Rendezvous revisited: The search for fast-intercept solutions

Eric M. Edlund[a)]
Department of Physics, SUNY Cortland, Cortland, New York, 13045

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Orbital interception scenarios typically involve a chaser that is actively maneuvered to encounter an inertial target and may be undertaken for a variety of purposes, including docking spacecraft or colliding with an asteroid for planetary defense studies. Viable intercept trajectories are constrained by the free-fall path of the target and by auxiliary conditions such as the available time or fuel budget. Whereas a constraint on the time to intercept is central to the (extensively studied) Lambert problem, a less common but more visually compelling constraint is that of the available fuel for intercept. This was the basis of a recent study [E. M. Edlund, Am. J. Phys. **89**, 559–566 (2021)], which analyzed one of the two families of possible intercept solutions that were identified. The second family, studied in more detail here, describes intercepts at all points in the orbit and has the interesting property that it admits fast-intercept solutions. This work concludes the analysis of this problem; it develops a general condition that describes both families of intercepts, presents representative solutions, and considers the sensitivity of these solutions to errors in the control parameters.
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I. INTRODUCTION

Long before space travel was considered a possibility, there was great interest in the intercept problem, first made famous by Lambert in 1761. The Lambert problem, as it is now known, seeks the velocity of a body given astronomical measurements of its position at two times. The solution allows the position of the body to be determined at any later time, thereby providing great predictive capability. This problem spurred seminal developments in celestial mechanics and analysis by some of the best minds of the time.[1,2] There is a long and rich history of the literature stemming from the Lambert problem, which was reinvigorated in the 1950s with the development of spaceflight. Modern incarnations of this problem often have a goal of finding the thrust vector that will allow an actively maneuverable craft to intercept an inertial target (meaning a craft on a "free-fall" or "ballistic" path) at a specific time.

A number of recent articles have focused on interesting and insight-building problems involving orbital dynamics, including analysis of the Lambert problem using a search method[3] and using the Hohman transfer in introductory physics courses.[4] An analysis of close-proximity rendezvous using the Clohesy–Wiltshire equations was presented in Ref. 5, a set of multi-thrust methods for achieving escape velocity from an initially circular orbit was given in Ref. 6, and a detailed analysis of Kepler's problem that examines all possible paths between two points in space was provided in Ref. 7. Reference 8 approached the intercept problem by considering a constraint of a specified Δv, which can be thought of as a constraint on the quantity of available fuel. Therefore, it was argued that this particular variation is an excellent problem for undergraduate students, because (in contrast to a constraint on the intercept time) the velocity constraint is more readily visualized and developed deeper intuition for motion on elliptical trajectories. A simple HTML-Javascript simulator was provided to help visualize and gamify this study of orbital dynamics.

While the work of Ref. 5 identified two possible families of intercept solutions, it analyzed only the first family in which intercept/rendezvous occurs after an integer number of chaser orbits. However, the second family of intercept solutions is particularly interesting, because it allows for fast intercepts that occur before the target has completed a full orbit. Such fast-intercept maneuvers may be relevant to planetary defense against civilization-threatening asteroids or comets where a short, but not pre-determined, time may be of the essence.[9] The Planetary Defense Coordination Office, a division within NASA, tracks known threats and develops mitigation plans.[10] As part of that effort, NASA's DART mission successfully intercepted the asteroid Dimorphos, the smaller of a double-asteroid pair, on September 26 of 2022 to test deflection by kinetic impact.[11] Other recent developments in this line of work include space debris collectors[12] and an actively maneuvering Russian satellite thought to be a satellite hunter of sorts.[13]

One can, of course, find solutions to the intercept problem using a "guess-and-check" method, where initial parameters are guessed, the trajectories are checked (using something like the HTML-javascript program distributed with Ref. 5), and then the parameters are iterated until an acceptable solution is found. This approach is effective but falls short of what is typically expected of a physics analysis in at least three important ways. First, such calculations necessarily rely on an external tool to plot the trajectories and, therefore, outsources the physics analysis to someone else. Second, guess-and-check solutions typically require many iterations and are not very efficient, especially if one wants to examine a wide range of parameters. Third, when a solution is finally realized, one has no way of determining whether it is in any way ideal or optimal. In contrast, an analytic solution requires greater initial effort, but also rewards with physical insight and provides great flexibility to efficiently explore parameter dependency and the sensitivity of solutions to errors.

This paper proceeds with some preliminaries and a recap of important results in Sec. II, followed by a formal definition of the problem and derivation of the intercept condition in Sec. III, with a discussion of solutions in Sec. IV A and sensitivity of solutions in Sec. IV B. A brief analysis of the

