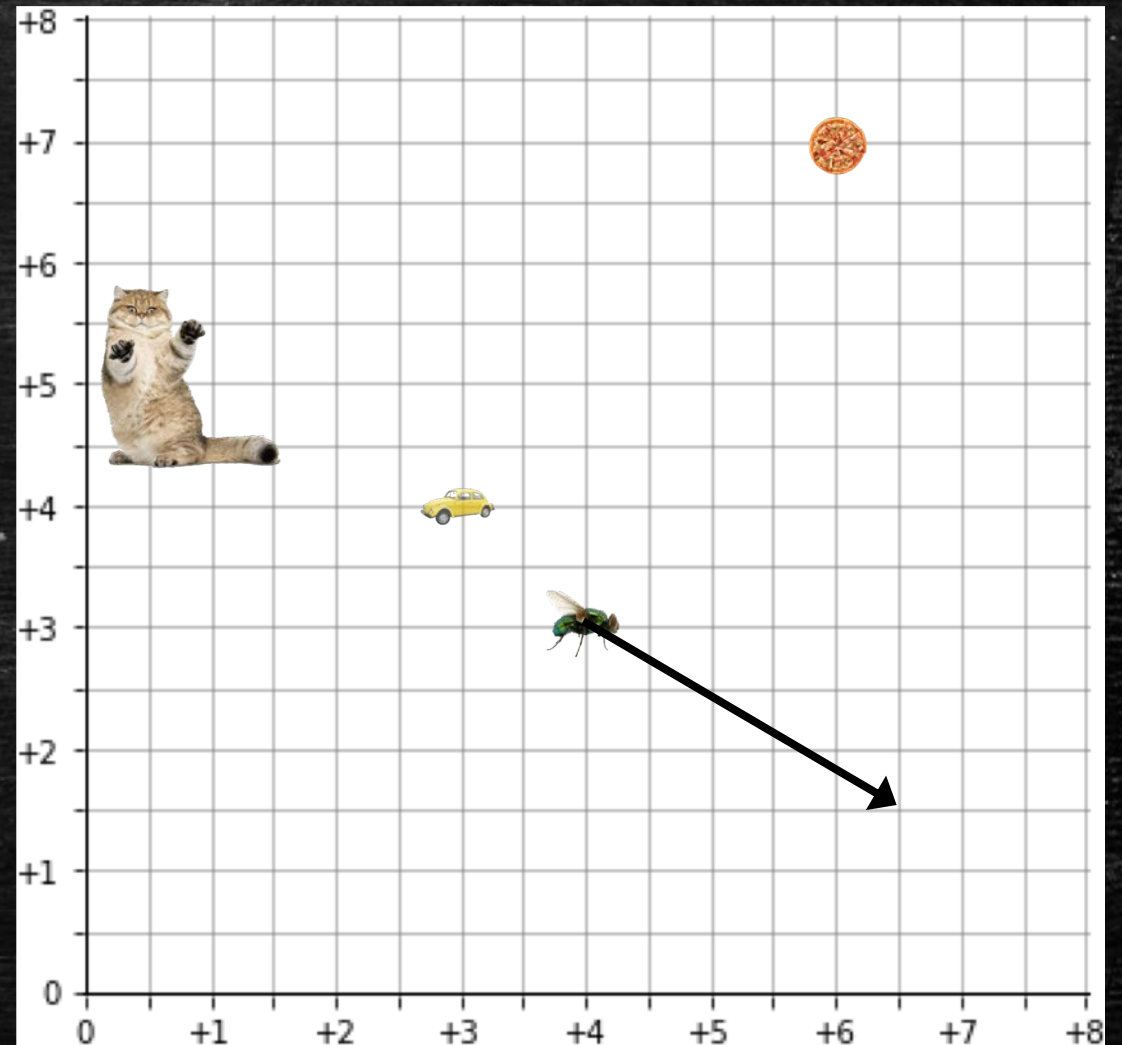




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  - (a) the x & y components of the velocity,
  - (b) the relative displacement of the fly, and
  - (c) the final position of the fly?
- Answers:
  - (a)  $v_x = +8.6$  mi/hr,  $v_y = -5.2$  mi/hr
  - (b)  $\Delta x = +2.5$  mi,  $\Delta y = -1.5$  mi





