Interception and rendezvous: An intuition-building approach to orbital dynamics

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The problem of rendezvous, the meeting of spacecraft in orbit, is an important aspect of mission planning. We imagine a situation where a chaser craft, initially traveling on the same circular orbit as its target and separated from it by a known distance, must select an initial thrust vector that will allow it to meet the target (interception) followed by a second thrust vector that will allow it to match velocities with the target (rendezvous). The analysis presented here provides solutions to this problem in simple algebraic forms while offering many rich challenges that support intuition-building exercises for students across a range of skill levels. An html-javascript orbit calculator is made available with this manuscript as a supporting visual aid and can be used to test the analysis and explore the consequences of different orbital intercept solutions.

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I. INTRODUCTION

In his book *Carrying the Fire*, former Apollo 11 astronaut Michael Collins discusses the counter-intuitive process required for rendezvous in orbit. For example, to close a gap with a target directly ahead, pilots must slow their craft to drop into a lower altitude orbit that advances their angular position relative to the target.[1] As we will see, this maneuver is just one of many possible solutions to the interception problem. Collins describes in great detail the extensive training undertaken to make such situations second nature, including countless hours in mock-ups and simulators, the meticulous study of equipment, and the rehearsal of procedures for both planned and contingent operations. In preparation for the Apollo 11 mission, Collins trained for no less than eighteen different cases for rendezvous of the modules, since co-planar conditions could not be guaranteed. Prior to the lunar landing of Apollo 11, the Apollo 10 mission completed a lunar orbit wherein the astronauts conducted a co-planar rendezvous of the lunar module and the command and service module.

Between the earliest docking maneuvers of the Gemini and Apollo missions and the present, there have been many such feats, under diverse conditions and with a wide range of craft, both manned and unmanned. The first unmanned docking event took place in 1967 between two variants of the Soviet Soyuz spacecraft.[2] SpaceX successfully docked its Crew Dragon Module with International Space Station (ISS) twice in 2020, and the Chang'e 5 robotic probe landed on the lunar surface and returned to orbit where it docked with and transferred lunar rock samples to the orbit-return vehicle. In most of these scenarios, one of the craft (e.g., the ISS) is on an unpowered trajectory and is considered the target, while the other craft, being capable of active trajectory modification (e.g., the Crew Dragon Module), is considered the chaser. For missions to the ISS, transition onto the target's orbit for a docking maneuver typically involves initial insertion of the chaser into a holding orbit, often with a two-stage transfer through an intermediate phasing orbit, all of which is typically completed close to the orbital plane of the target.[3,4] A Hohmann transfer or bi-elliptic maneuver is often used to intercept the target. The total time for completing the rendezvous and docking of manned spacecraft with the ISS can be less than one day to as much as two days.[3]

Intermediate-level classical mechanics textbooks discuss orbital dynamics, but the mechanics of orbital intercept and rendezvous are notably absent from such discussions, which often include little more than a brief discussion of the Hohmann transfer.[5–8] Advanced, discipline-specific texts focusing on orbital dynamics often couch these discussions in the language of differential equations and three-dimensional vectors, resulting in a complete but complicated analysis that is quite challenging for undergraduate students.[4,9–11] The approach taken in this paper differs in that it avoids using differential equations[4,9,10,12–14] and instead focuses on geometric considerations.[15,16] By reducing the mathematical overhead, it emphasizes other important aspects of the general problem-solving framework, including the definition and mathematical interpretation of the intercept and rendezvous conditions, motion on elliptical orbits, the role of conserved quantities, and the utility of approximate solutions.

The central goal of this analysis is the discovery of the thrust vector for the chaser, defined in terms of a magnitude and angle that will enable it to intercept the target. Rendezvous is a second condition wherein the craft are at the same location and have matched velocities.[9] This analysis is restricted to co-orbital initial conditions that do not represent the situation of most intercept and rendezvous maneuvers, though this situation might arise, for example, when relocating craft that were placed in co-orbital trajectories for prior purposes or for practice of orbital maneuvers as was done on the Apollo 10 mission. Within these limitations, the analysis presented here seeks all possible solutions for orbital intercept given co-orbital initial conditions, including solutions of multiple and disparate number of orbits between the two craft. Simulations were and continue to be an important aspect of flight training,[1] and when used appropriately, allow the user repeated opportunities to apply theory to complete specific tasks. An auxiliary html-javascript orbit calculator accompanying this paper provides a visual aid that can be used to explore the consequences of various trajectory changes, including sensitivity to control parameters.

