A Macroscopic Model of Quantum Mechanical Systems

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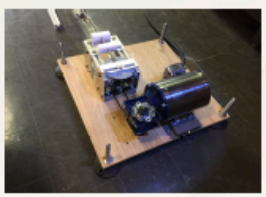
Abstract

This research uses a macroscopic system of pendula and springs to model microscopic quantum mechanical systems. A series of coupled pendula, spanning 20 feet, was designed and constructed in conjunction with a mechanism that drives oscillations at variable frequencies. A spatially varying electric potential, as may be found in an atom for example, is represented by a variation in the length of the pendula. This non-uniformity leads to wave tunneling, the process whereby quantum particles can pass through forbidden regions, and illustrates the nature of radioactive decay, among other phenomena.

The Experimental Apparatus

The machine


Drive Mechanism


Coupled Pendulum

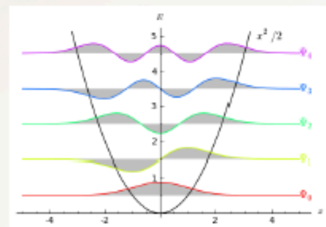
The mechanism driving the system is a 0.5 horsepower electric motor. This motor is capable of providing various frequencies to the system, necessary to observe the system with different inputs. The coupled pendulum is composed of iron masses suspended by fishing line. Plastic blocks are used to provide different effective lengths of the pendula.

Planned experiments/measurements

We plan to take measurements of the system at various frequencies and see how the system changes. We plan to observe resonant frequencies corresponding to the largest amplitude displacements of the system as well as observe quantum tunneling behaviors. With respect to quantum tunneling, we plan to install a very large potential well to the system and try to observe tunneling phenomena.

Quantum Dispersion

In a quantum mechanical model, particles are treated at as a probability distribution that propagates and diffuses through space in a wave-like manner. The structure of this probability wave depends on the potential and, in general, multiple solutions exist, corresponding to different energies. Strangely, particles have a non-zero probability of existing in regions where they would have negative kinetic energy (classically).


Source: Physics Stack Exchange

Schrödinger Equation in one dimension:

$$i\hbar\frac{\partial}{\partial t}\Psi = -\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2}\Psi + V\Psi \qquad \text{(Eq. 1)}$$

In the following, we consider ``local'' solutions of the form:

$$\Psi(x,t) = \Psi_0\cos(kx - \omega t + \phi) \qquad \text{(Eq. 2)}$$

With this Fourier representation we can make the following transformations of the differential equation:

$$\frac{\partial}{\partial t} \rightarrow -i\omega \qquad \frac{\partial}{\partial x} \rightarrow ik \qquad \text{(Eqs. 3, 4)}$$

From which we get the following dispersion relation:

$$\omega = \frac{\hbar}{2m}k^2 + \omega_0 \quad \text{with} \quad \omega_0 = \frac{V}{\hbar} \qquad \text{(Eqs. 5, 6)}$$

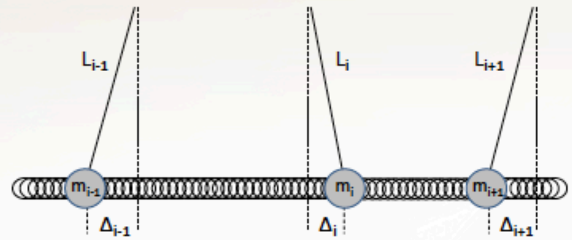
Or, taking the frequency as specified, we can solve for the wavenumber.

$$\mathrm{k} = \left(\frac{2m}{\hbar}\right)^{1/2}(\omega - \omega_0)^{1/2} \qquad \text{(Eq. 7)}$$

Wave Tunneling Effect:
The presence of the square root in Eq. 7 and Eq. 13 means that the wavenumber becomes imaginary when ω is less than ω_0.

Coupled-pendula Dispersion

Our system is a series of spring-coupled pendula.



The net force on each mass is a sum of the effects of a pendulum and

$$m_i\frac{\partial^2}{\partial t^2}\Delta_i = m_i\frac{g}{L_i}\Delta_i + k_{sp}(\Delta_{i+1} - 2\Delta_i + \Delta_{i-1}) \qquad \text{(Eq. 8)}$$

In the limit of a continuous distribution of mass, this equation becomes

$$\rho\frac{\partial^2}{\partial t^2}\Delta = \rho\frac{g}{L}\Delta + T_0\frac{\partial^2}{\partial x^2}\Delta \qquad \text{(Eq. 9)}$$

Using a similar Fourier analysis, the dispersion relation is:

$$\omega^2 = \omega_0^2 + v^2k^2 \quad \text{with} \quad \omega_0^2 = \frac{g}{L} \quad v^2 = \frac{T_0}{\rho} \qquad \text{(Eqs. 10, 11, 12)}$$

Solving for the wavenumber, we see a relationship similar to that of the quantum system:

$$k = \left[\frac{1}{v}(\omega + \omega_0)^{1/2}\right](\omega - \omega_0)^{1/2} \qquad \text{(Eq. 13)}$$