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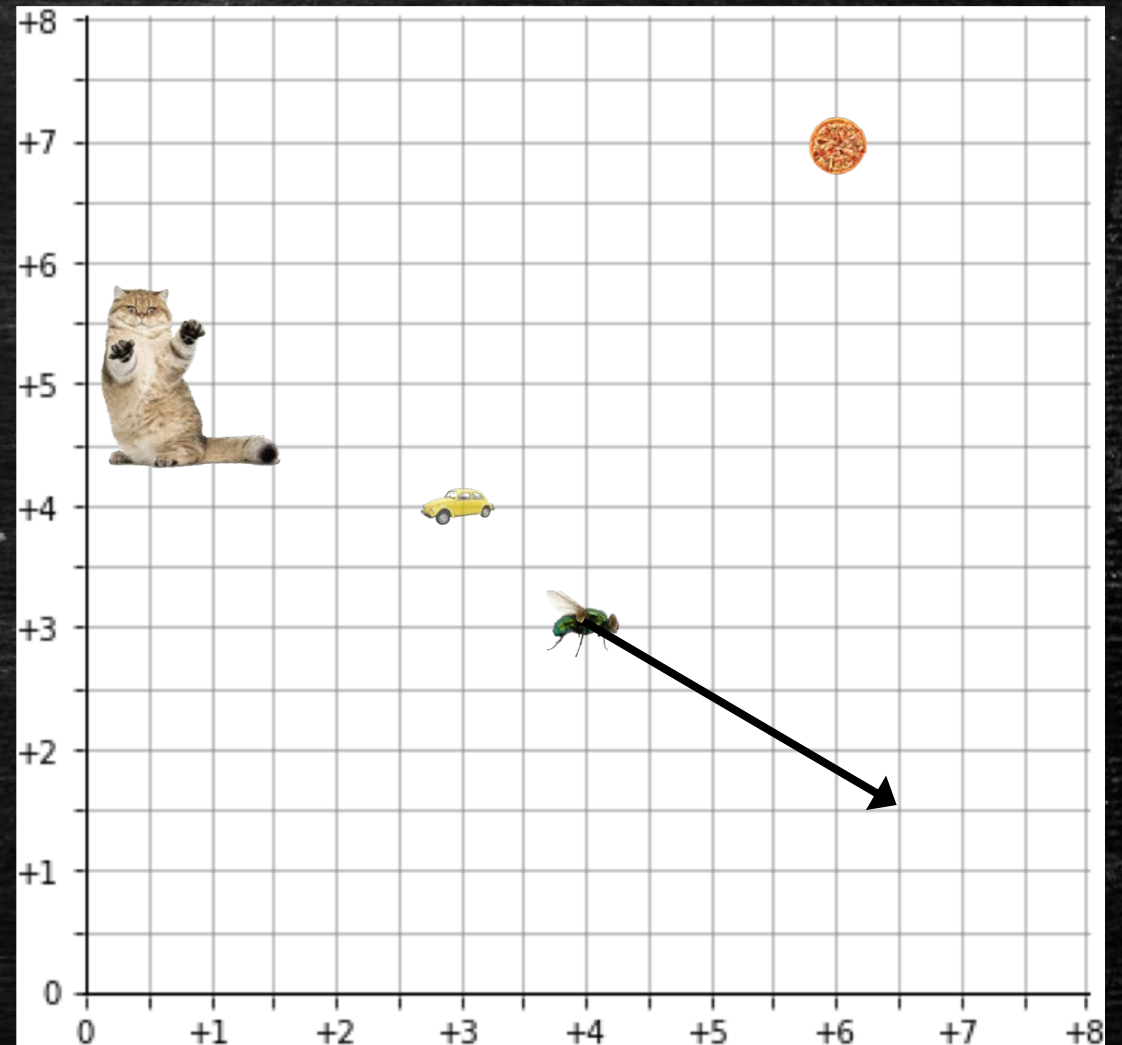
- (a) the x & y components of the velocity,
- (b) the relative displacement of the fly, and
- (c) the final position of the fly?

- Answers:

