Introduction to vectors

My doctor told me I should stay away from vectors. Can I be excused?

No.

Introduction

- We are now using vectors to describe the motion of objects in terms of position, velocity and acceleration. As vectors, these quantities take the following forms:

$$\vec{r} = x\hat{x} + y\hat{y} + z\hat{z}$$

$$\vec{v} = v_x\hat{x} + v_y\hat{y} + v_z\hat{z}$$

$$\vec{a} = a_x\hat{x} + a_y\hat{y} + a_z\hat{z}$$

$\hat{x}, \hat{y}, \hat{z}$ have no units, they just indicate direction
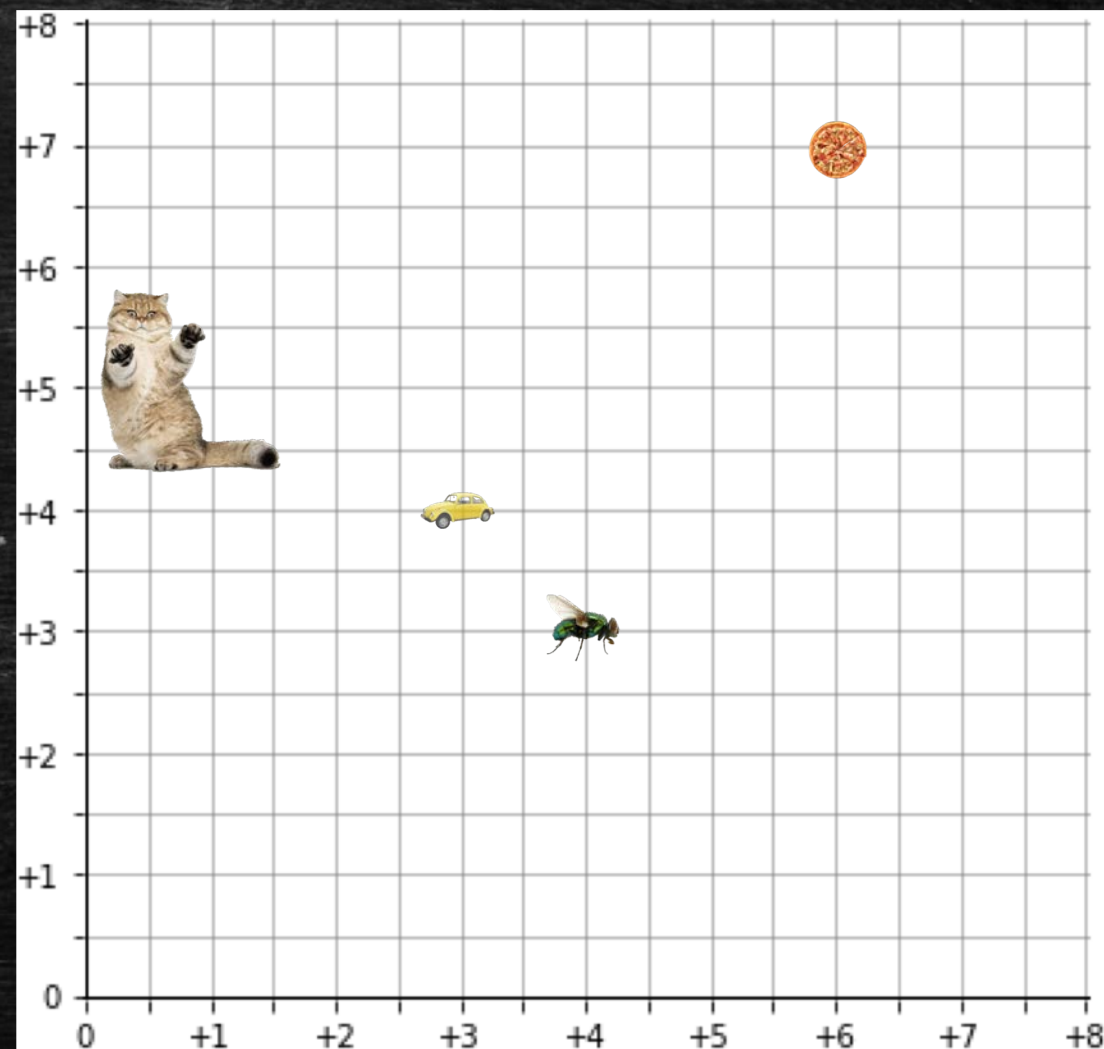
x, y, z have units of length

v_x, v_y, v_z have units of length/time

a_x, a_y, a_z have units of length/time^2

Question 1

- (a) What are the coordinates of the beetle?

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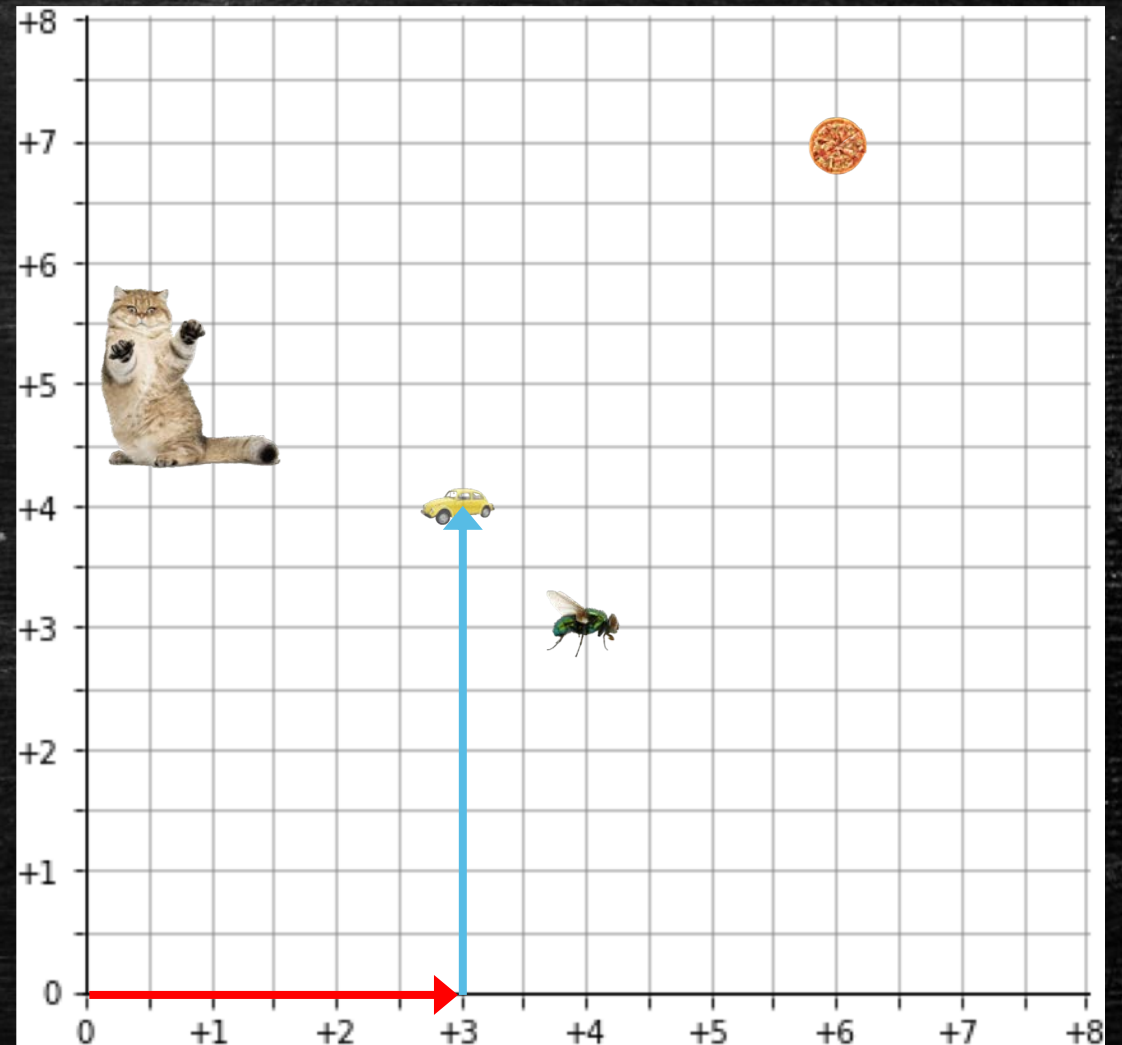
- Answer:

x_{beetle} = +3.0 miles

y_{beetle} = +4.0 miles

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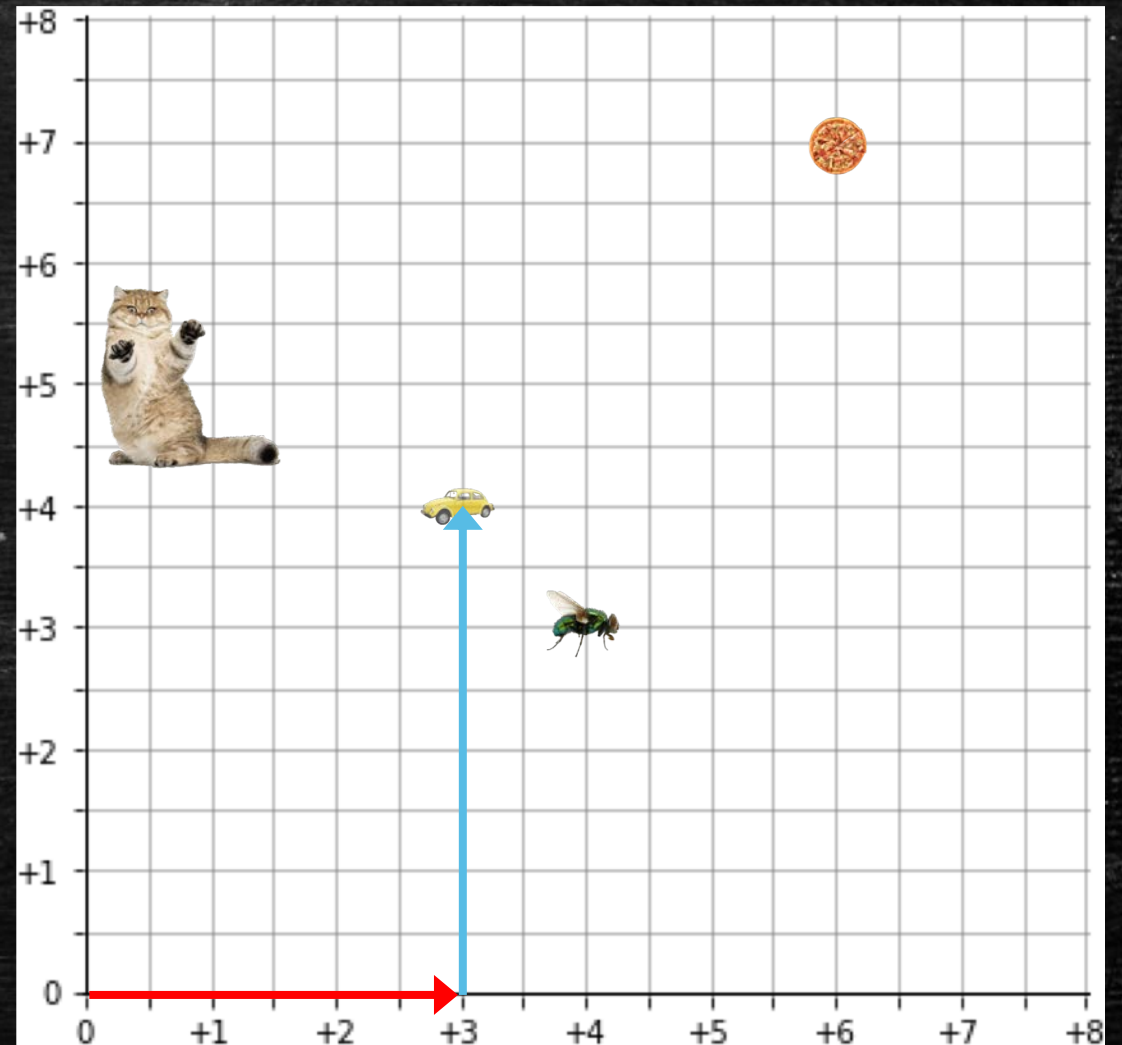
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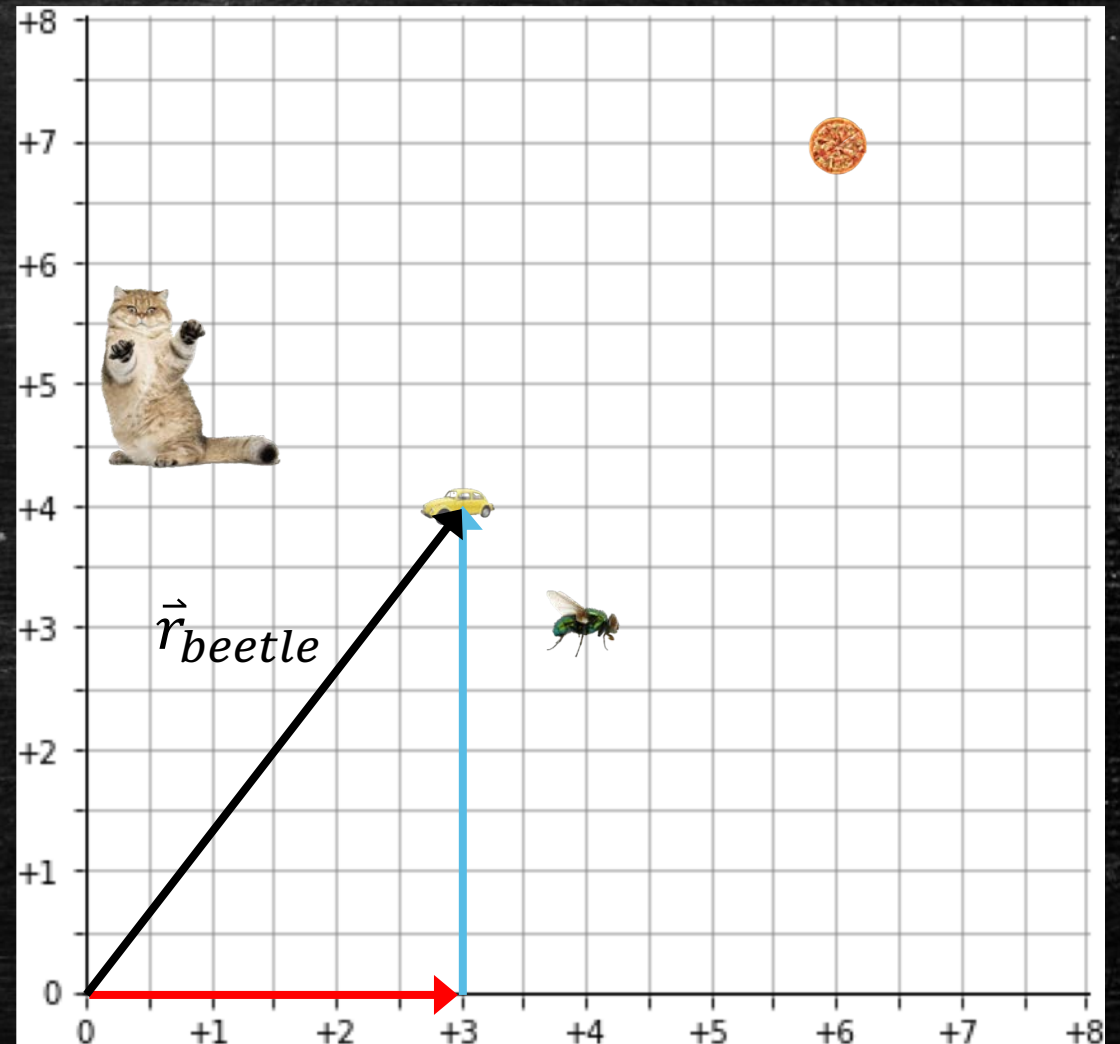y_{beetle} = +4.0 miles

