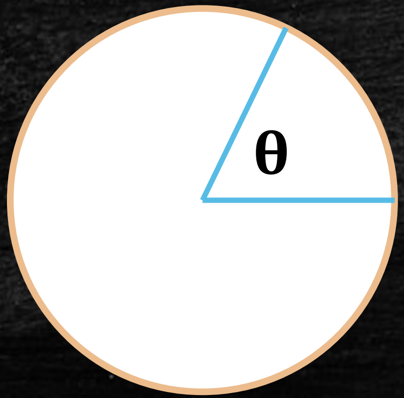
Wheels and moment of inertia

Fundamental definition

- Rotation (for this class) will consider motion on a circular path where the speeds may be changing in time.
- Can measure angular position in three different units:

$$1 \text{ revolution} = 360° = 2\pi \text{ radians}$$

- Fundamental relationship for a circle:



$$s = \theta r$$

for $\theta = 2\pi$ we have:

$$s = 2\pi r = \text{circumference}$$

Linear to rotational conversions

- We can convert between linear and rotational quantities using the following definitions:

Distance & angle: $s = \theta r$

Velocity & angular velocity: $v = \omega r$

Acceleration & angular acceleration: $a = \alpha r$

Introduction

Linear equations

$x = x_0 + v_0 t + \frac{1}{2}at^2$

$v = v_0 + at$

$x = x_0 + \frac{1}{2}(v_0 + v)t$

$v^2 = v_0^2 + 2a(x - x_0)$

Rotational equations

$\theta = \theta_0 + \omega_0 t + \frac{1}{2}\alpha t^2$

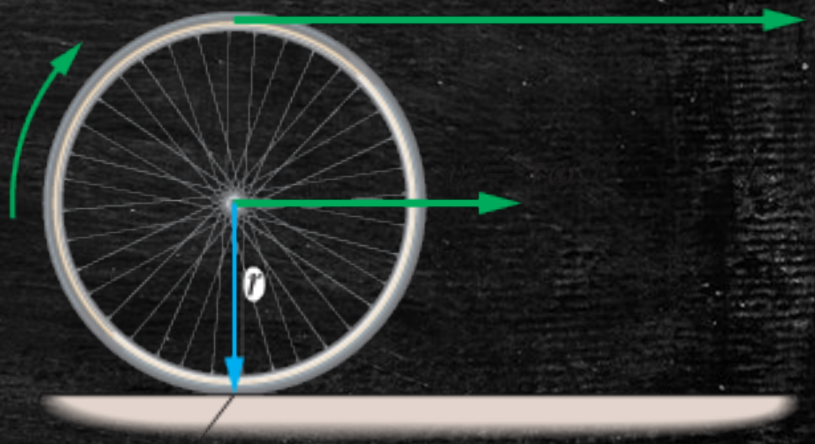
$\omega = \omega_0 + \alpha t$

$\theta = \theta_0 + \frac{1}{2}(\omega_0 + \omega)t$

$\omega^2 = \omega_0^2 + 2\alpha(\theta - \theta_0)$

Part 1: motion of wheels (without slipping)

- If wheels rotate, how is is it that we were able to use static friction when considering how cars accelerate or decelerate?



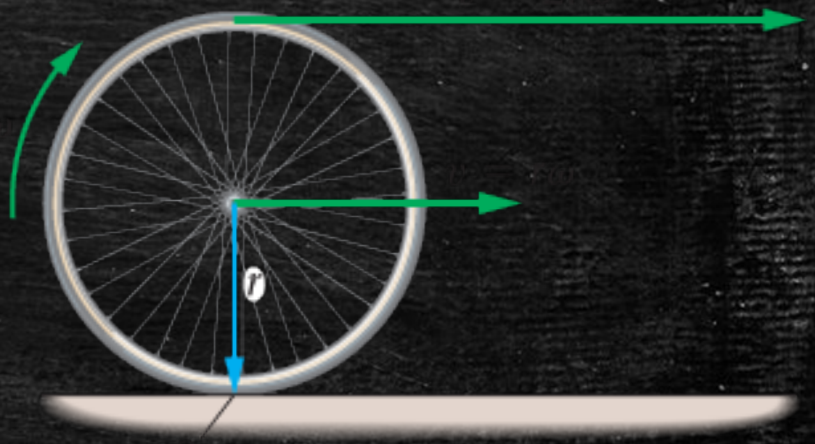
Part 1: motion of wheels (without slipping)