(a)  $v_x = +8.6$  mi/hr,  $v_y = -5.2$  mi/hr

(b)  $\Delta x = +2.5$  mi,  $\Delta y = -1.5$  mi

(c)  $x_{\text{final}} = +6.5$  mi,  $y_{\text{final}} = +1.5$  mi



## Question 6

All distances measured in miles

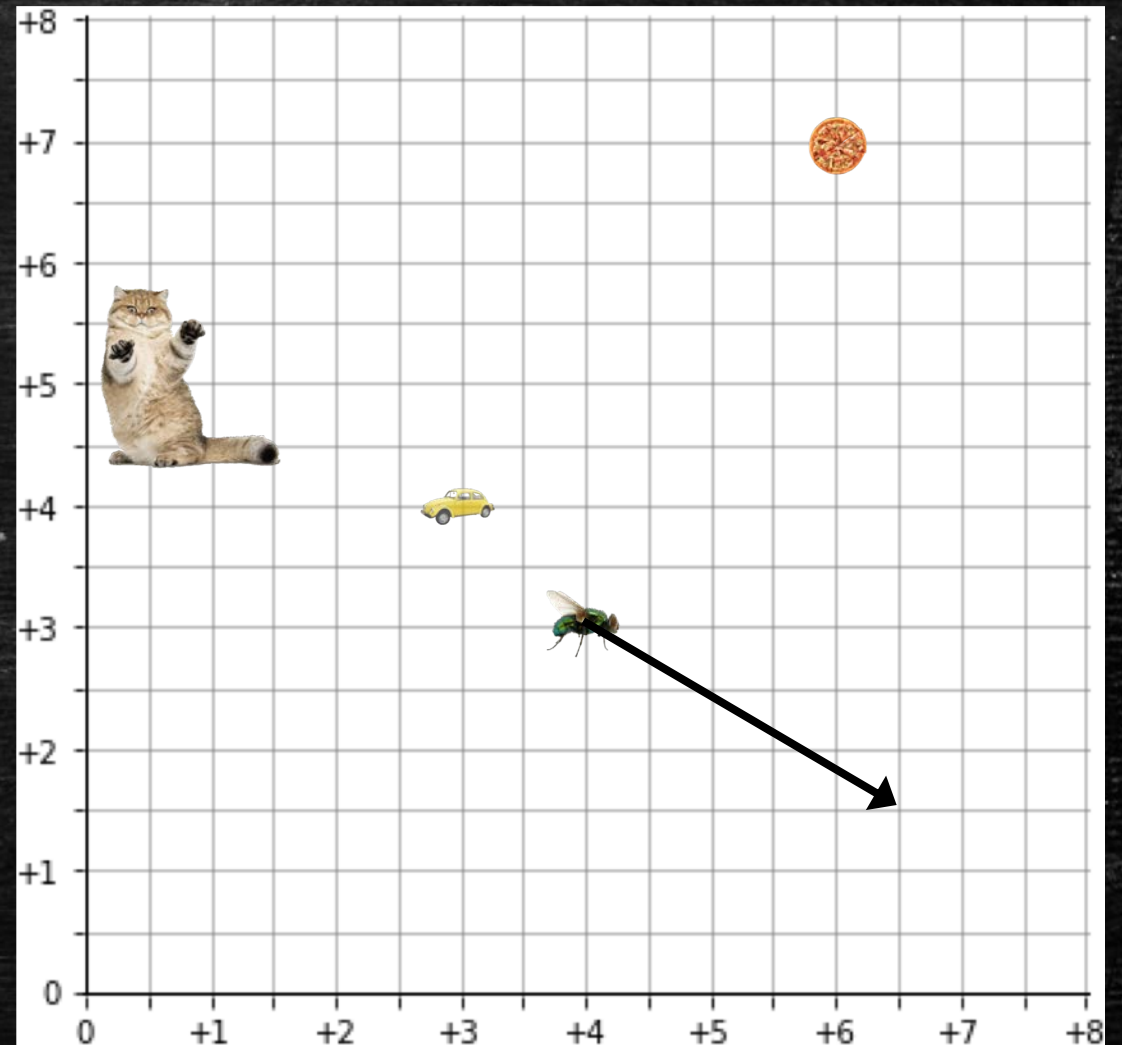
- Express all of the previous answers as vectors.

- Answers:

$$(a) \vec{v} = \left( +8.6 \frac{\text{mi}}{\text{hr}} \right) \hat{x} + \left( -5.2 \frac{\text{mi}}{\text{hr}} \right) \hat{y}$$