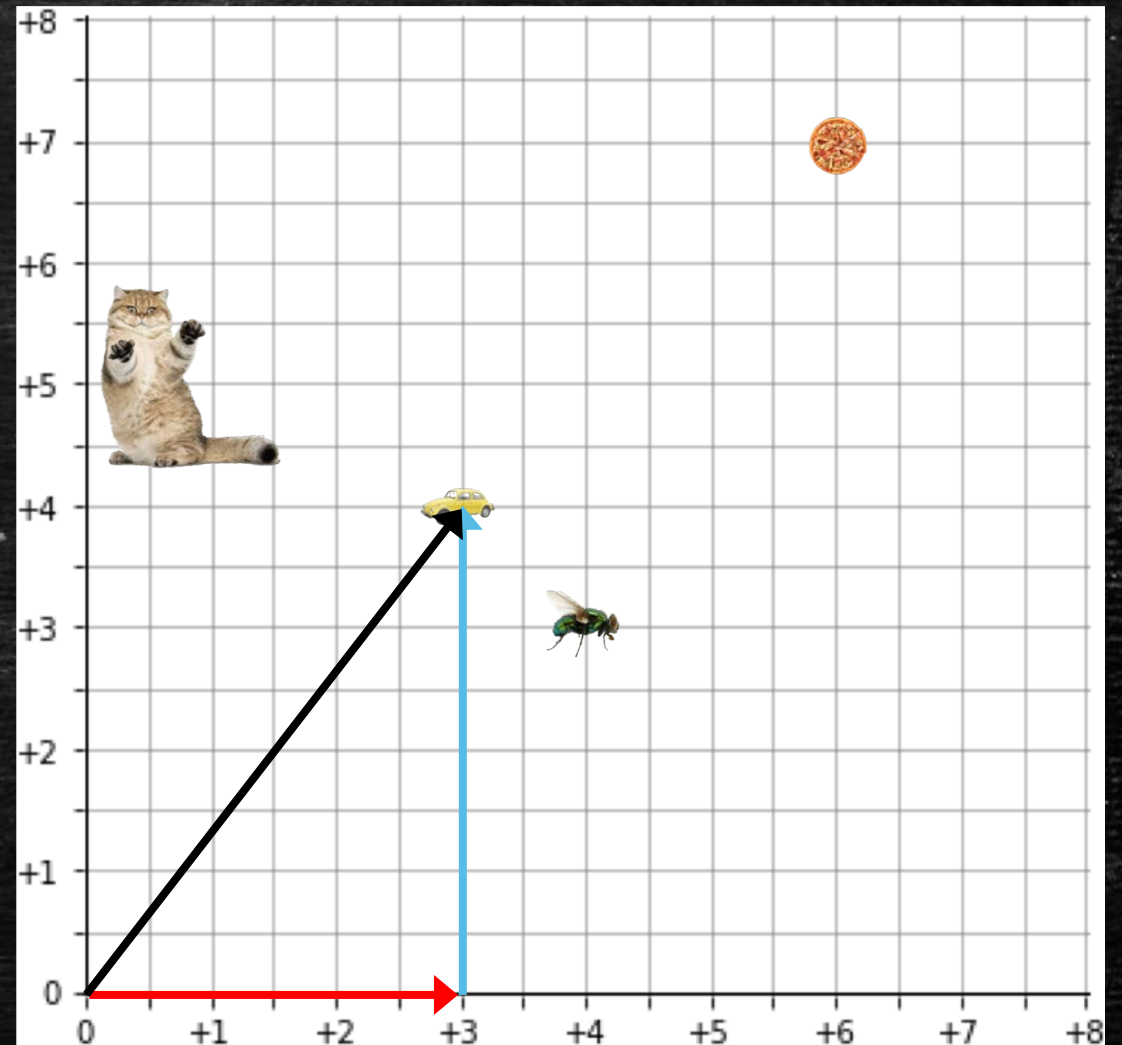- (b) What is the vector that defines the position of the beetle?

- Answer:

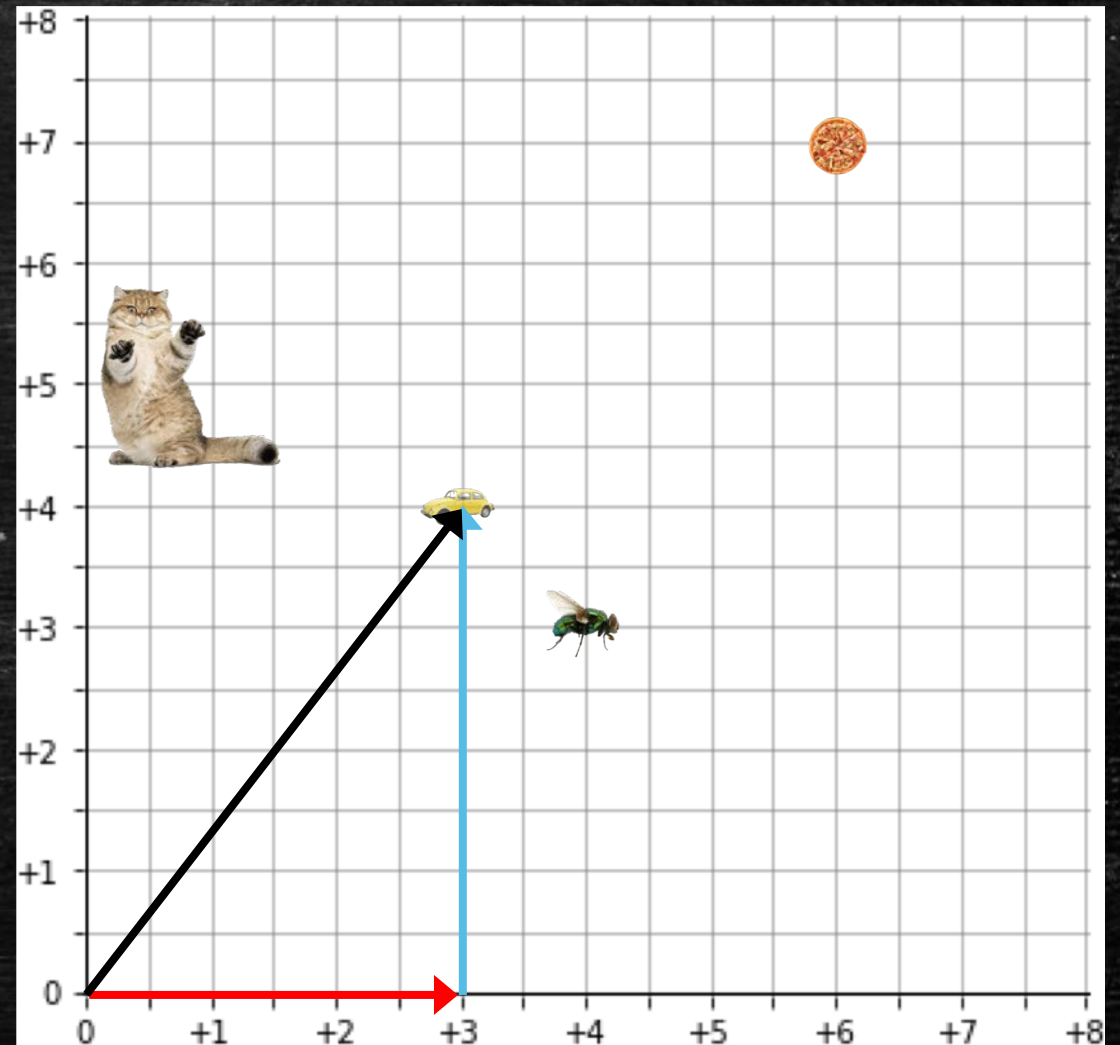$\vec{r}_{beetle} = (+3.0 \text{ miles})\hat{x} + (+4.0 \text{ miles})\hat{y}$

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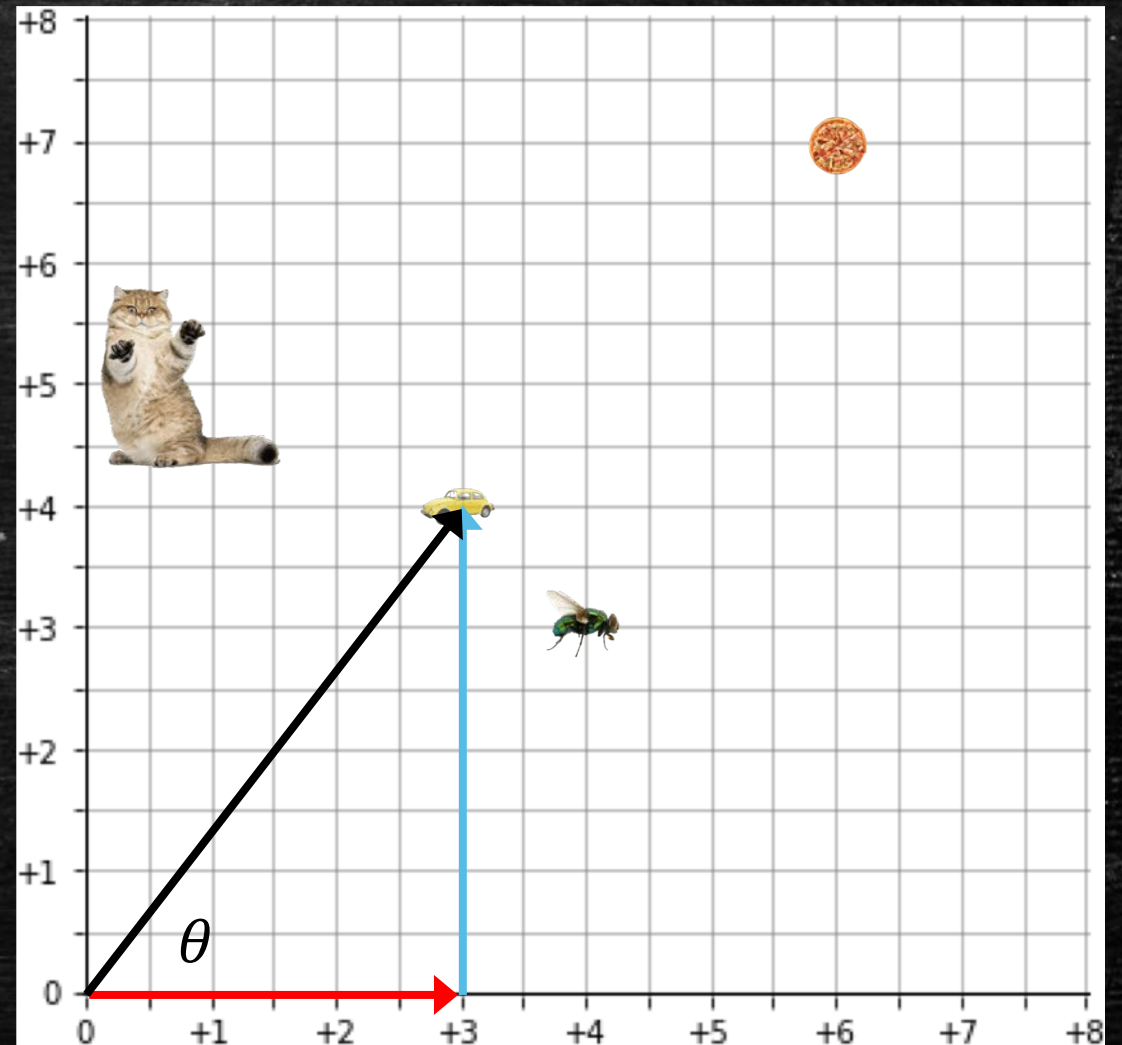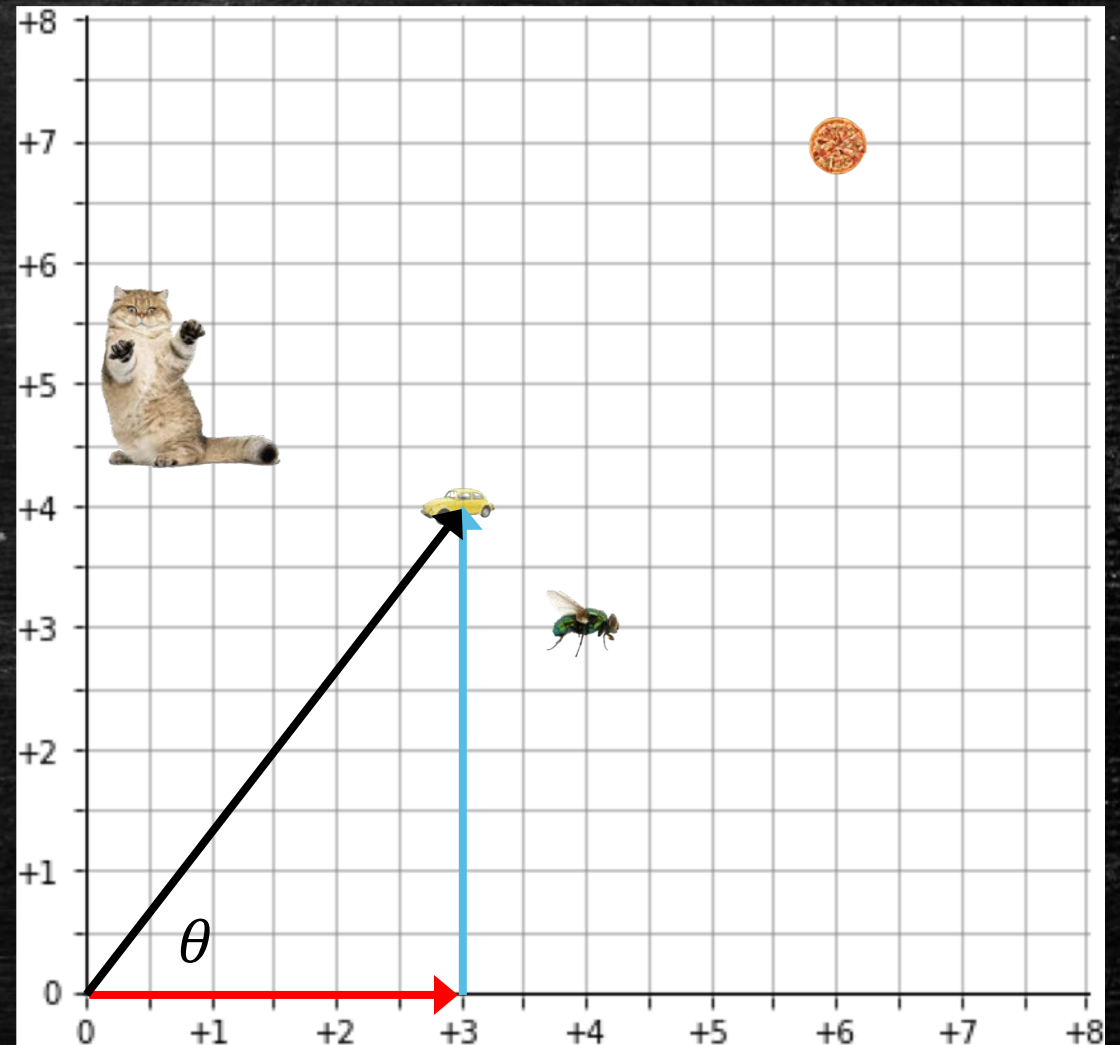
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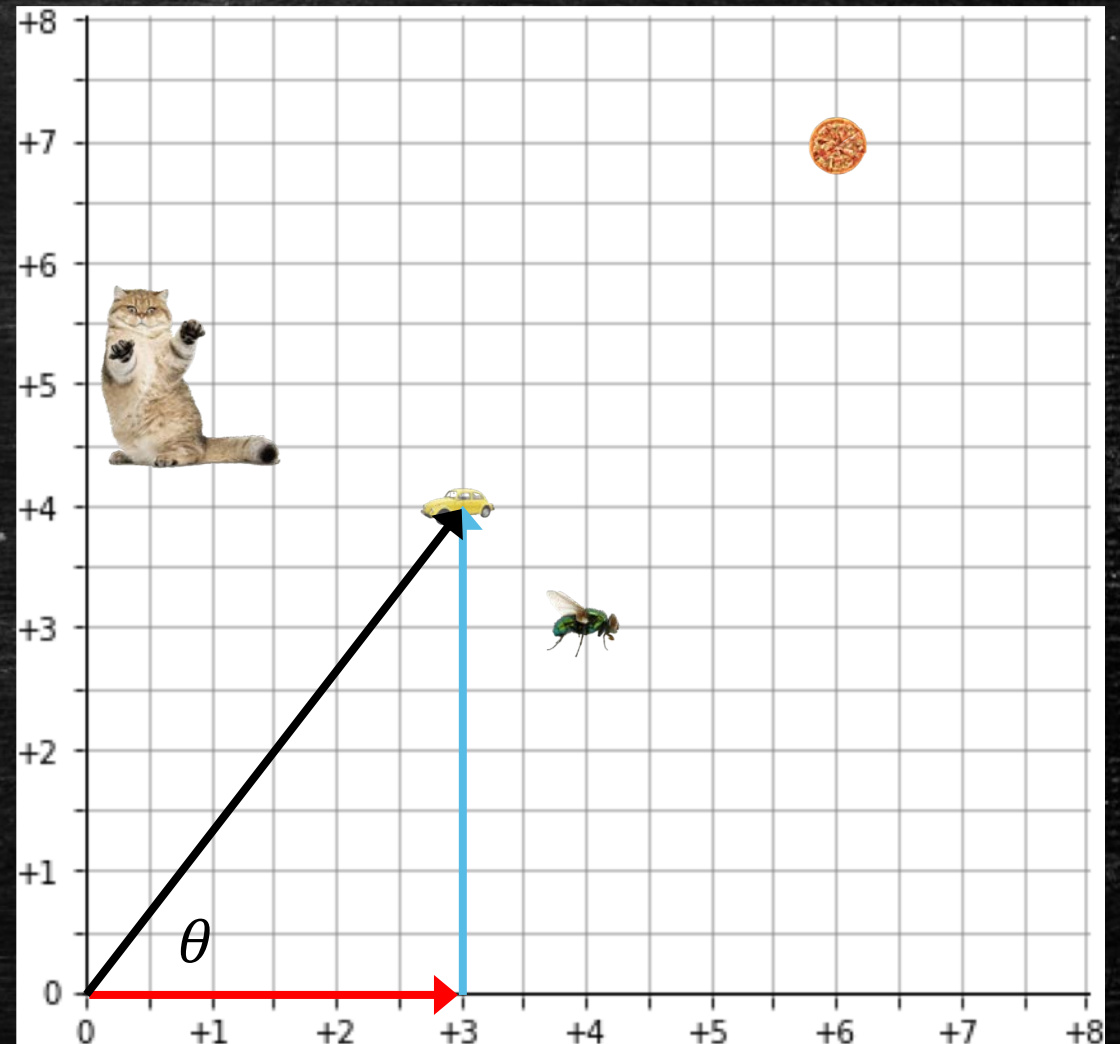
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- Discussion: what is special about the red and blue arrows that allows us to use the Pythagorean theorem to solve for the length of the position vector?

