

Observation of Reversed Shear Alfvén Eigenmodes between Sawtooth Crashes in the Alcator C-Mod Tokamak

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Groups of frequency chirping modes observed between sawtooth crashes in the Alcator C-Mod tokamak are interpreted as reversed shear Alfvén eigenmodes near the $q = 1$ surface. These modes indicate that a reversed shear q profile is generated during the relaxation phase of the sawtooth cycle. Two important parameters, q_{\min} and its radial position, are deduced from comparisons of measured density fluctuations with calculations from the ideal MHD code NOVA. These studies provide valuable constraints for further modeling of the sawtooth cycle.

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The sawtooth cycle has long been recognized as a robust feature of tokamak plasmas [1]. The cycle is composed of an abrupt crash in the central electron temperature and a relaxation, or reheat, phase (see Fig. 1, $t \geq 0.25$ s). The sawtooth crash is a process whereby the magnetic field lines interior to the $q = 1$ surface (where $q = rB_\phi/R_0B_\theta$ is the magnetic safety factor, r is the minor radial coordinate, B_ϕ is the toroidal magnetic field, R_0 is the tokamak major radius, and B_θ is the poloidal magnetic field) reconnect to lines outside of the $q = 1$ surface. In this process the magnetic field “frozen-in” laws are broken leading to rapid radial transport wherein the temperature, density, and current profiles are flattened [2–4]. Following the cessation of magnetic reconnection, the plasma relaxation phase is characterized by a nearly linear increase in the central electron temperature and an inward diffusion of parallel current formerly displaced by the reconnection. Understanding the evolution of the current profile and its relation to sawtooth phenomena is important, especially for future burning plasma experiments such as ITER, where an accurate model of the physics of the crash event and the subsequent relaxation process is crucial for developing predictive capability for sawtooth control. In this Letter we report observations of frequency chirping Alfvénic modes excited during the sawtooth cycle in Alcator C-Mod tokamak plasmas, herein identified as reversed shear Alfvén eigenmodes (RSAEs, also Alfvén cascades) located near the $q = 1$ surface. RSAEs should be of great interest for future burning plasma experiments both for their use as a form of MHD spectroscopy and because Alfvénic modes may negatively impact the confinement of the energetic ions in the plasma core [5]. Importantly, the presence of RSAEs near the $q = 1$ surface is interpreted as evidence of a hollow current profile during the sawtooth cycle.

The phase contrast imaging (PCI) diagnostic [6], a type of internal reference beam interferometer, is an outstanding tool for the study of core localized Alfvénic activity. The output signal of the PCI system is a 1D image, decomposed

into 32 elements of approximately 4.5 mm chord separation in the direction of the major radius, which is linearly proportional to the line integral of the electron density perturbations along the beam path. The reporting of $q = 1$ RSAEs in Alcator C-Mod, and also in JET [7,8] (see also [9]), is due to the availability of diagnostics sensitive to small perturbations in the plasma core. In a few Alcator C-Mod experiments the amplitude of the $q = 1$ RSAEs was large enough allow for mode number identification with the magnetic pickup (Mirnov) coils.

Typical plasma parameters in Alcator C-Mod ($R_0 = 0.68$ m, $a = 0.22$ m, $\kappa \leq 1.85$) in which these modes are seen are $B_\phi = 5.4$ T, $I_p = 600$ – 800 kA and $n_{e0} \lesssim 1.5 \times 10^{20}$ m⁻³. Up to 5 MW of auxiliary heating is provided by ion-cyclotron resonance heating (ICRH) at the fundamen-