$$(b) \Delta \vec{r} = (+2.5 \text{ mi}) \hat{x} + (-1.5 \text{ mi}) \hat{y}$$

$$(c) \vec{r}_{final} = (+6.5 \text{ mi}) \hat{x} + (+1.5 \text{ mi}) \hat{y}$$

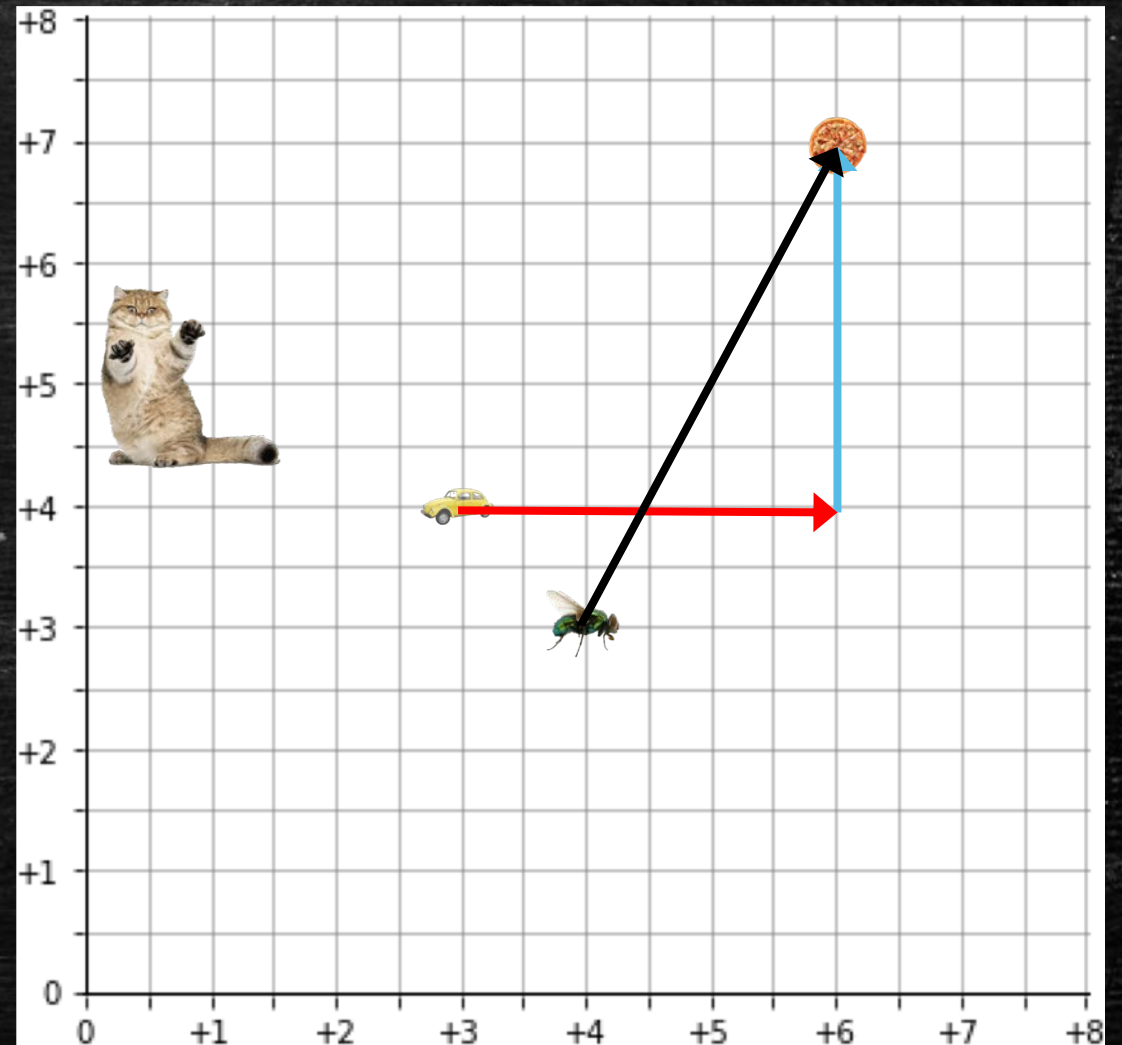




# Question 7

All distances measured in miles