Cartesian & Polar coordinates

- Notice that we defined the position of the beetle with two pieces of information: the **x coordinate** and the **y coordinate**.

- We can also describe the position with two other pieces of information, which is the distance from the origin and the angle relative to the x-axis.

- These two forms carry the same information and are therefore different but equivalent representations of position.

Cartesian coordinates

x_{beetle} = +3.0 miles

y_{beetle} = +4.0 miles

Polar coordinates

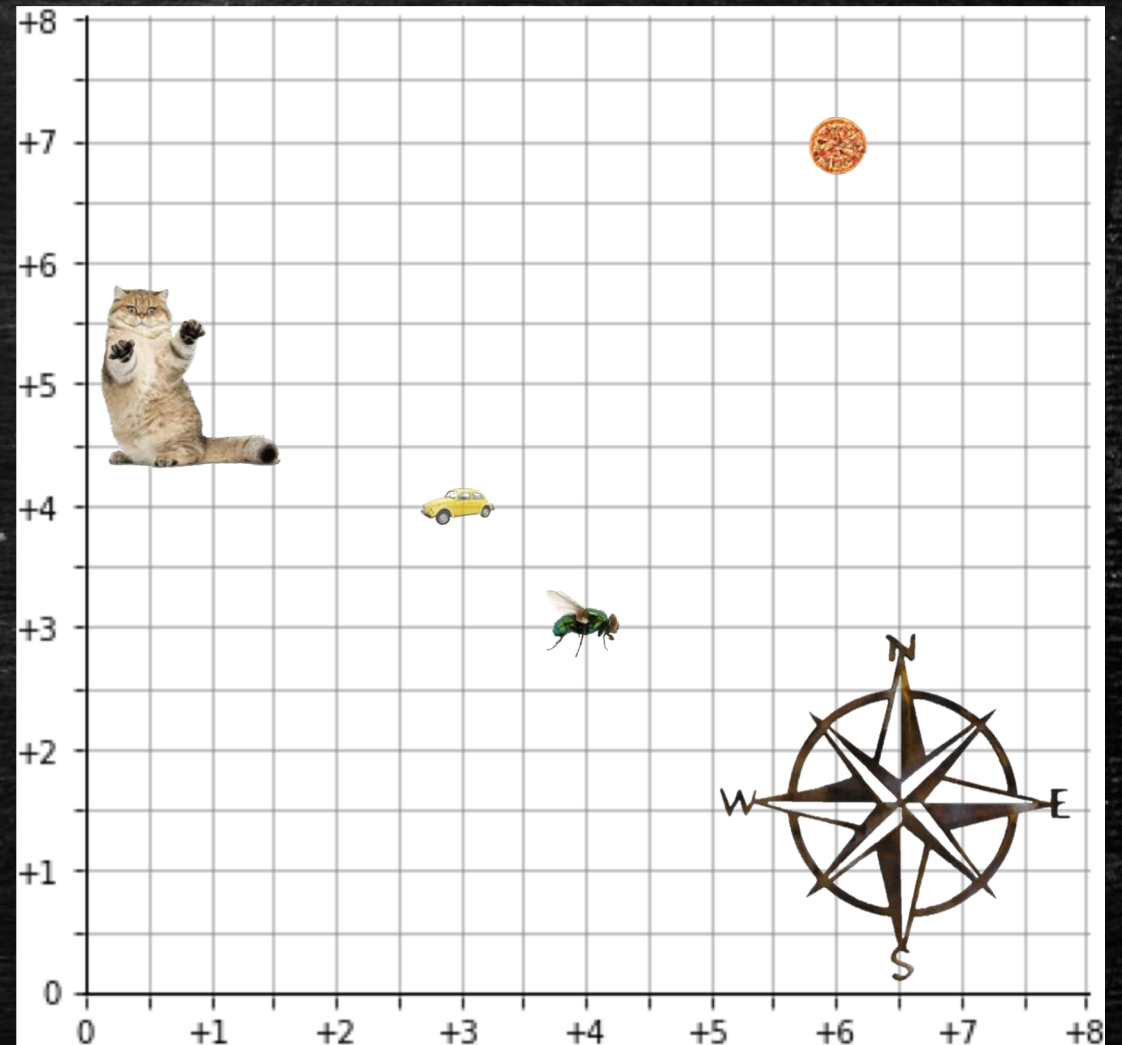d = 5.0 miles

θ = 53.1°

Question 3

- If north is up and east is to the right, describe the directions of the other objects relative to the beetle in terms of cardinal directions (roughly).
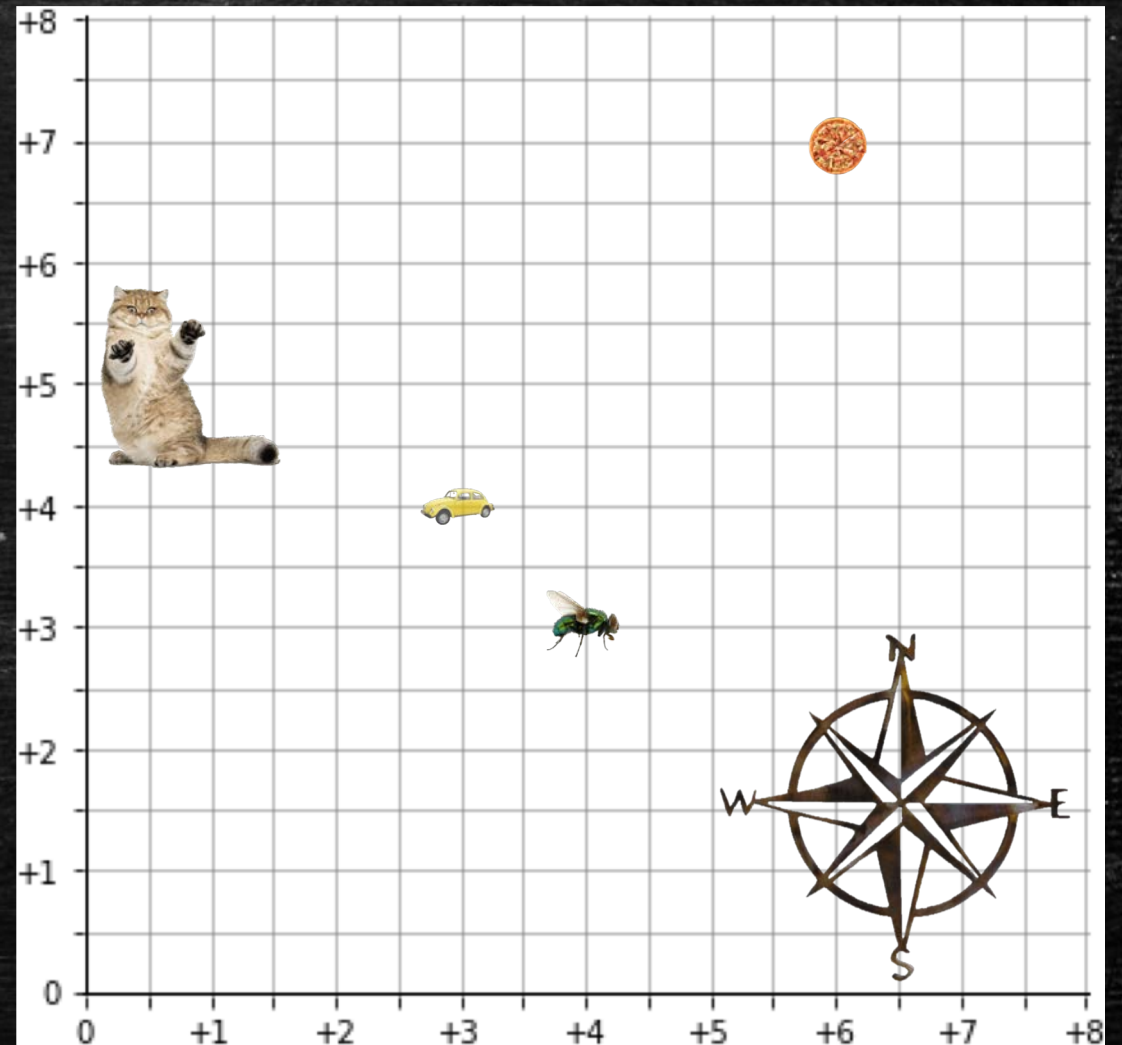
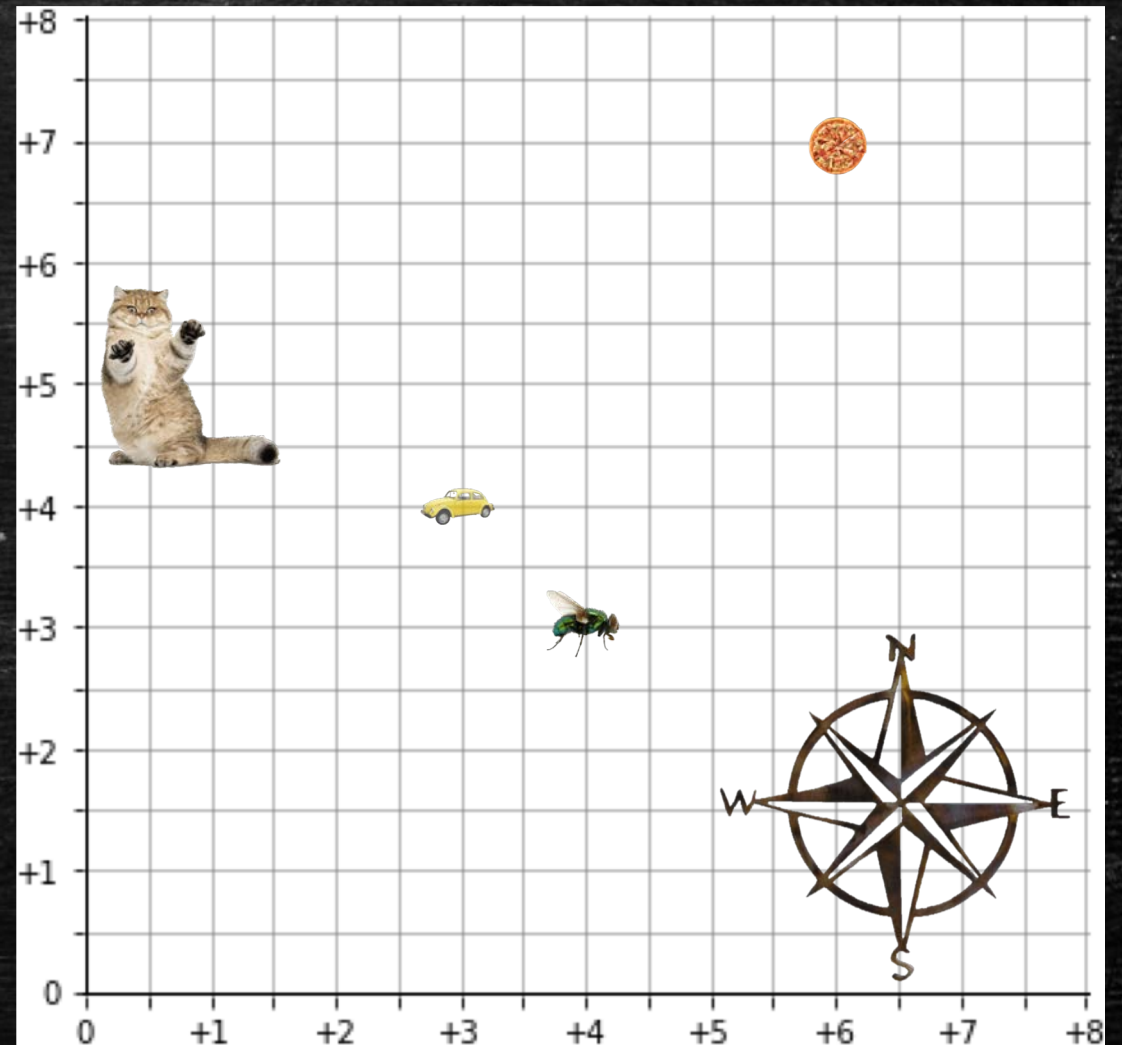# Question 3