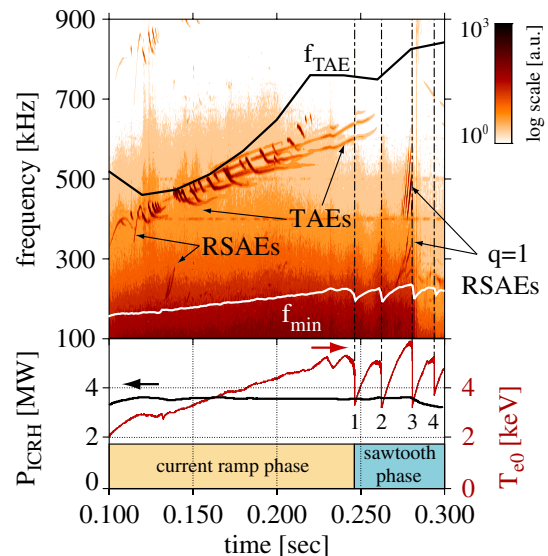


FIG. 1 (color online). PCI spectrogram showing the range of modes observed in the current-ramp and early sawtoothing phase. The numbers below the T_{e0} trace denote the sawtooth crash number relative to the beginning of the shot. $T_i/T_e = 0.8$ has been used for the calculation of f_{\min} .

tal cyclotron frequency of the H^+ minority species in a deuterium majority plasma ($n_H/n_D \approx 0.05$), creating central electron temperatures in the range of 3–6 keV [6]. Analysis of many experiments shows that $q = 1$ RSAE activity is seen in about a third of all sawtoothing L-mode shots with central electron densities below about $1.5 \times 10^{20} \text{ m}^{-3}$ and on-axis ICRH, with a strong positive correlation with electron temperature. The experiments presented here were optimized for the investigation of RSAEs during the current-ramp phase of the discharge ($t < 0.25$ sec), where a maximum in the current density exists transiently off axis as the Ohmic current diffuses toward the plasma core [10]. Application of early ICRH (turned on at 0.08 sec after plasma creation) was used to heat the plasma to retard the current penetration and create a fast ion population to drive the RSAEs at the $q = 2$ and higher surfaces. By extending the ICRH pulse to the current flat-top phase, multiple groups of frequency chirping modes similar to the RSAEs excited during the current ramp were observed following the second sawtooth crash and continued, at varied amplitudes, for many cycles thereafter.

RSAs are known to exist near the minimum of the q profile (q_{\min}) in a reversed shear equilibrium and are driven unstable by the spatial gradient of the fast ion density from neutral beams, fusion born alpha particles, or in the case of Alcator C-Mod, an ICRH driven minority ion species [11,12]. RSAEs and other Alfvénic modes have been used for “MHD spectroscopy”; that is, the inference of equilibrium parameters such as q_{\min} or q on axis (q_0) from frequency spectra [12,13]. The characteristic frequency sweep of the RSAEs has been well qualified in previous experimental [11,12], computational [14], and theoretical [15,16] studies. An approximate dispersion relation for the RSAEs can be found in Ref. [16],

$$\omega = \left[\frac{2T_e}{R_0^2 M_i} \left(1 + \frac{7T_i}{4T_e} \right) + \frac{V_A^2}{R_0^2} \left(\frac{m}{q_{\min}} - n \right)^2 \right]^{1/2} + \omega_{\Delta}, \quad (1)$$

where R_0 is the tokamak major radius, M_i is the majority ion mass, $V_A = B/\sqrt{\mu_0 n_i M_i}$ is the Alfvén velocity, B is the modulus of the magnetic field, n_i ($\approx n_e$) is the ion density,

m is the poloidal mode number, n is the toroidal mode number, and the ω_{Δ} term includes corrections for fast ion pressure and finite pressure gradients and can be neglected in the current experiments because these terms are small near the $q = 1$ surface. It should be noted that though Eq. (1) was derived in the limit of high q and high n , comparison with the ideal MHD code NOVA [17] has shown that it is accurate within 5%, and for the analysis presented here Eq. (1) is sufficient to determine mode numbers from the rate of the frequency chirping. The first term in brackets in Eq. (1) derives from the geodesic deformation of the Alfvén continuum and provides the minimum frequency offset (f_{\min}). The Alfvénic nature of the RSAEs is represented by the second term in brackets, and being inversely proportional to q_{\min} it is largely responsible for the frequency sweep as the equilibrium evolves and q_{\min} decreases. The maximum frequency of the RSAEs is set by the toroidicity-induced Alfvén eigenmode (TAE) frequency, $f_{\text{TAE}} = V_A/4\pi q R_0$. The minimum frequency and TAE frequency (taking $q = q_{\min}$) are plotted in Fig. 1 for reference. Toroidal mode numbers of 2–5 in the ion-diamagnetic direction have been directly measured by magnetic pickup coils, shown in Fig. 2. The $n = 6$ –9 mode numbers are inferred through their frequency spectra and rate of frequency chirp, which is expected to scale approximately proportional to n when the Alfvénic contribution is dominant [see Eq. (1)].