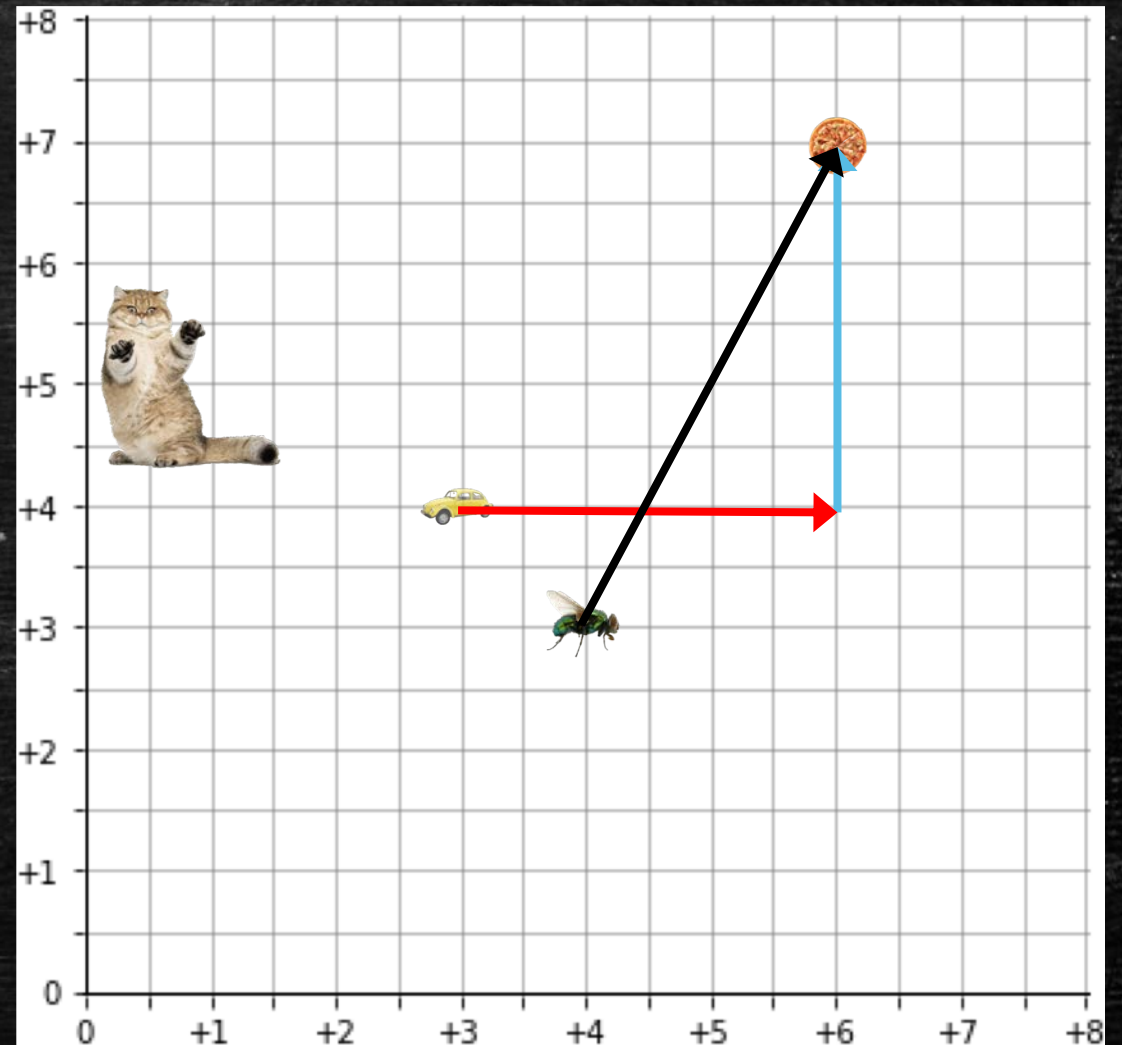
- (a) If the beetle can travel with an average speed of 12.0 mph through traffic on city streets, and the beetle and the fly leave their current positions at the same time, which one arrives at the pizza first?