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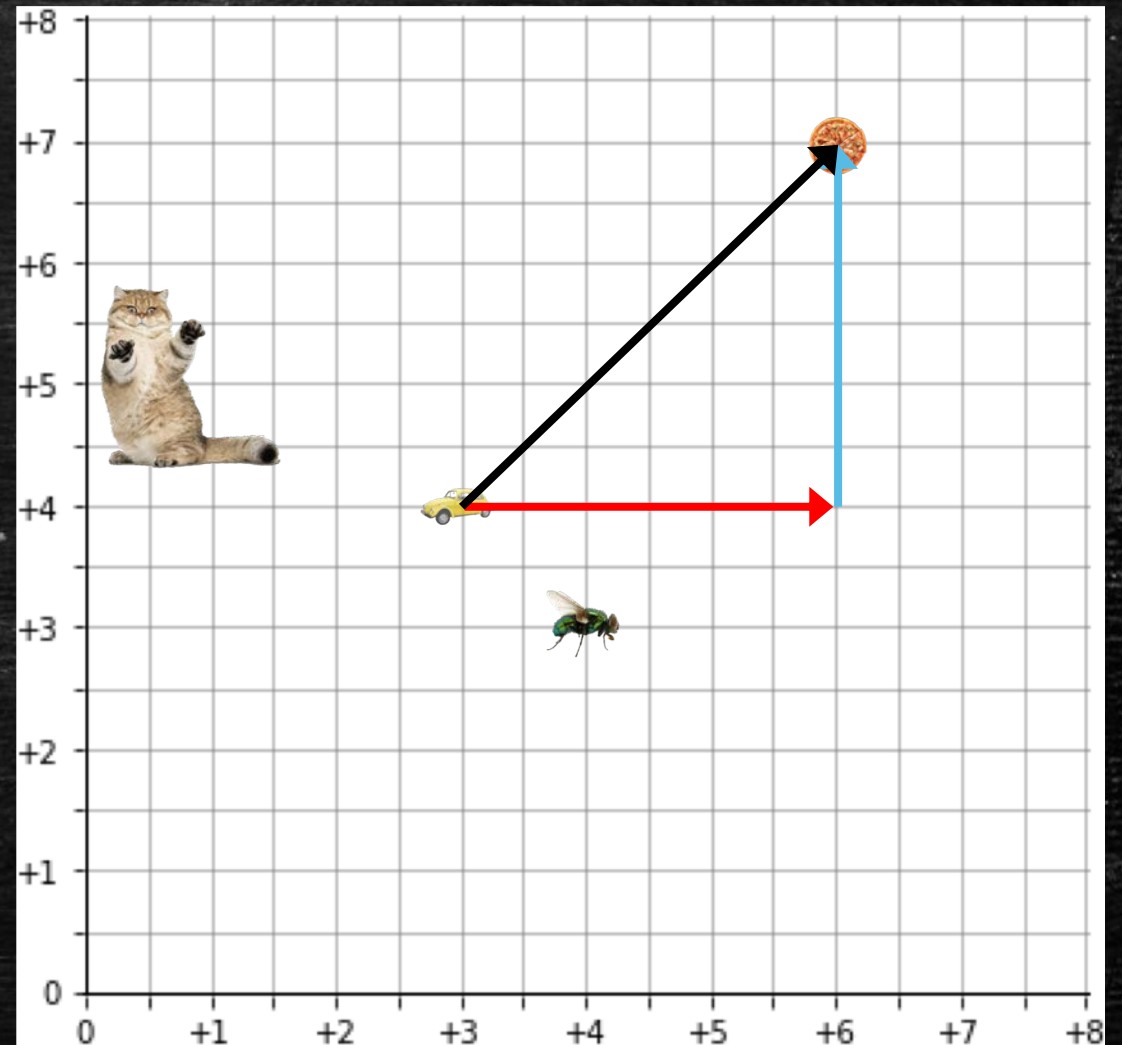 cat: NW or WNW

  pizza: NE

  fly: SE

Question 3

- If north is up and east is to the right, describe the directions of the other objects relative to the beetle in terms of cardinal directions (roughly).

- Answer:

  cat: NW or WNW

  pizza: NE

  fly: SE

- Discussion: Is it possible to define a position for the cat?

Question 4

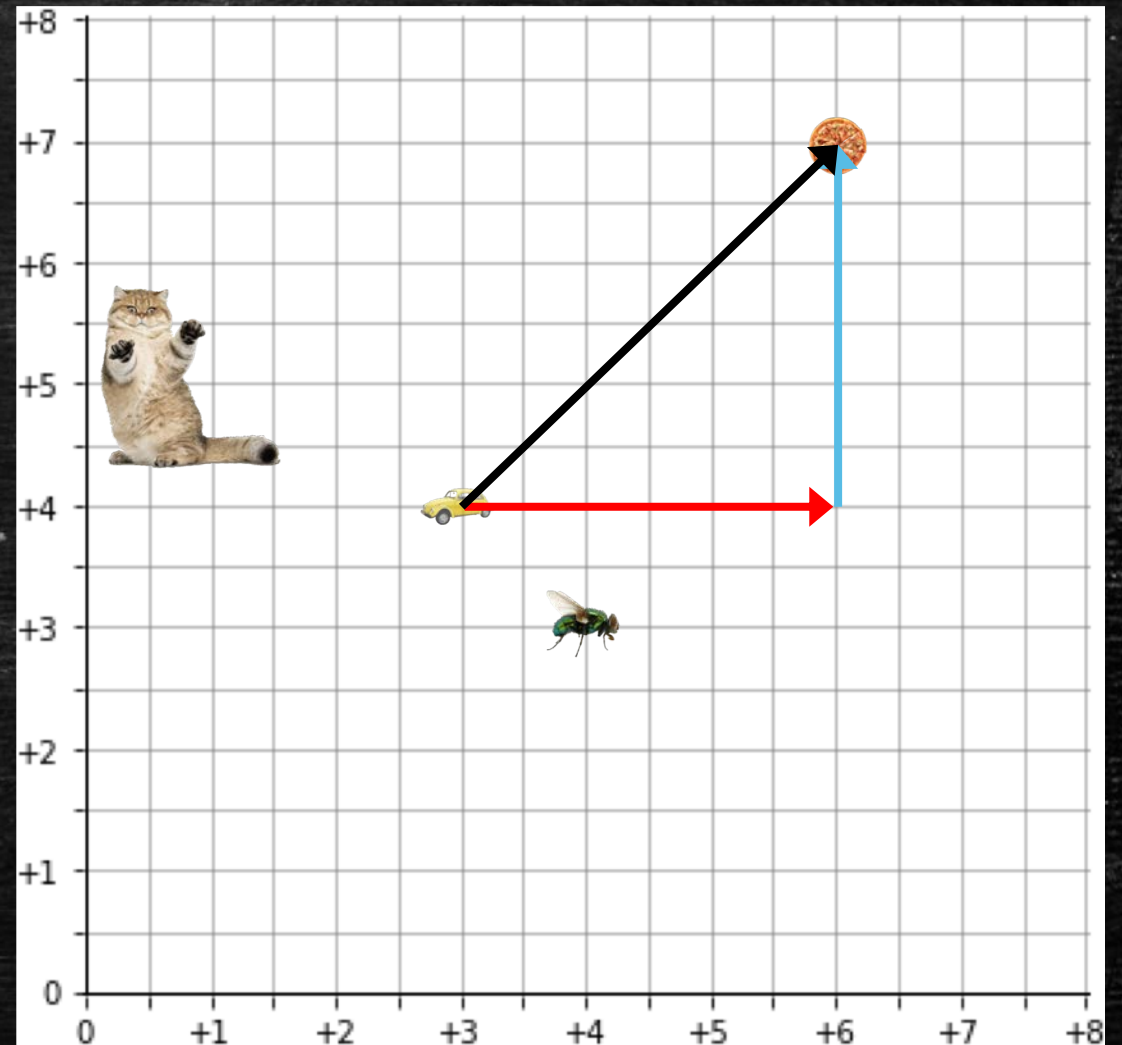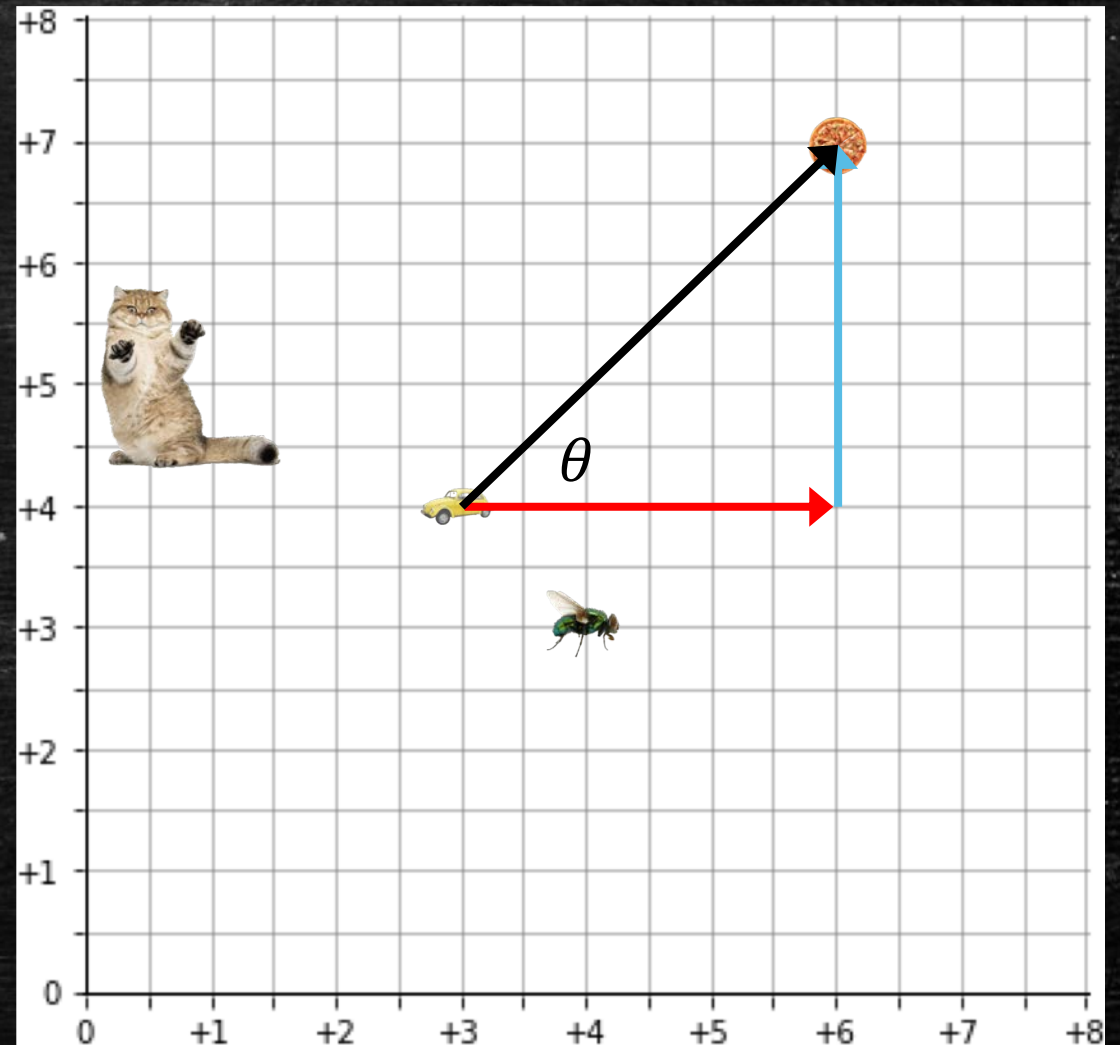- (a) If you could drive straight to the pizza, how far would you have to travel?