The ideal MHD code NOVA, employing a nonvariational method for solution of the eigenmode equations in full toroidal geometry, is used to model RSAEs with $q_{\min} \approx 1$. NOVA has been used formerly to explain a range of observations, including TAE behavior during sawtooth crashes in JT-60 [18], the antiballooning structure of RSAEs in TFTR [19], and mode coupling in DIII-D [20]. The analysis presented in this Letter is an extension of the MHD spectroscopy technique which shows how both the value of q_{\min} , its radial position (r_{\min}), and to some extent the value of q_0 (q at the magnetic axis), can be deduced from the spatial structures of the $q = 1$ RSAEs. When possible, experimental data are used in the NOVA analysis. The electron temperature profiles are taken from the electron cyclotron emission diagnostic, the deuterium majority

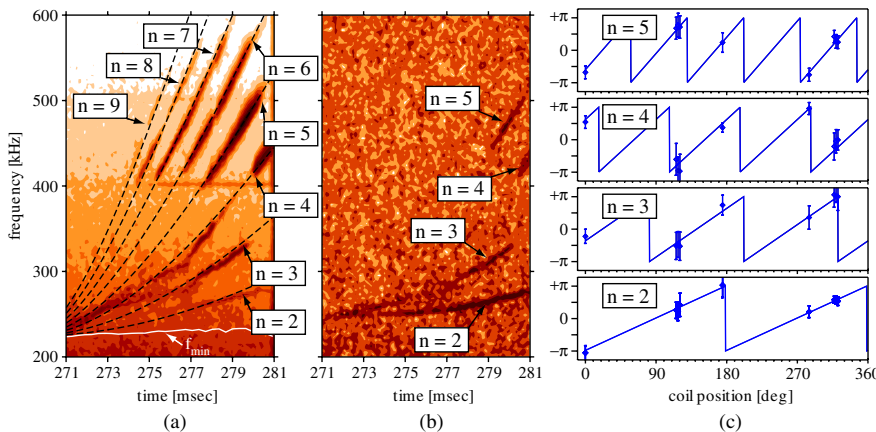


FIG. 2 (color online). Experimental data for RSAEs following the second sawtooth crash showing (a) a composite PCI spectrogram, (b) Mirnov coil spectrogram, and (c) measured mode phases as a function of toroidal angle. The mode numbers in (a) are derived from comparison with Eq. (1), shown here as the dashed lines.

ion temperature is calculated from the neutron flux ($T_i/T_e \approx 0.8$), and electron density profiles from the Thomson scattering system. The q profile is derived from a combination of edge constraints using the equilibrium code EFIT [21], which is based on edge magnetic measurements, determination of the $q = 1$ surface by the sawtooth inversion radius in the soft-x-ray data, and a core solution determined through analysis of the PCI data. Within the ideal MHD framework NOVA has produced $q = 1$ RSAE solutions for toroidal mode numbers from 2 through 9 whose spectra are in good agreement with the PCI observations, shown in Fig. 3 as the dashed overlay. NOVA shows that q_{\min} decreases to approximately 0.92 prior to the third sawtooth crash, and a linear extrapolation to the beginning of the sawtooth period estimates q to be 1.02, suggesting a complete reconnection process has occurred. In the present case the RSAEs are not observed in the first half of the sawtooth cycle and hence we have no quantitative information on the q profile during this time ($t = 0.262\text{--}0.270$ sec in Fig. 3). The stability of these modes is calculated with the kinetic extension of the NOVA code, NOVA-K [22], which shows that the $n = 3$ RSAE solution can be unstable for fast ion distributions with tail temperatures above 100 keV with a central β_H of 0.5%, $\beta_H/\beta_0 \approx 0.2$ (where $\beta_0 = 2\mu_0 p_0/B^2$ is the ratio of the kinetic pressure to magnetic pressure and the subscript “H” refers to the “hot” ion component), all reasonable values for the heating scheme used [23]. An example of a possible core q profile based on this analysis and the associated $n = 3$ eigenmode are presented in Fig. 4.