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- Answer: the fly

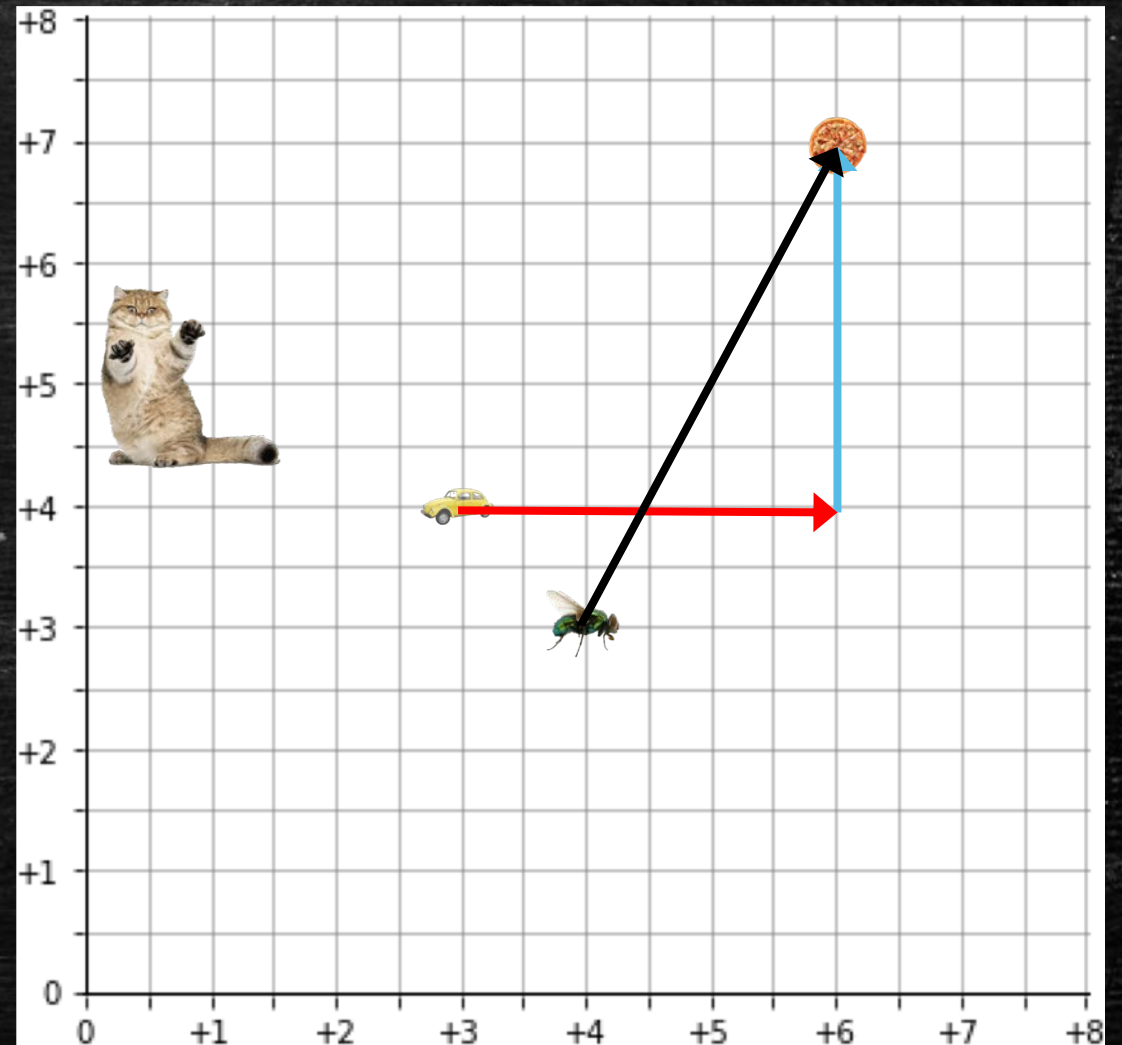




## Question 7

All distances measured in miles

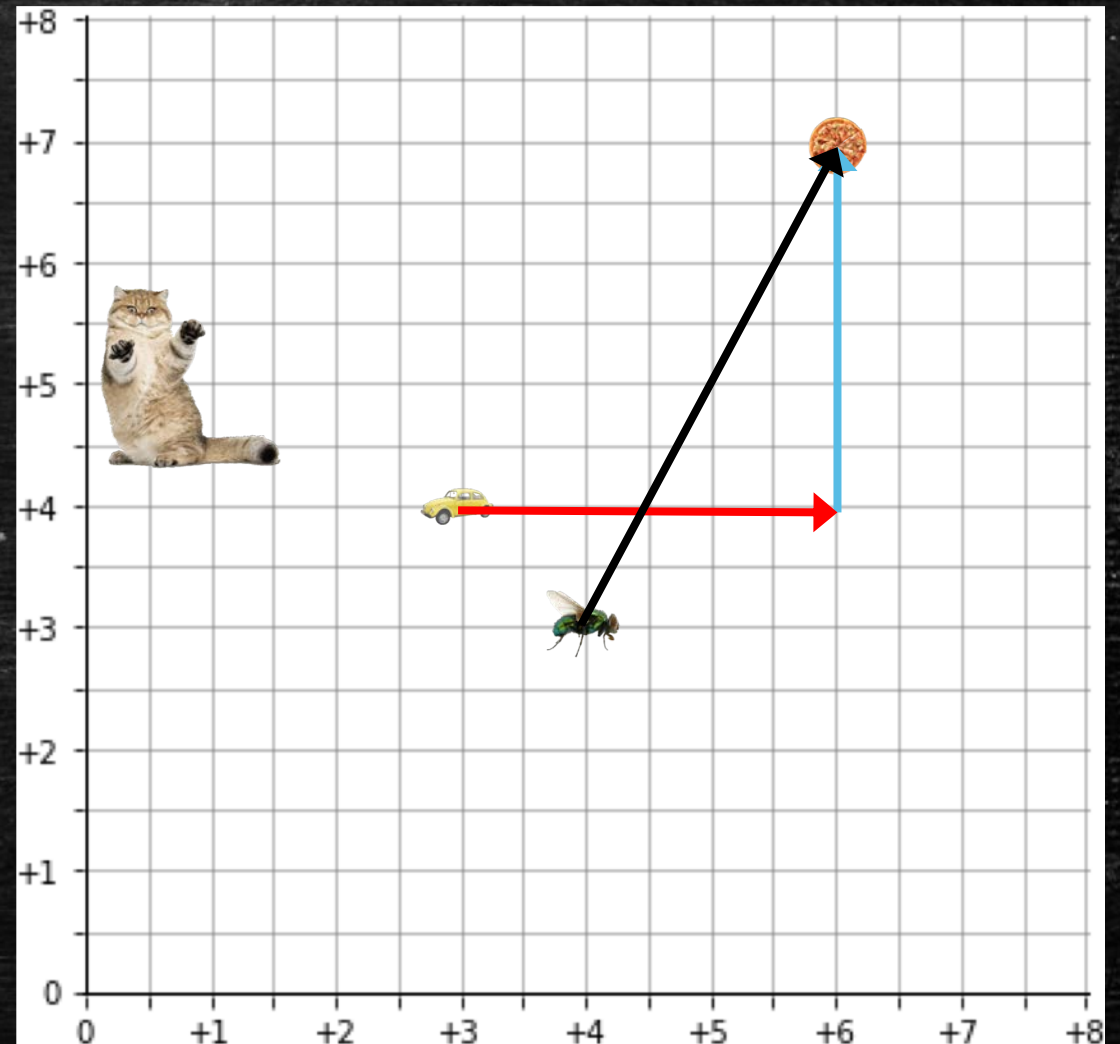
- (a) If the beetle can travel with an average speed of 12.0 mph through traffic on city streets, and the beetle and the fly leave their current positions at the same time, which one arrives at the pizza first?
- Answer: the fly
- (b) How fast would the beetle need to be able to travel in order to just beat the fly to the pizza?



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- Answer: the fly
- (b) How fast would the beetle need to be able to travel in order to just beat the fly to the pizza?
- Answer: speed  $> 13.4$  mi/hr