- If wheels rotate, how is is it that we were able to use static friction when considering how cars accelerate or decelerate?
- Rolling without slipping means that the point of contact at the road is stationary.
- This can only be true if the rotational speed is equal to the forward speed of the vehicle.

$v_{rotation} = \omega r$ (always true)

$v_{translation} = \omega r$ (true if no slipping)

Rotation + Translation



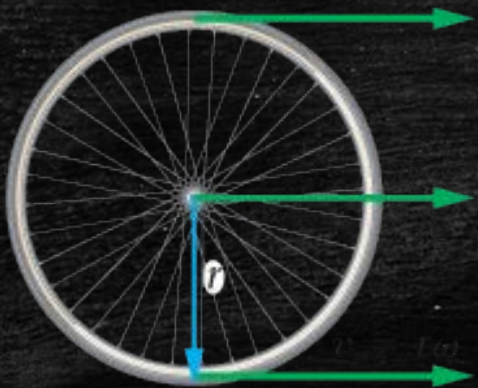
Part 1: motion of wheels (without slipping)

Rotation

$v_{rotation} = \omega r$

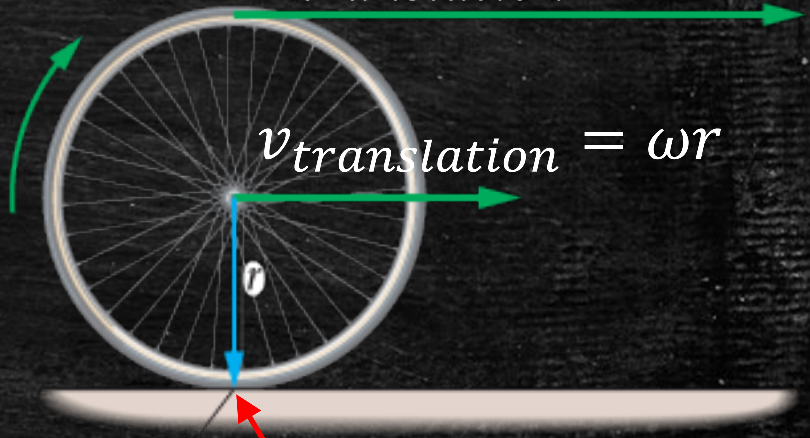
$v_{rotation} = \omega r$

Translation



$v_{translation} = v_{rotation}$

Rotation + Translation



$v_{translation} = 2\omega r$

$v_{translation} = \omega r$

$v_{translation} = 0$

Part 1: motion of wheels (without slipping)

Rank from smallest to largest angular velocity.

A train engine has a wheel diameter of 36” and travels at 40 mph.

A Beetle has a tire diameter of 20” and travels at 65 mph.

A monster truck has a tire diameter of 66” and travels at 20 mph.