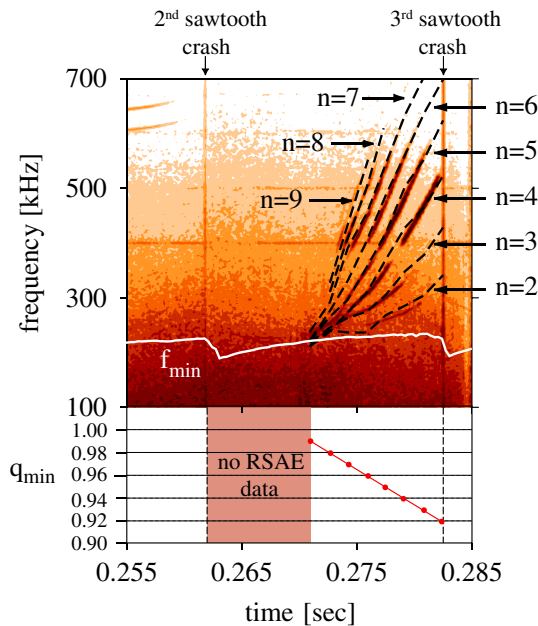


FIG. 3 (color online). PCI spectrogram showing the modes following the second sawtooth crash. NOVA results of the frequency spectra are overlaid in dashed line and have been used to infer the evolution of q_{\min} from 0.99–0.92 over the period of RSAE activity.

A synthetic PCI signal can be created from the two dimensional density perturbation calculated by NOVA by integrating the perturbations along a PCI viewing chord and convolving it with the system response function. A complex pattern of peaks and nodes is seen in the PCI signal and is attributed to integration through regions of positive and negative density fluctuations which tend to cancel. The spacing of the peaks has been studied in the synthetic PCI signal and varies linearly with the value of r_{\min} used in the simulation. It is the dominance of the value of r_{\min} in determining the spatial structure of the synthetic diagnostic signal that allows this parameter to be determined. The dependence of the value of q_0 has also been explored using parameter scans in NOVA. Our studies suggest that very low shear, represented here by the quantity $\delta q \equiv q_0 - q_{\min}$, is required to generate the range of observed RSAEs. The $n = 4, 5, 6$ RSAEs appear to be especially sensitive to this parameter and cease to exist when the δq exceeds 0.02. At this time there is no additional experimental measurement of the value of q_0 in Alcator C-Mod. Two free parameters are used in the comparison of the experimental and synthetic PCI signals. Though the channel to channel spacing of the PCI signal is known very accurately, the position of the array in major radius is known only to ± 1 cm. To account for this the experimental data are shifted in major radius when comparing to the synthetic diagnostic signal to minimize the rms difference and find the best fit. The second free parameter is the absolute value of the synthetic diagnostic signal which accounts for the fact that NOVA calculates relative fluctuation levels. The best-fit solution is found by scanning the synthetic diagnostic signal amplitude until the rms difference with the experimental data is minimized.

A comparison of the synthetic and experimental PCI signals is presented in Fig. 5 for an $n = 3$ RSAE, where