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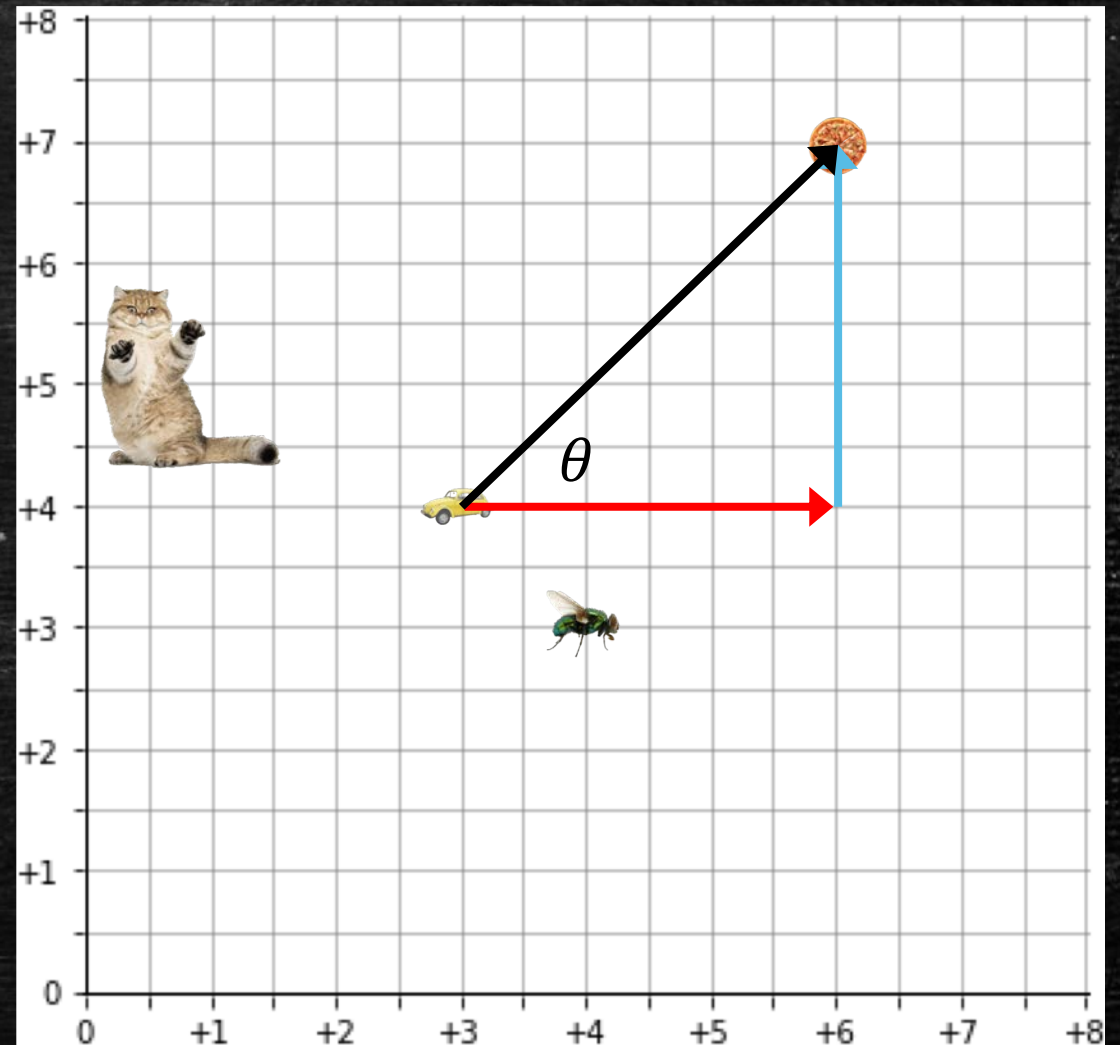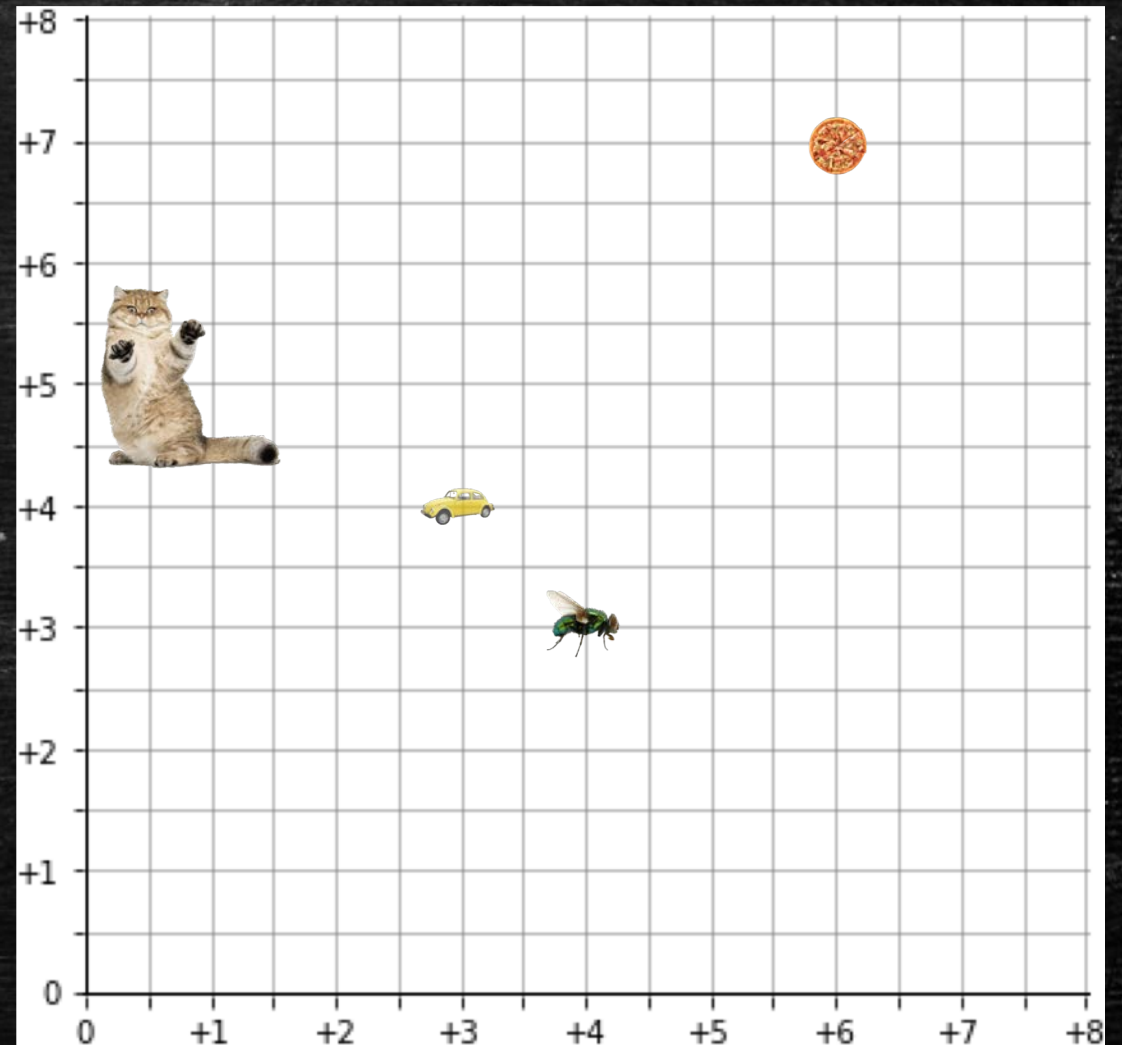
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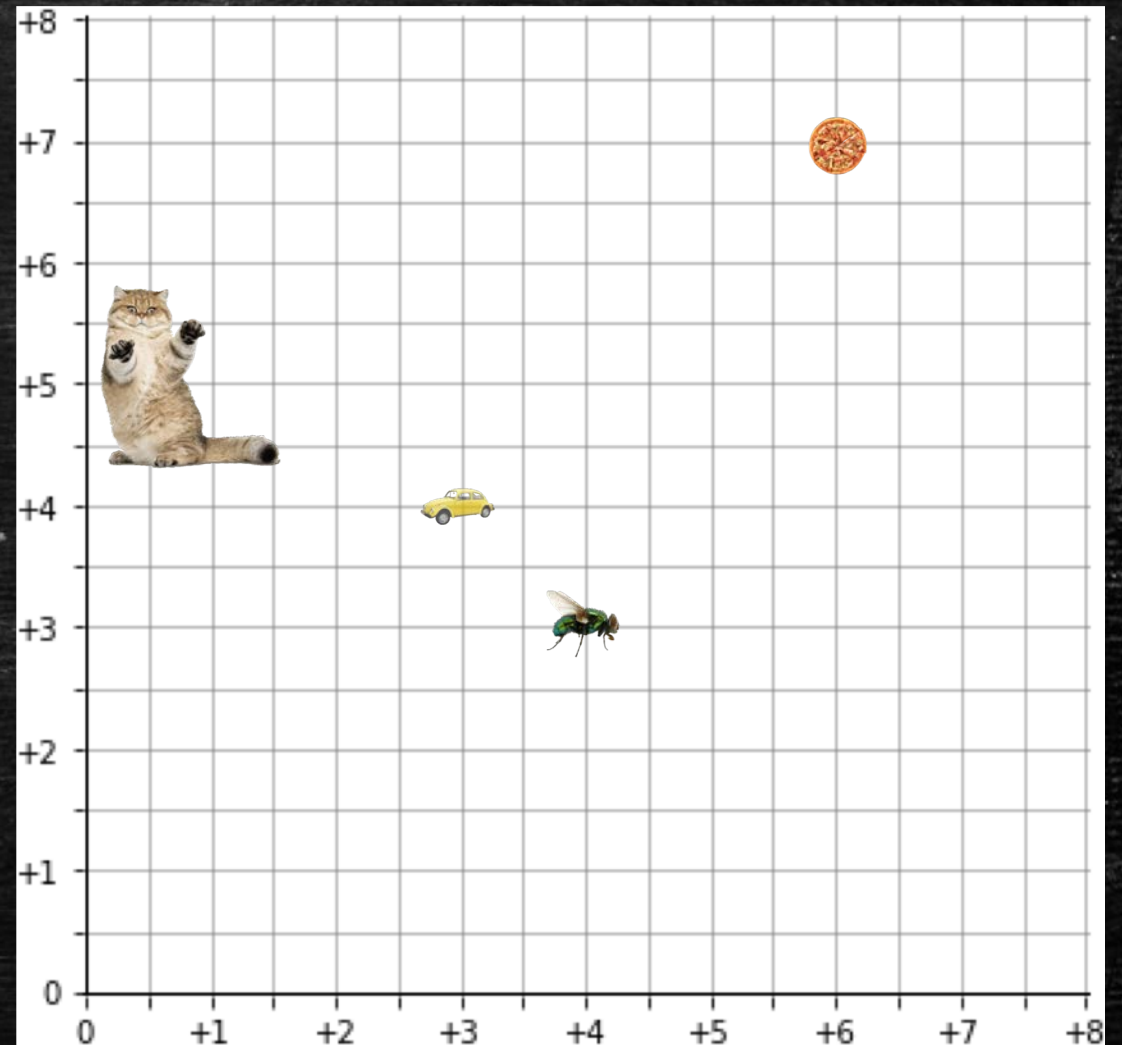
- Answer: θ = 45.0°

Bonus Round

- Q1: Does the cat look playful, scared, or mean?

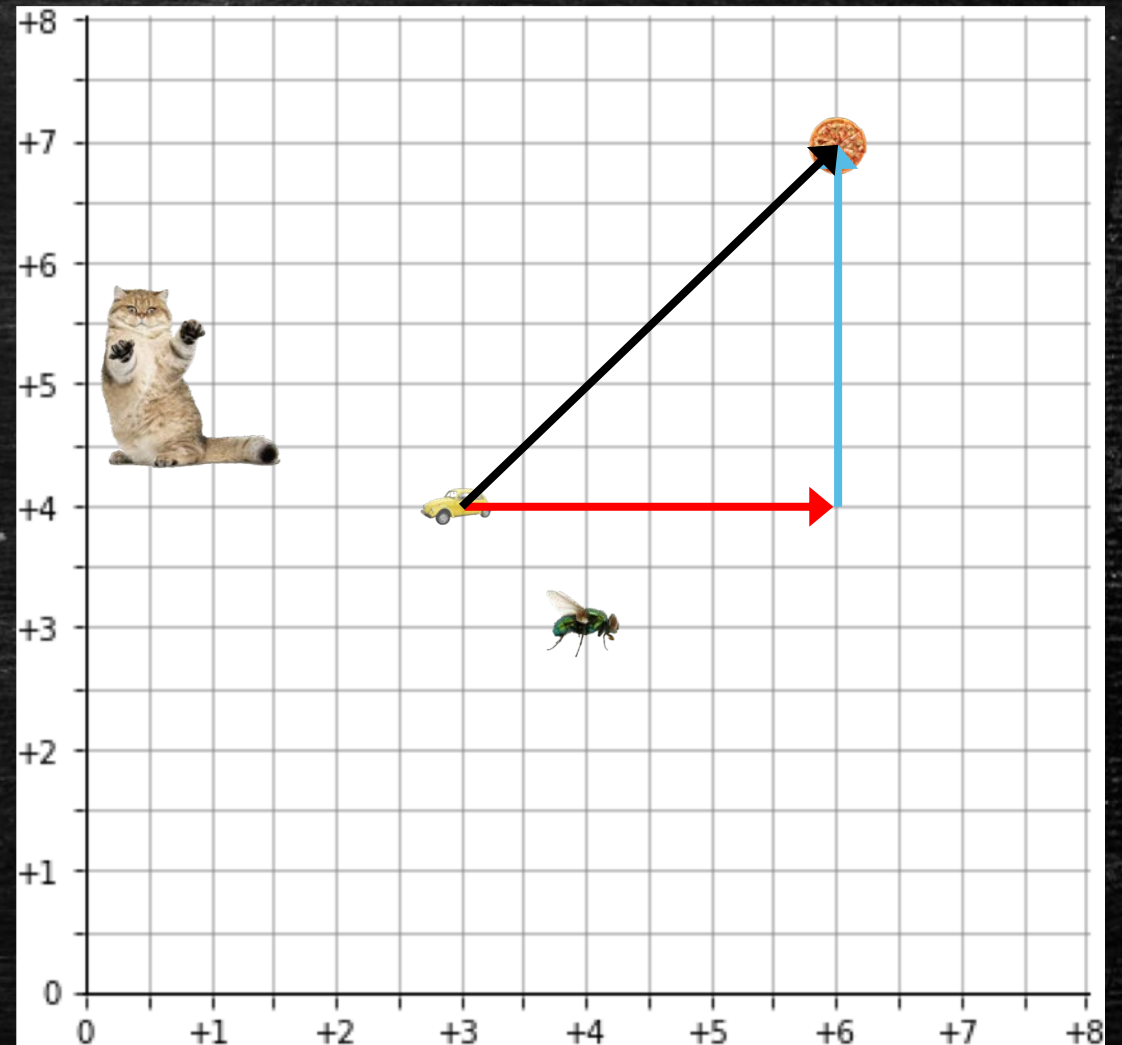- Q2: Write a formula that relate these cat emotions to each other.