Part 2: Moment of inertia

mass of m
large radius

mass of m (same)
small radius

Part 2: Moment of inertia

mass of m
large radius
small $\boldsymbol{\omega}$

mass of m (same)
small radius
large $\boldsymbol{\omega}$

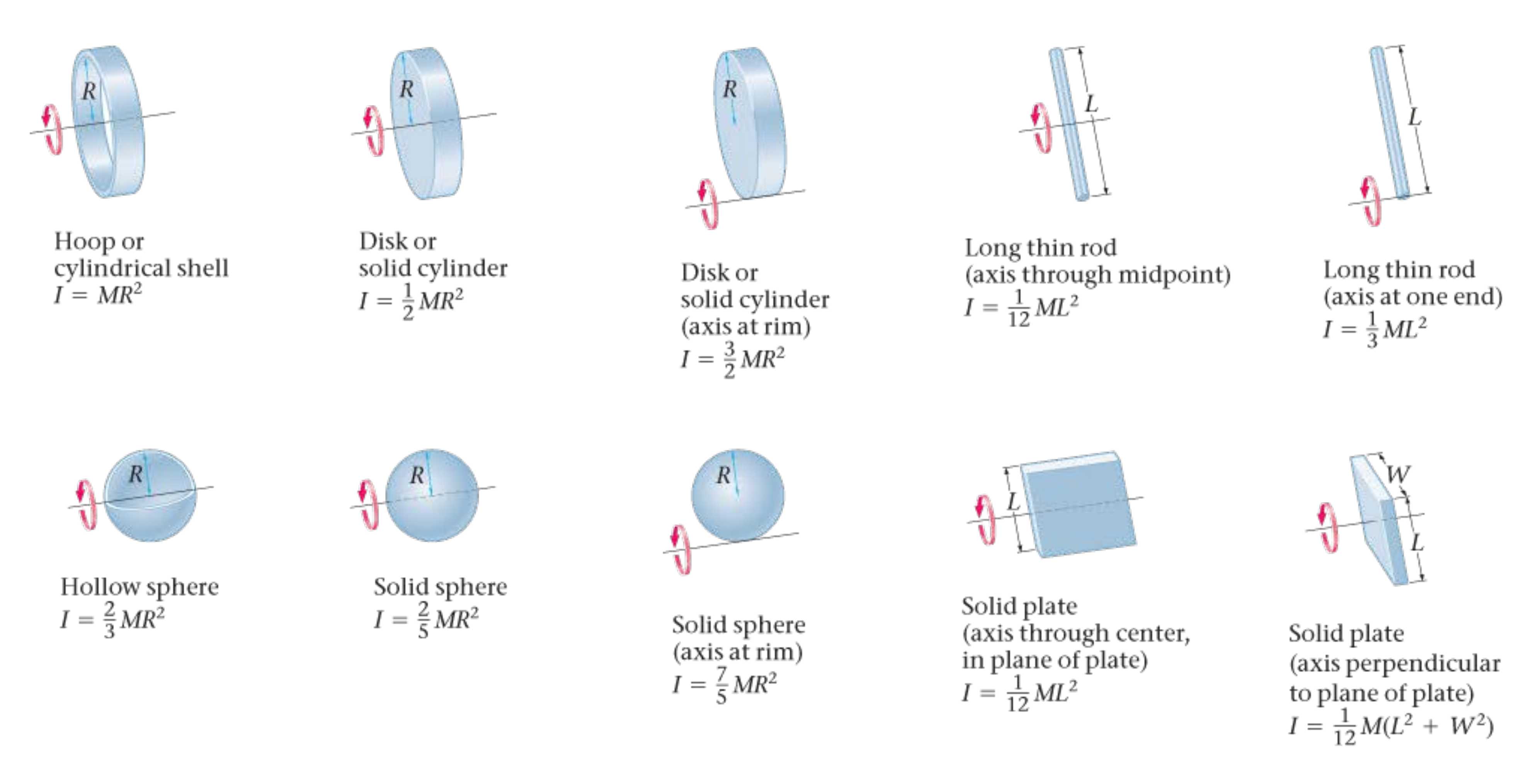
Part 2: Moment of inertia

- When there are no external influences causing a change in speed (no torques), then we can say that the following quantity is conserved

$$L = I\omega$$

where I is the moment of inertia (like mass).

Part 2: moment of inertia



Part 2: parallel axis theorem

- We can consider an object rotating about an axis that is **not** through its center of mass. To do so, we use the following formula:

$$I = I_{CM} + MD^2$$

where D is the distance from the center of mass to the new rotation axis.

For example: I_{CM} for a wheel is $\frac{1}{2}MR^2$. If we shift the rotation to the edge, that is a distance of D=R from the center so we have:

$$I_{wheel,edge} = \frac{1}{2}MR^2 + MR^2 = \frac{3}{2}MR^2$$