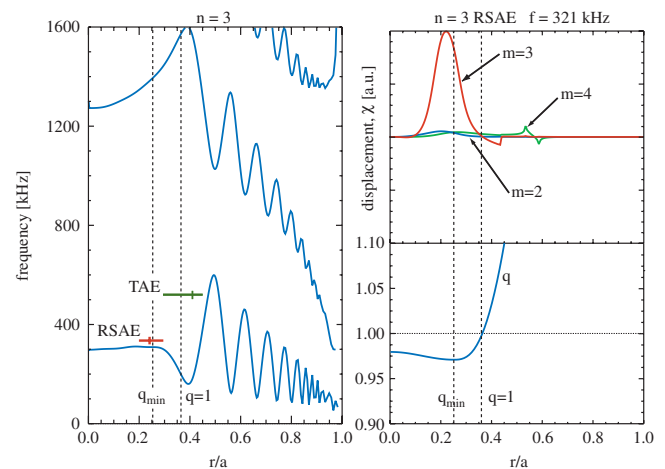


FIG. 4 (color online). Left: The $n = 3$ Alfvén continuum calculated for the q profile shown at bottom right, with $q_0 = 0.96$ and $q_{\min} = 0.95$ at $r/a = 0.25$ corresponding to the conditions near $t = 0.275$ s in Fig. 3. Top right: An $n = 3$ RSAE solution with the typical single dominant poloidal mode. The q profile is shown on the bottom right for reference.

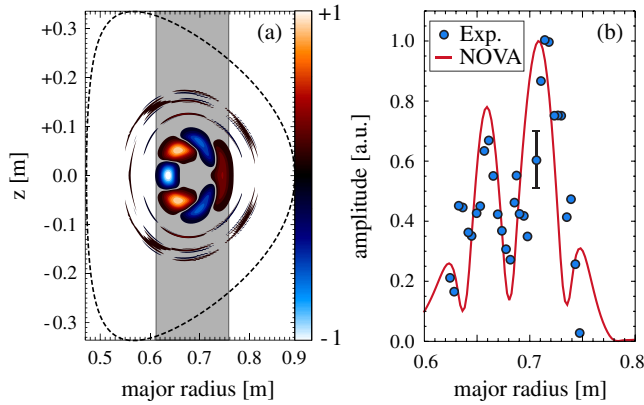


FIG. 5 (color online). (a) Real part of the 2D density perturbation structure for an $n = 3$ RSAE calculated by NOVA with the PCI beam path marked by the gray background, and (b) the comparison of the measured radial structure from PCI (blue circles) to the synthetic diagnostic result using the NOVA solution (red line).

the synthetic PCI signal represents the best-fit solution. Similar calculations have been performed for $n = 4, 5, 6$ and together determine r_{\min}/a to be near 0.20. The magnitude of the central shear (δq) was scanned over the range $\delta q = 0.01-0.05$ for each of the modes and at each value of r_{\min} . The conclusion from this study is that RSAE solutions are found only for $\delta q \leq 0.02$ at $r_{\min}/a = 0.20$ and exist up to $\delta q = 0.04$ for larger r_{\min}/a . Larger values of shear force the RSAEs into the continuum which causes them to become heavily damped. As a final note we would like to mention that the presence of $q > 1$ following the sawtooth crash may allow for down-chirping RSAEs to exist. This possibility has been explored with NOVA with the result that relatively undamped and unstable solutions have been discovered [24]. The existence of the down-chirping solutions seems to require a local maximum in q near the core, equivalent to a peak in the Alfvén continuum, around which the RSAEs form.

In summary, we have shown that the frequency chirping modes excited during the sawtooth cycle are consistent with the behavior of $q = 1$ RSAEs. Calculations performed with the code NOVA, using a weakly reversed shear q profile with $q_0 - q_{\min} \lesssim 0.02$, are able to produce RSAE solutions for the range of mode numbers identified in the experiments. Calculations of the growth rates show that the RSAEs can be unstable in the presence of an energetic hydrogen ion tail produced with minority ion-cyclotron resonance heating. Using the measured frequency spectra of the RSAEs as a diagnostic for q , the NOVA calculations show that q is likely greater than unity immediately following the crash and may decrease to values in the range of 0.92–0.95. The constraints derived from the analysis of the RSAEs show remarkable agreement with recent results using equilibrium reconstructions from motional Stark effect constrained EFIT for the sawtooth phase of plasmas in the DIII-D tokamak [25]. Future experiments would

benefit from multiple core perturbation diagnostics and energetic ion loss detectors to study the relation between the energetic ions, Alfvén eigenmodes, and sawtooth crashes.

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