Bonus Round

- Q1: Does the cat look playful, scared, or mean?

- Answer: mean (see claws)

- Q2: Write a formula that relate these cat emotions to each other.

- Possible answer:

    playful + scared = mean

Question 5

- How are the direct path (black) and the city-block path (red & blue) similar and different?
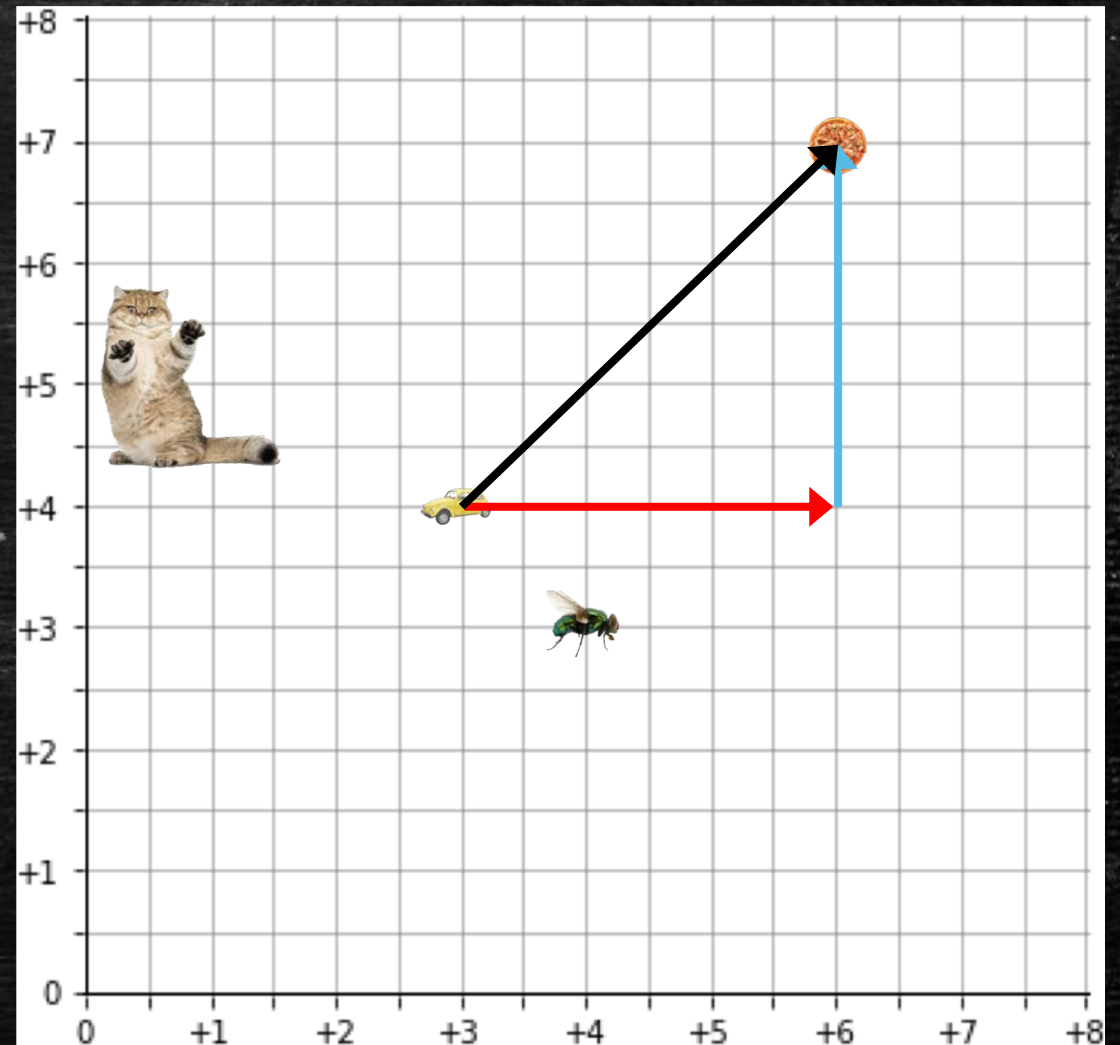
Question 5

- How are the direct path (black) and the city-block path (red & blue) similar and different?

- Answer:

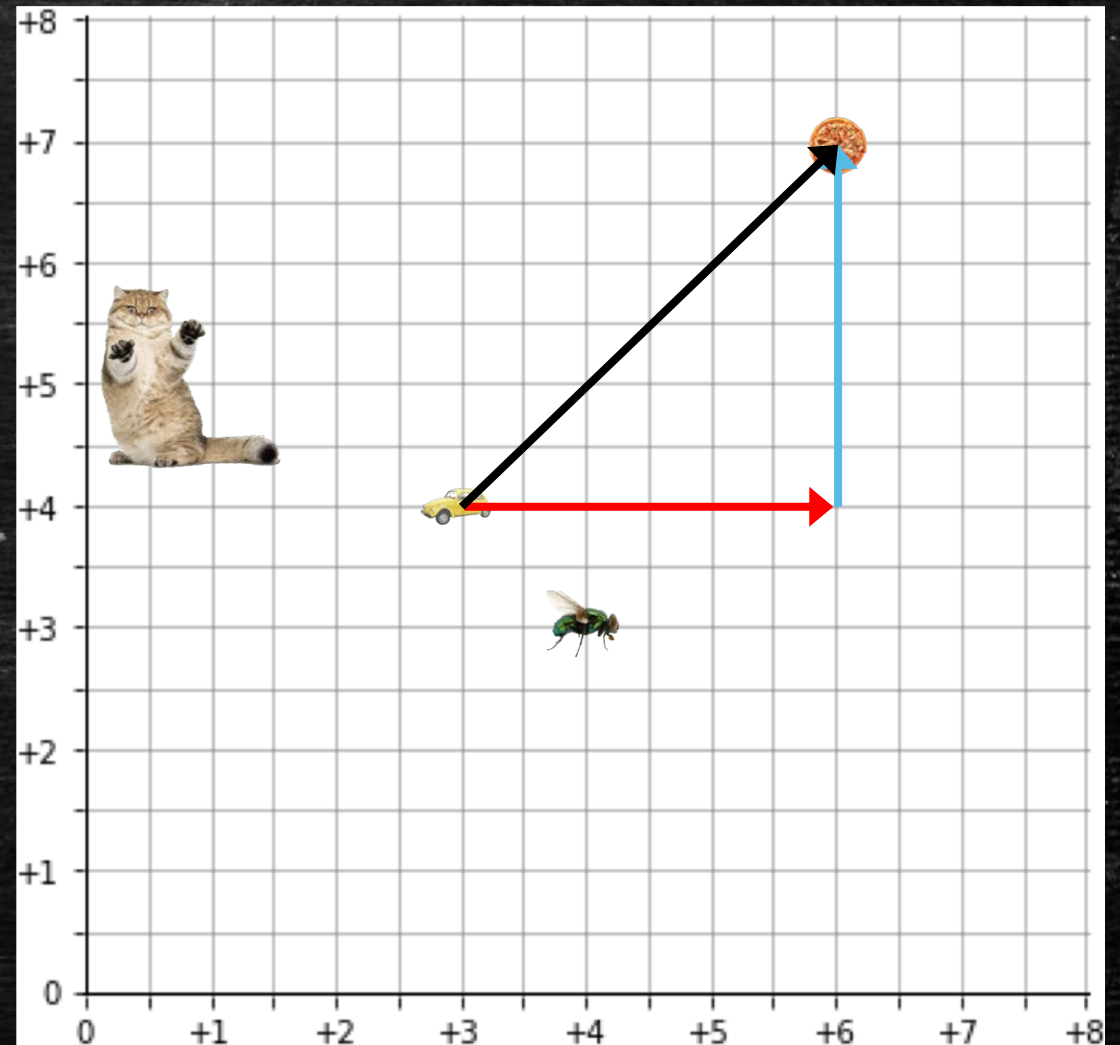<u>Similar</u>: both paths start and end at the same points.

Question 5

▪ How are the direct path (black) and the city-block path (red & blue) similar and different?

▪ Answer:

Similar: both paths start and end at the same points.

Different: the direct path has a total distance of 4.2 miles whereas the city-block path has a total distance of 8.0 miles.
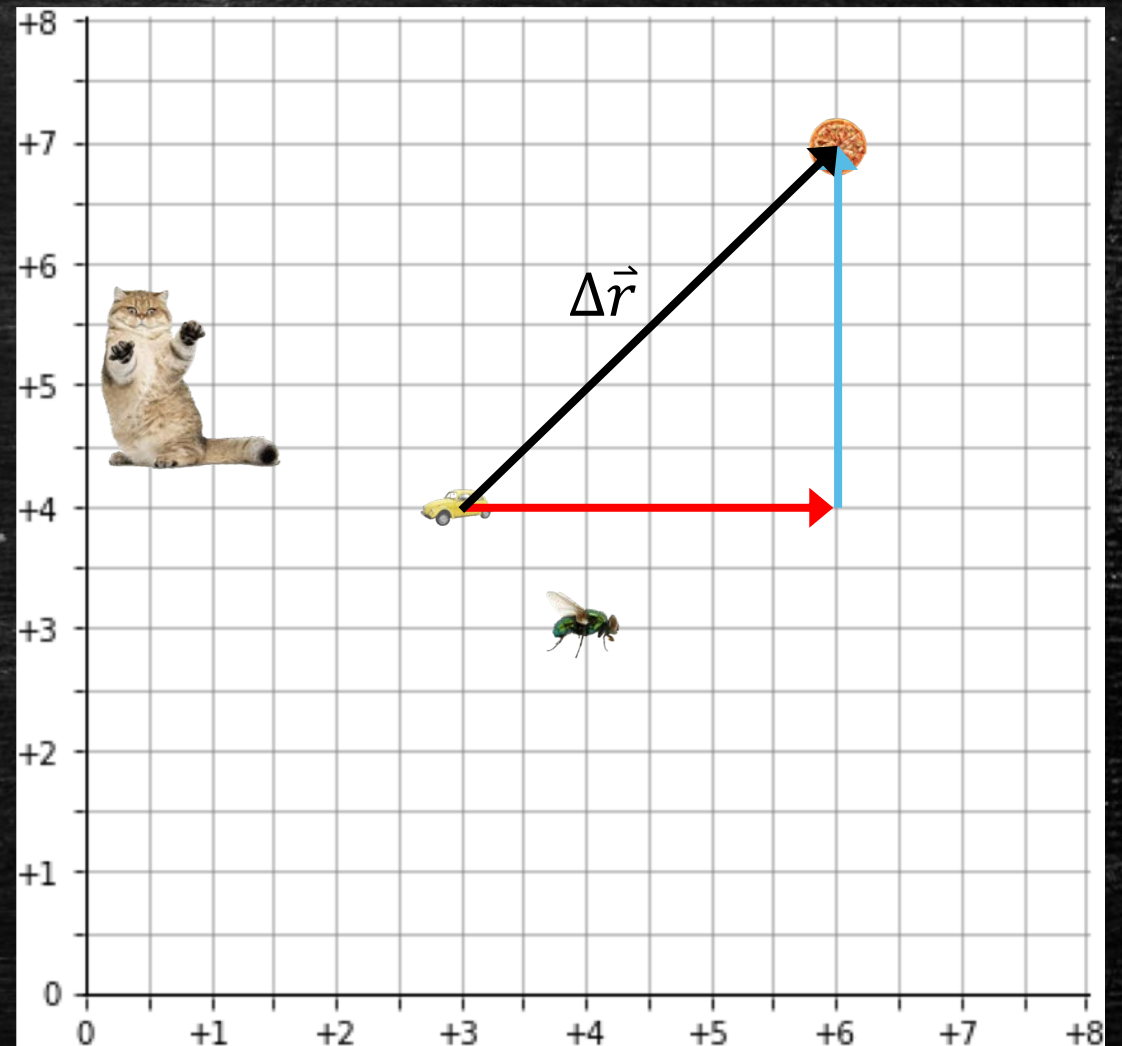
Question 6

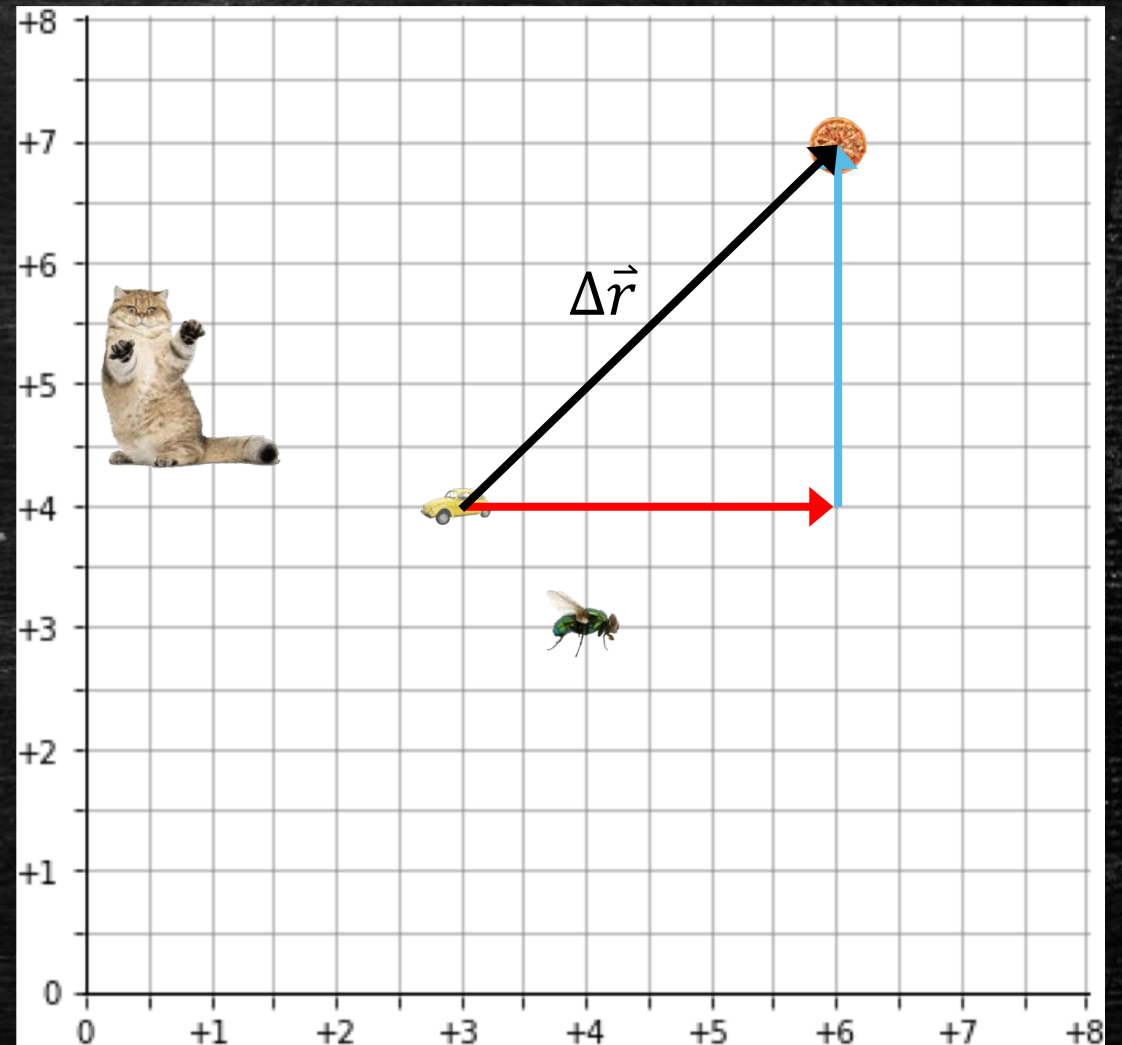- (a) Describe the position of the pizza relative to the beetle in components.

