Electronics Chapter 4 Homework
4.2, 4.6, 4.7, 4.15, 4.21, 4.25

4.2 $f(t) = 5\sin 2\pi t$ is already a fourier series, it is written as a sum (1 term) of sin waves w/ integer multiples of ω_0 (2π in this case).

4.6 see companion document

4.7 a) the bandwidth is where the gain drops below $1/\sqrt{2}$, this is @ about $f = 7\,Hz$

b) $$F_{in}(t) = \frac{4}{\pi}\sin(2\pi f_0 t) + \frac{4}{3\pi}\sin(2\pi 3 f_0 t) + \frac{4}{5\pi}\sin(2\pi 5 f_0 t)$$

$$+ \frac{4}{7\pi}\sin(2\pi 7 f_0 t) + \frac{4}{9\pi}\sin(2\pi 9 f_0 t)$$

where $f_0 = 1\,Hz$

$$F_{out}(t) = \frac{4}{\pi}\Big[\sin 2\pi f_0 t + \frac{1}{3}\sin 2\pi 3 f_0 t + \frac{1}{5}\sin 2\pi 5 f_0 t$$

$$+ \frac{1}{7}\left(1 - \frac{1}{4}(6\,Hz - 7\,Hz)\sin(2\pi 7 f_0 t)\right)$$

$$+ \frac{1}{9}\left(1 - \frac{1}{4}(6\,Hz - 9\,Hz)\sin(2\pi 9 f_0 t)\right)\Big]$$

$$= \frac{4}{\pi}\Big(\sin 2\pi f_0 t + \frac{1}{3}\sin 2\pi 3 f_0 t + \frac{1}{5}\sin 2\pi 5 f_0 t$$

$$+ \frac{3}{28}\sin(2\pi 7 f_0 t) + \frac{1}{36}\sin 2\pi 9 f_0 t\Big)$$

c) see companion document

~~4.21 In a first~~

4.15 see companion document

4.21 In a first order system

$$\tau \frac{dX_{out}}{dt} = -X_{out} + K\, x_{in}$$

and $\frac{dx_{out}}{dt}$ can be approximated as

$$\frac{dx_{out}}{dt} = \frac{\Delta X_{out}}{\Delta t} = \frac{X_n - X_{n-1}}{0.1s}$$

from the graph in the companion document

$$\tau = 2.86\,s \quad \text{and}$$

$$\frac{K\,x_{in}}{\tau} = 1.8$$

$$K\,x_{in} = 1.8 \times 2.86s = 5.14$$

to find the static sensitivity we need x_{in},
assuming $x_{in} = 1$

$$K = 5.14$$

4.25 $F_{ext} = 20N \sin(0.75t)$

$m = 10\ kg$

$k = 12\ N/m$

$b = 10\ Ns/m$

$\omega = 0.75\ \frac{rad}{s}$

$\omega_n = \sqrt{\frac{K}{m}} = \sqrt{\frac{12N/m}{10\,kg}} = \sqrt{1.2}\ \frac{rad}{s}$

$= 1.095\ rad/s$

$\xi = \frac{b}{2\sqrt{km}} = \frac{10}{2\sqrt{120}} = \frac{1}{2\sqrt{1.2}}$

$$\frac{X_0}{F_i/k} = \frac{X_0}{20N/12N/m} = \frac{X_0}{5/3\,m} = \frac{1}{\sqrt{\left[1-\left(\frac{\omega}{\omega_n}\right)^2\right]^2 + 4\xi^2\left(\frac{\omega}{\omega_n}\right)^2}}$$

$$= \frac{1}{\sqrt{\left[1-\left(\frac{9/16}{1.2}\right)\right]^2 + \frac{1}{1.2}\left(\frac{9/16}{1.2}\right)}}$$

$$= \frac{1}{\sqrt{0.609 + 0.391}} = 1$$

$$X_0 = 5/3 \text{ m} \simeq 1.67 \text{ m}$$

$$\phi = -\arctan\left(\frac{2\xi}{\frac{\omega_n}{\omega} - \frac{\omega}{\omega_n}}\right) = -\arctan\left(\frac{\frac{1}{\sqrt{1.2}}}{\frac{\sqrt{1.2}}{3/4} - \frac{3/4}{\sqrt{1.2}}}\right)$$

$$= -\arctan\left(\frac{1}{1.2 \times 4/3 - 3/4}\right)$$

$$= -\arctan\left(\frac{1}{0.85}\right)$$

$$= -50° = -0.27\pi$$

$$X(t) = 5/3 \text{ m} \sin\left(0.75\,t - 0.27\pi\right)$$

So the key is to find X_0 & ϕ_0 at the drive frequency.