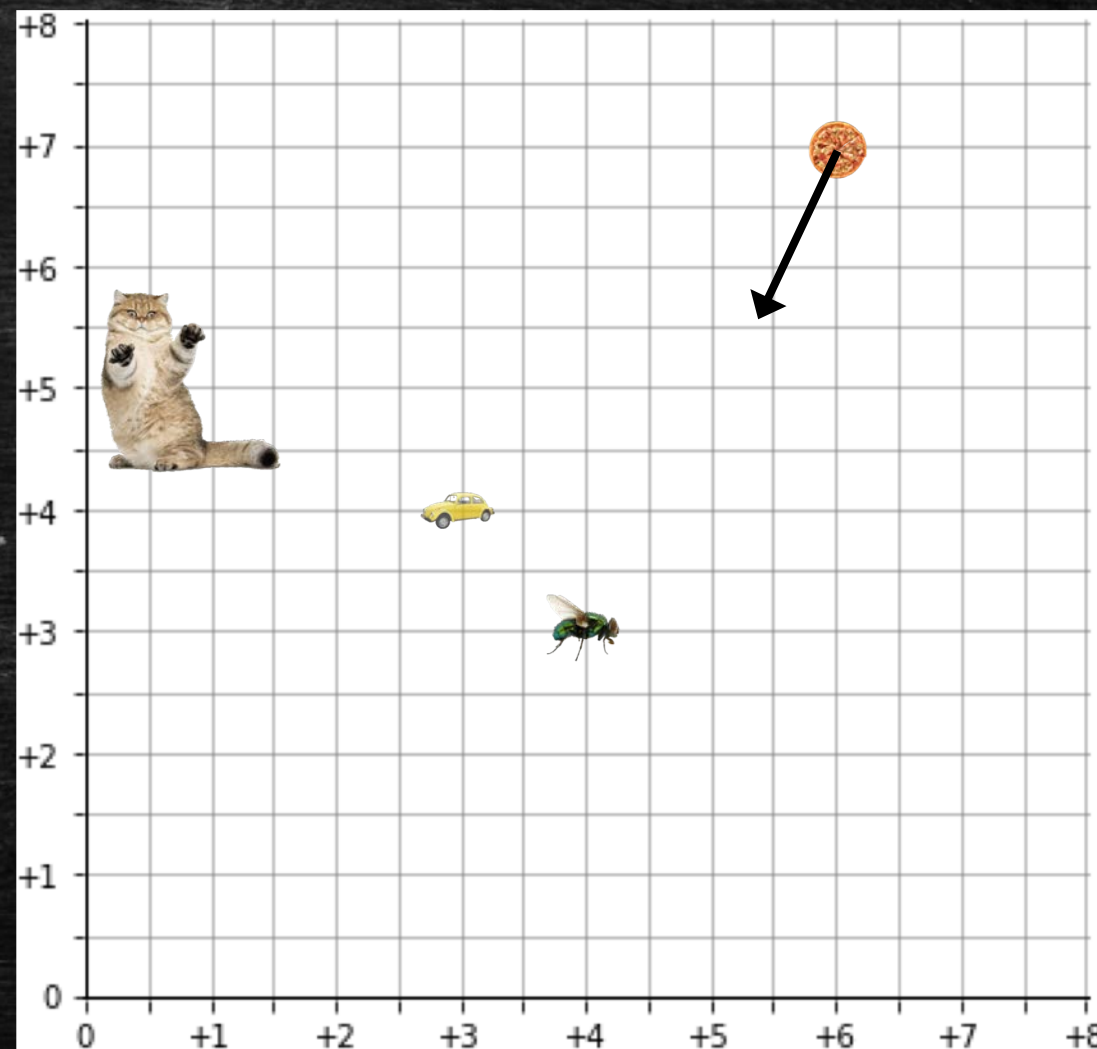




# Question 8

All distances measured in miles

- According to the fly, with what velocity is the pizza coming toward the fly?

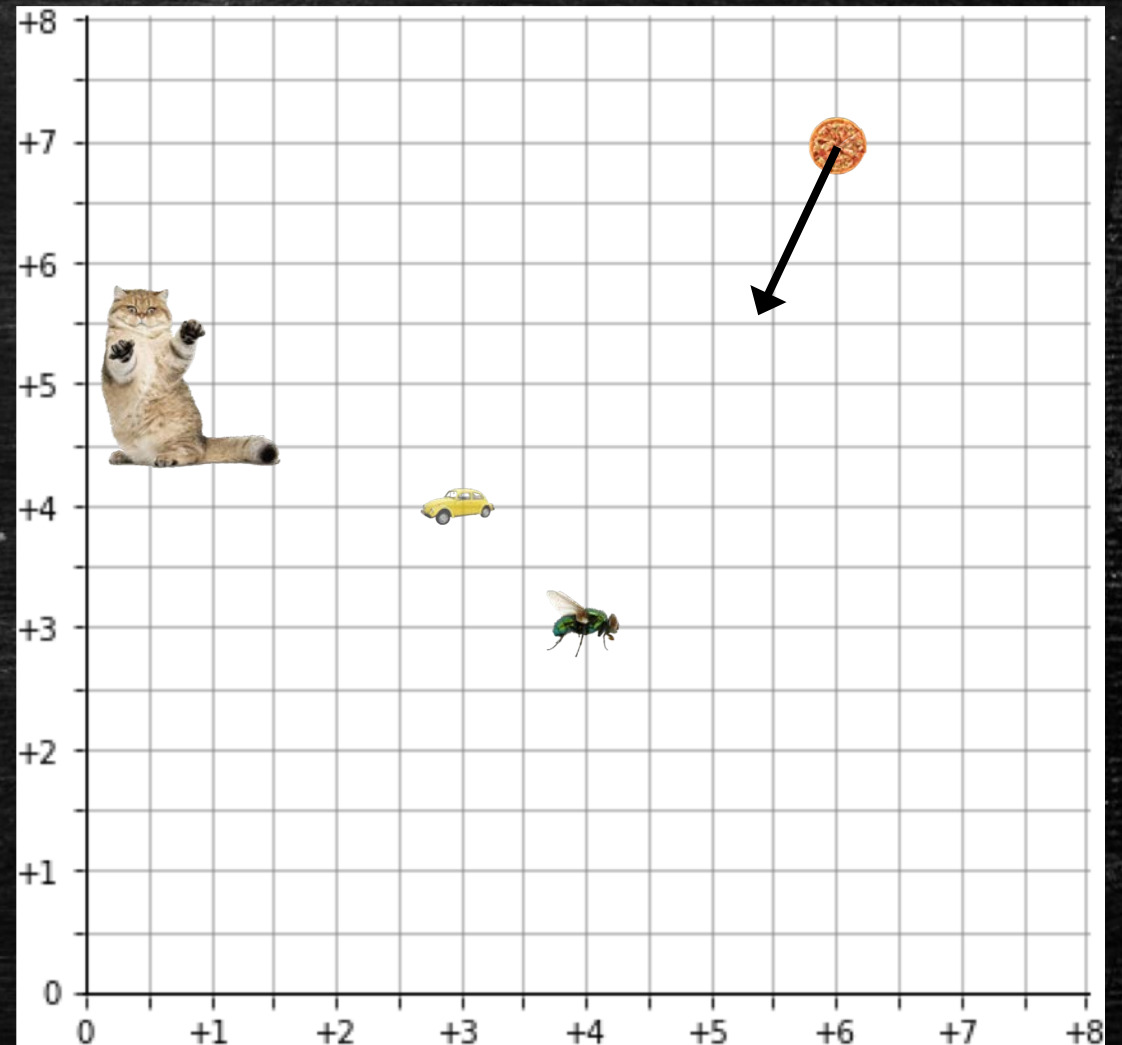


## Question 8

All distances measured in miles

- According to the fly, with what velocity is the pizza coming toward the fly?
- Answer:

$$\vec{v}_{relative} = \left(-4.5 \frac{mi}{hr}\right) \hat{x} + \left(-8.9 \frac{mi}{hr}\right) \hat{y}$$





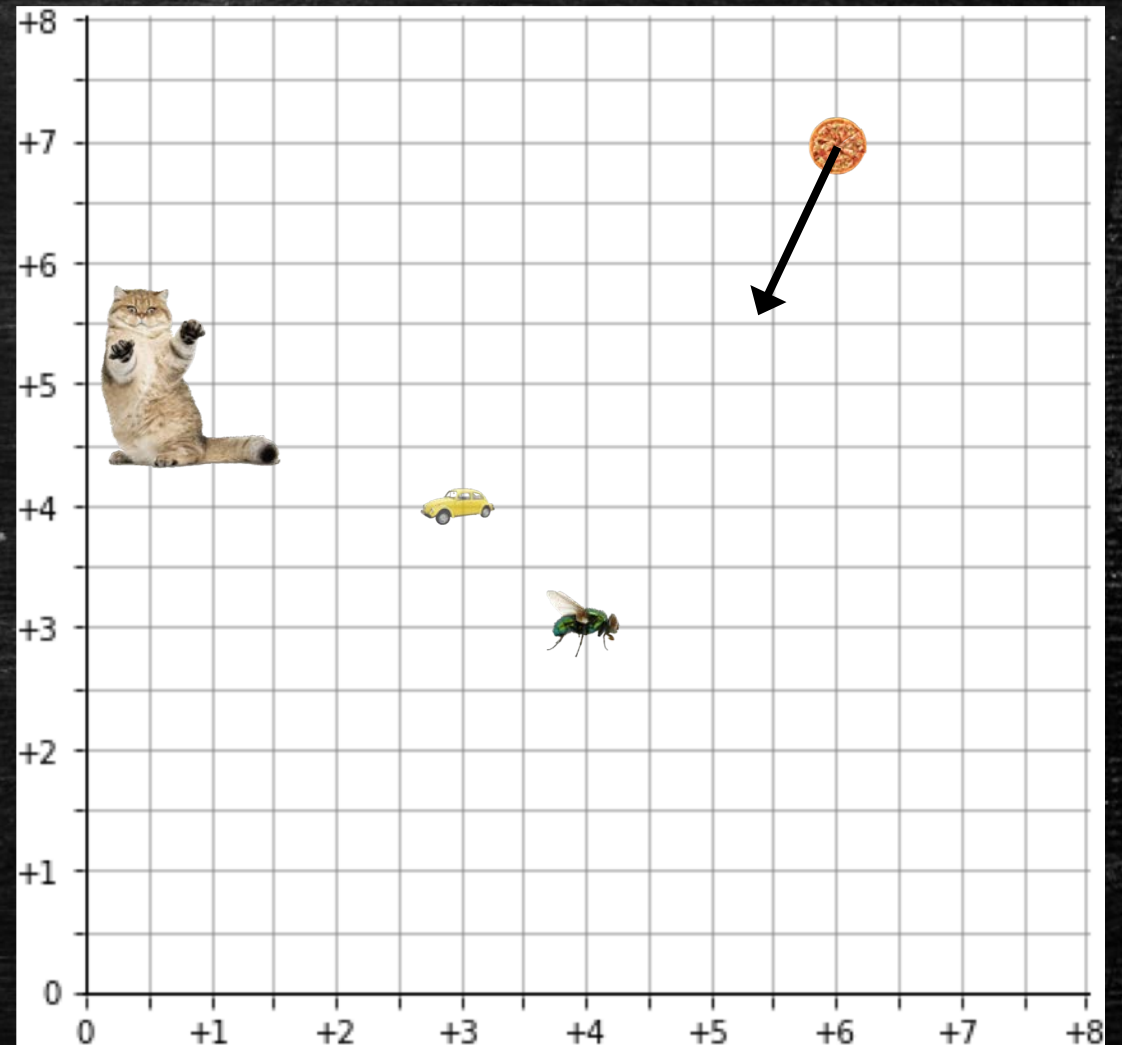
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Discussion: Why is it that the speed of the fly was determined to be 10 mph, but the sum of the velocity components is greater (in magnitude) than 10 mph?



# Group Work

All distances measured in miles

- Question: If the beetle can only make one turn on the way to the pizza (traveling on city blocks), what speed must it have to intercept the fly before it gets to the pizza? The beetle and the fly start moving at the same time.
- Consider the following:
  1. Identify the path of the beetle.
  2. Where will the beetle intersect the fly?
  3. Describe the position of the fly as a function of time.