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- (b) Using information already known, determine the components of the pizza's position vector.

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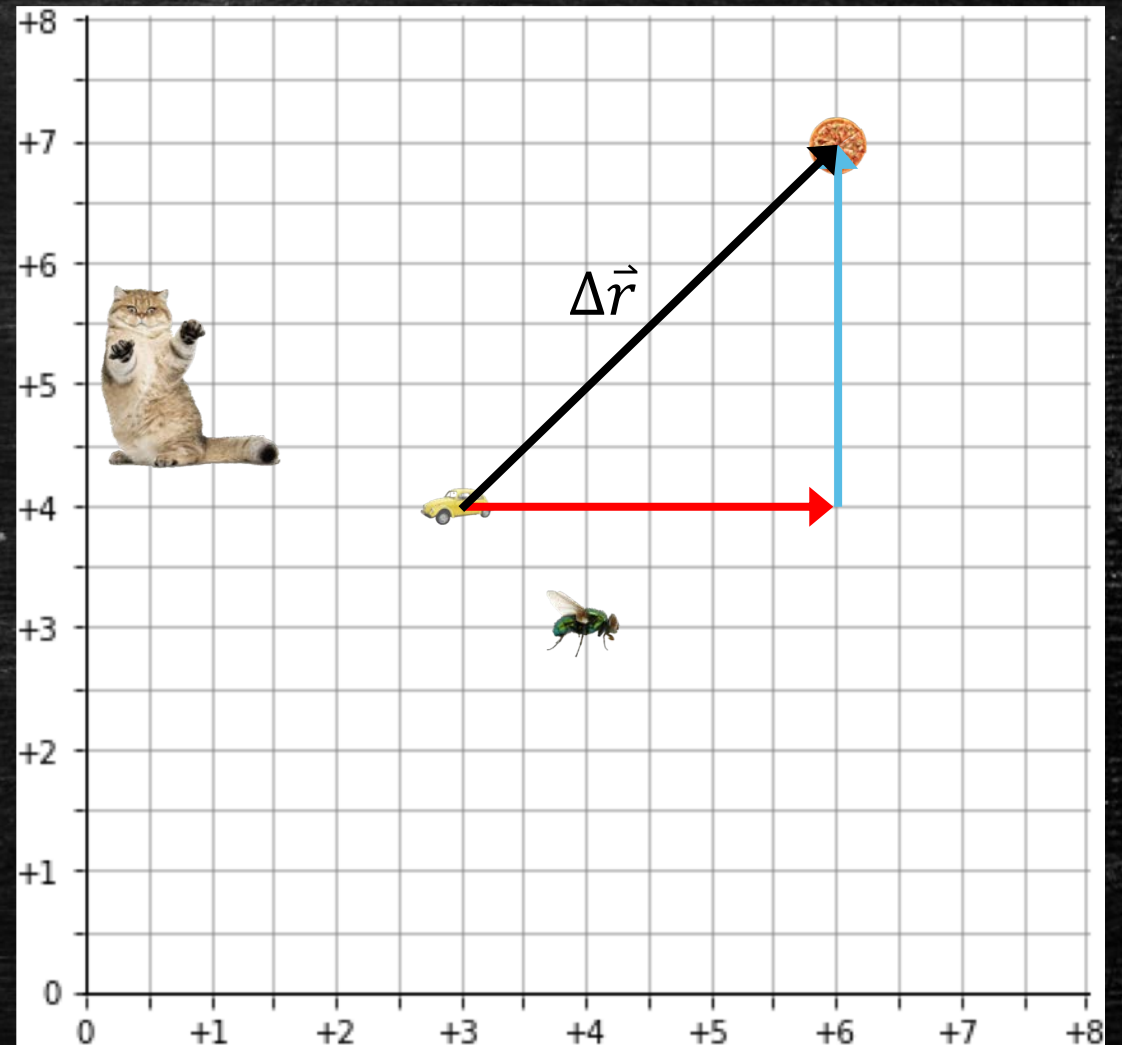
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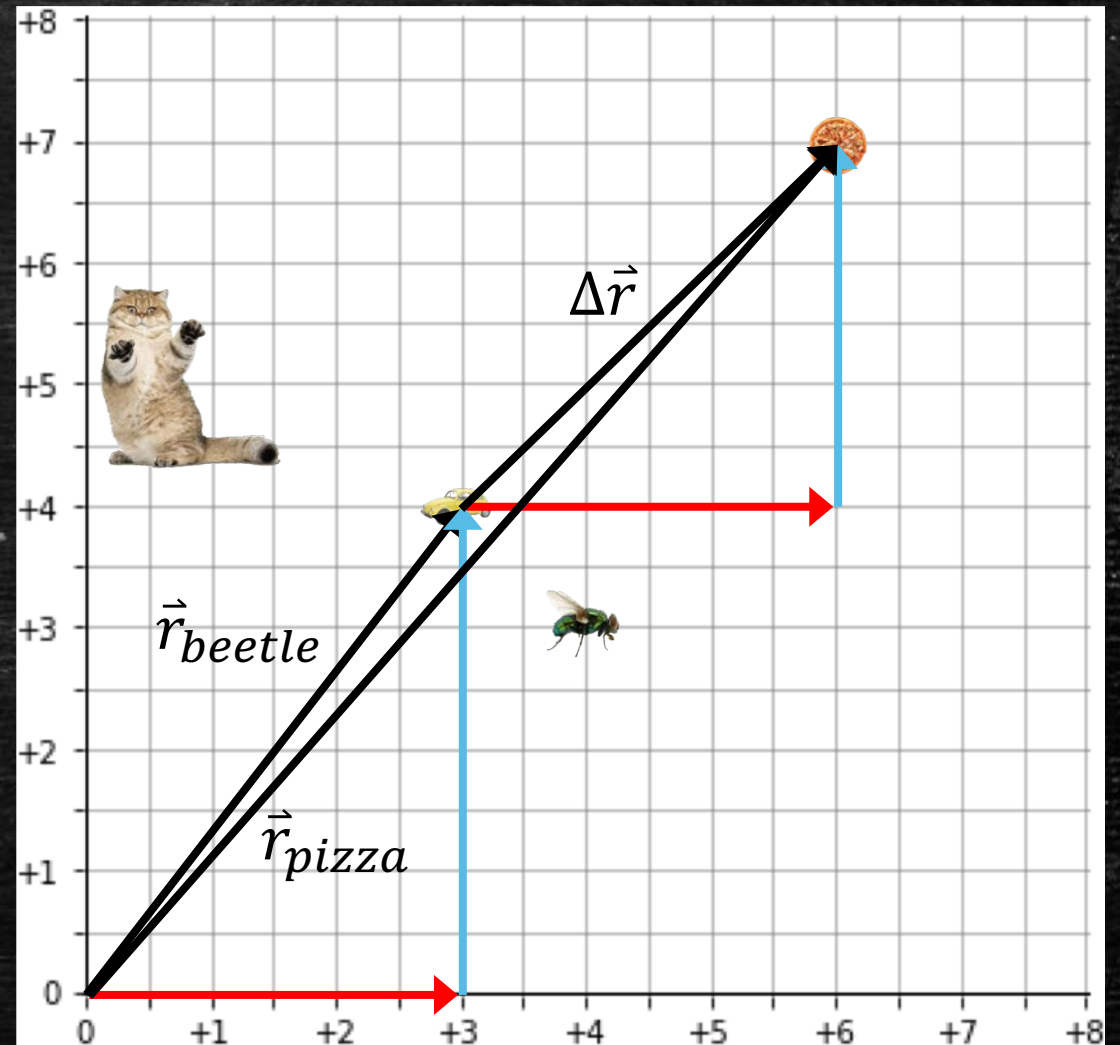
- Answer:

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- (b) Using information already known, determine the components of the pizza's position vector.

- Answer:

x_{pizza} = x_{beetle} + Δx = +6.0 miles

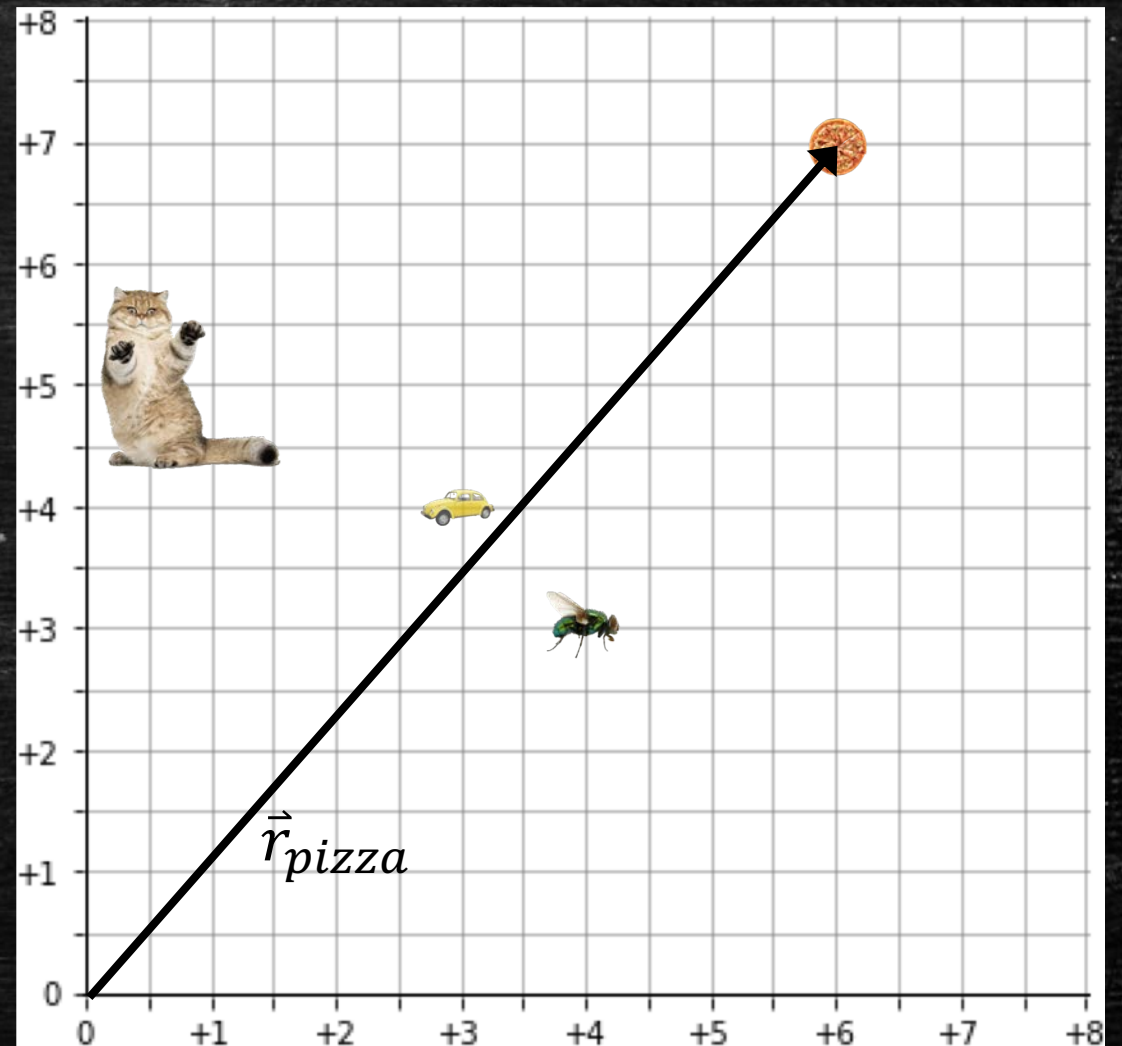y_{pizza} = y_{beetle} + Δy = +7.0 miles

Question 7

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d = 9.2 miles

θ = 49.4°
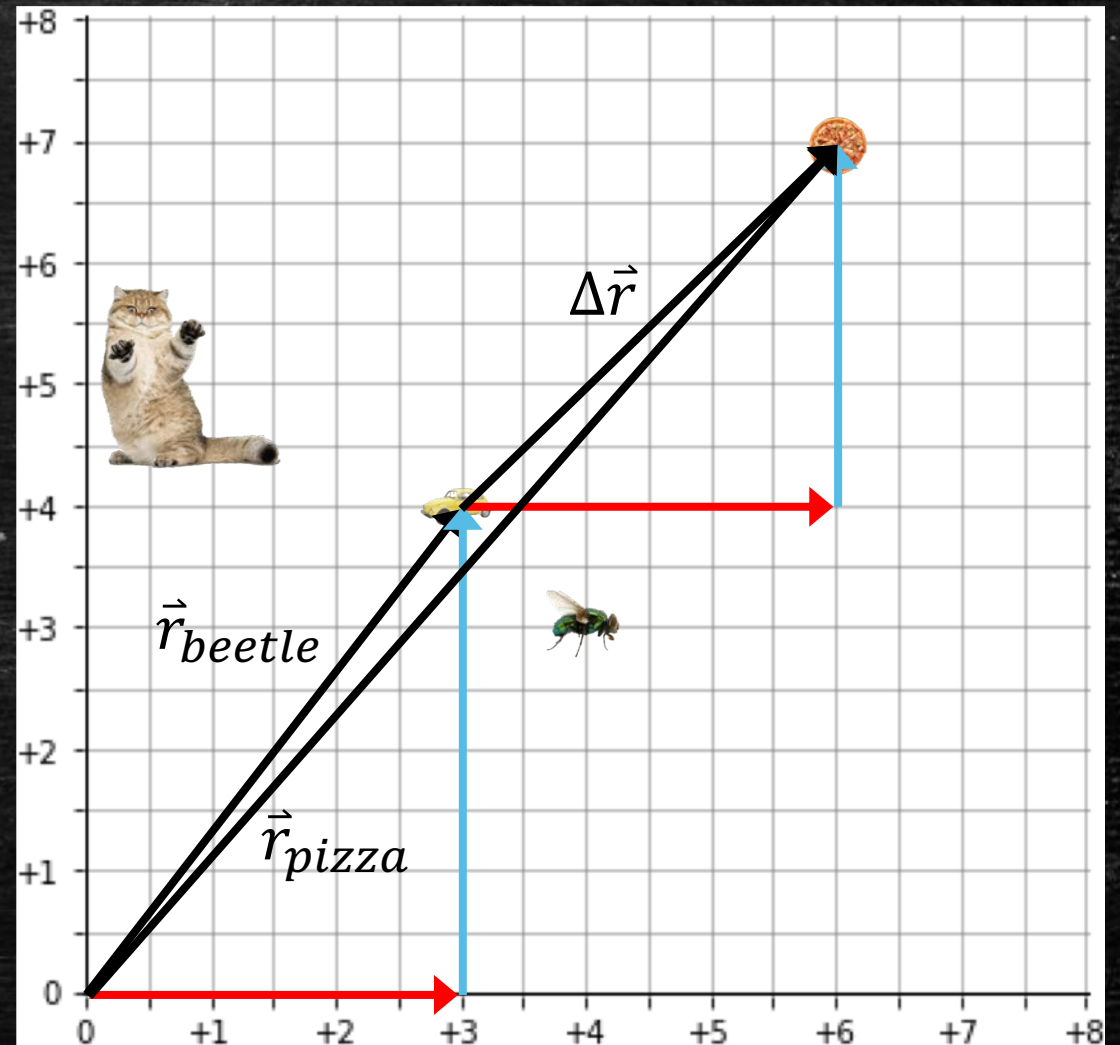
Question 7
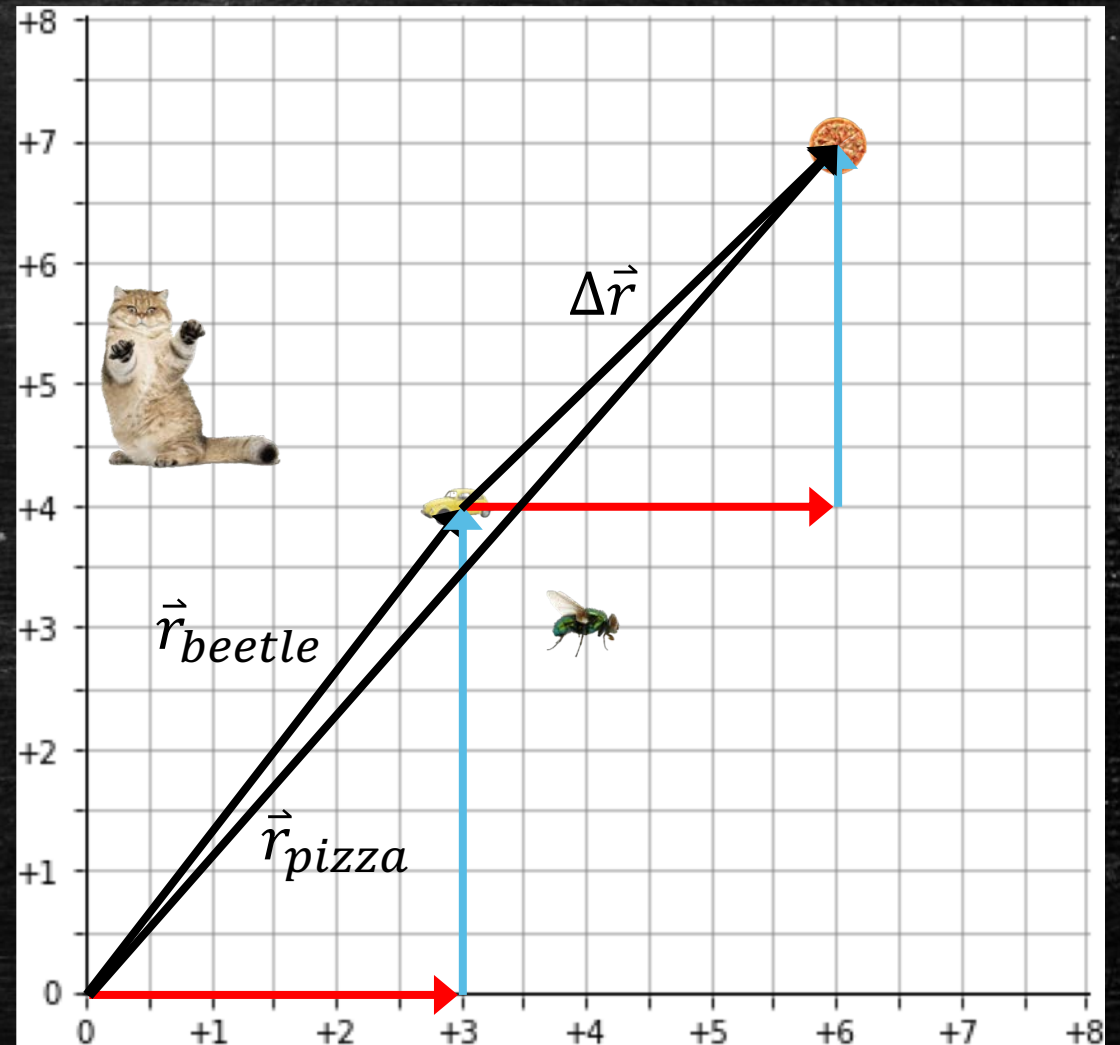
- What are the length and angle of the displacement vector of the pizza (measured from the origin)?

- Answer:

  d =9.2 miles

  θ = 49.4°

- Discussion: why can you not use the Pythagorean theorem to relate the lengths of the vectors in black?

Vector addition

- Vectors can be used to describe displacement along a path.

- When two paths start and end at the same place the net displacement must be the same, therefore

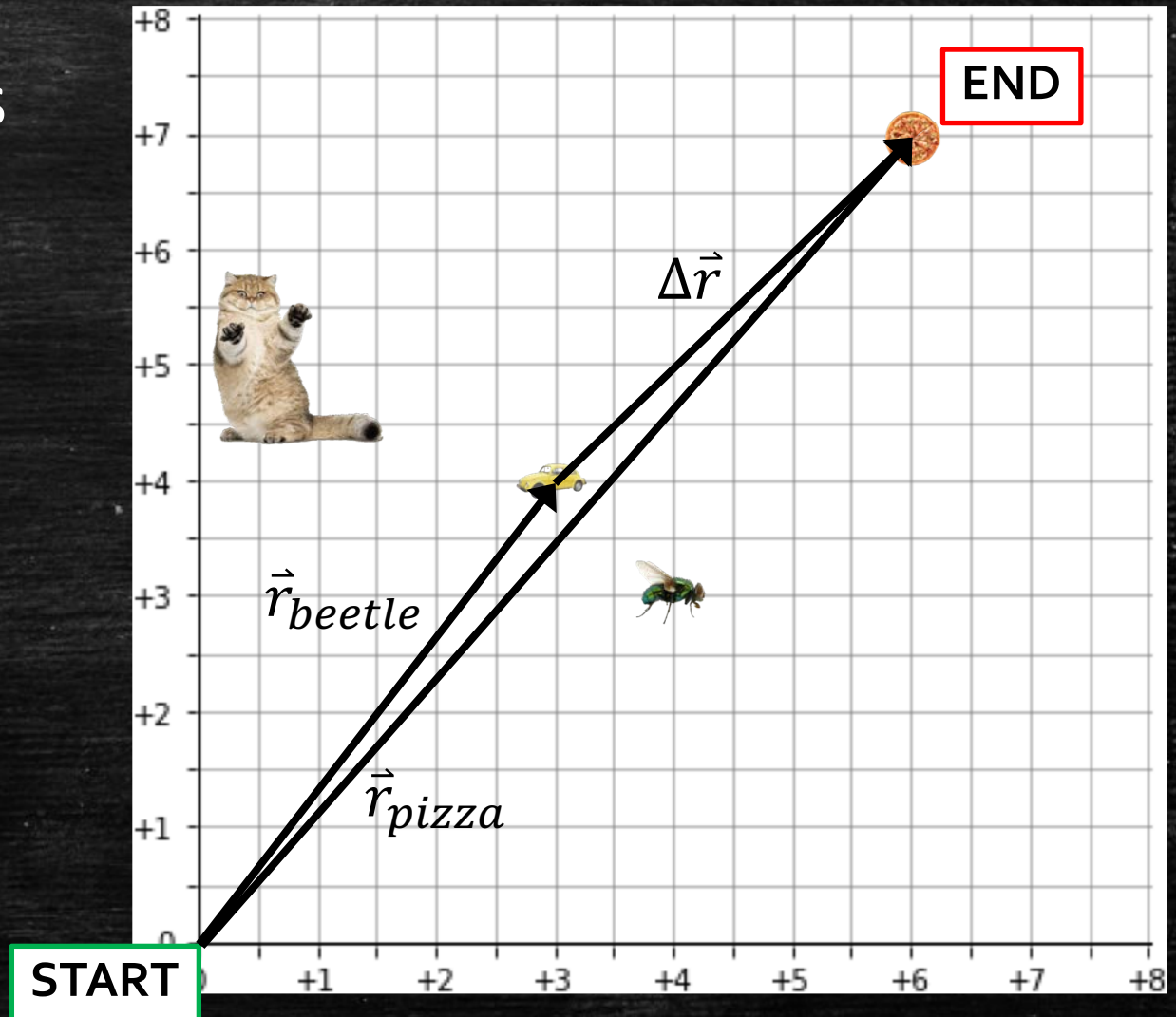$$\Delta\vec{x} + \Delta\vec{y} = \Delta\vec{r}$$
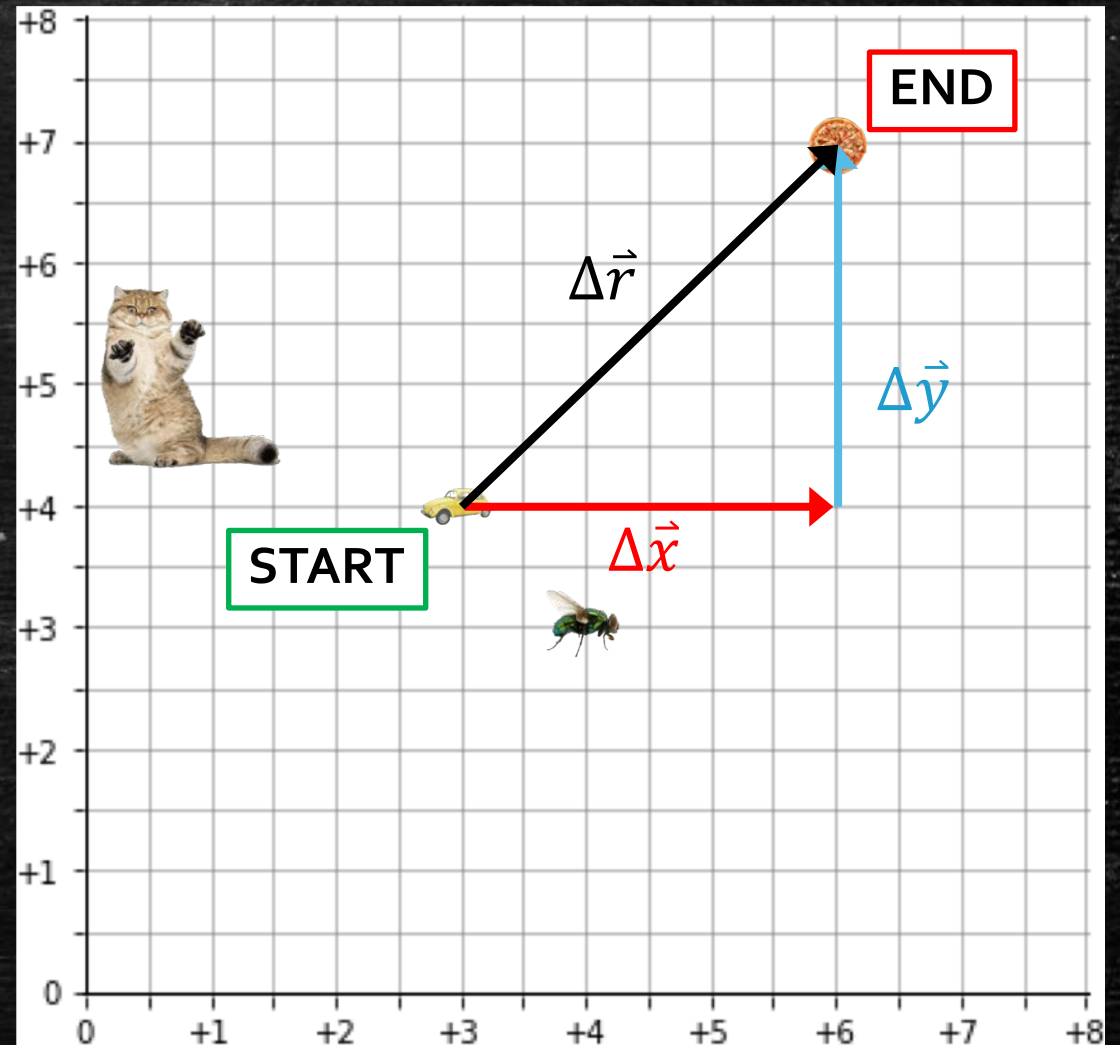
Vector addition

- This is also true of vectors that do not form a right triangle.

$$\vec{r}_{beetle} + \Delta\vec{r} = \vec{r}_{pizza}$$

Group Work

- Consider a round trip with two stops (three legs), where you first pick up a pizza, then get your cat, and finally return home (where you started).

1. Described in terms of x and y displacements, what are the components of travel for the first and second leg of this trip? Define these as x_1, y_1 and x_2, y_2?

2. By summing the components of leg 1 and leg 2, calculate the net (total) displacement vector for this trip.

3. Using only the prior information and the rules for vector summing, calculate the displacement vector for leg 3 needed to bring you home.

4. What is the total distance traveled?

5. If you use the Pythagorean theorem to calculate the hypotenuse using the distances of leg 1 and leg 3, do you get the distance of leg 2? Explain why this does or does not make sense.