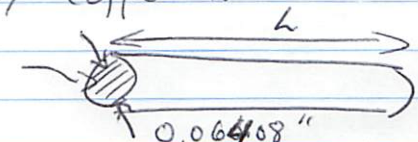
Electronics Home Work Sol^ns Problem set 1

2.1, 2.3, 2.10, 2.16, 2.17, 2.19, 2.21

2.1 The resistance of a piece of copper is

$$R = \rho \frac{L}{A}$$



$\rho = 1.7 \times 10^{-8}\ \Omega m$ (according to the text)

Note that this value is only correct if $T \simeq 20°C$, @ 200°C the resistivity is about twice as much (source Engineering toolbox.com)

$$R = \frac{1.7\times10^{-8}\ \Omega m \ 10^3 m}{\frac{\pi}{4}\left(0.06408^{\text{inch}} \times 2.54\times10^{-2}\frac{m}{\text{inch}}\right)^2}$$

$= 8.2\ \Omega$ (@20°C)

which is similar to the NEC value @ 75°C of 10.1Ω for solid wire and 10.3Ω for stranded wire (Chapter 9, table 8 NEC 2014)

2.3

$R_1 = 10\times10^2 \pm 5\% = 1\times10^3 \pm 50\ \Omega$

$R_2 = 25\times10^1 \pm 5\% = 250 \pm 1.25\Omega$

$$\frac{1}{R_{eq}} = \frac{1}{250\Omega} + \frac{1}{1000\Omega} = \frac{5}{1000\Omega} = \frac{1}{200\Omega}$$

$$R_{eq} = \frac{1}{\frac{1}{R_1}+\frac{1}{R_2}}$$

$$\delta R_{eq} = \sqrt{\left(\frac{\partial R_{eq}}{\partial R_1}\delta R_1\right)^2 + \left(\frac{\partial R_{eq}}{\partial R_2}\delta R_2\right)^2}$$

$$\frac{\partial R_{eq}}{\partial R_1} = \frac{-\,-\frac{1}{R_1^2}}{\left(\frac{1}{R_1}+\frac{1}{R_2}\right)^2} = \left(\frac{R_{eq}}{R_1}\right)^2$$

$$= \sqrt{\frac{R_{eq}^4}{R_1^4}\delta R_1^2 + \frac{R_{eq}^4}{R_2^4}\delta R_2}$$

$$\frac{\delta R_{eq}}{R_{eq}} = R_{eq}\sqrt{\frac{1}{R_1^2}\left(\frac{\delta R_1}{R_1}\right)^2 + \frac{1}{R_2^2}\left(\frac{\delta R_2}{R_2}\right)^2}$$

$$= \frac{R_{eq}}{R_1}\frac{\delta R_1}{R_1}\sqrt{1 + \frac{R_1^2}{R_2^2}} = 5\% \times\frac{200\Omega}{1000\Omega}\sqrt{1+4^2}$$

$$= 5\% \times \frac{\sqrt{17}}{5} = 4\%$$

2.3 cont. to find the tolerance we need to propagate the uncertainty in R_1, δR_1, and R_2 δR_2 into δR_{eq}.

by standard error propagation

$$\delta R_{eq} = \sqrt{\left(\frac{\partial R_{eq}}{\partial R_1}\right)^2 \delta R_1^2 + \left(\frac{\partial R_{eq}}{\partial R_2}\right)^2 \delta R_2^2}$$

$$\frac{\partial R_{eq}}{\partial R_1} = \frac{\partial \frac{1}{\frac{1}{R_1}+\frac{1}{R_2}}}{\partial R_1} = \frac{1/R_1^2}{\left(\frac{1}{R_1}+\frac{1}{R_2}\right)^2} = \left(\frac{R_{eq}}{R_1}\right)^2$$

so

$$\delta R_{eq} = \sqrt{\left(\frac{R_{eq}}{R_1}\right)^4 \delta R_1^2 + \left(\frac{R_{eq}}{R_2}\right)^4 \delta R_2^2}$$

or

$$\frac{\delta R_{eq}}{R_{eq}} = \frac{R_{eq}}{R_1}\sqrt{\frac{\delta R_1^2}{R_1^2} + \left(\frac{R_1}{R_2}\right)^2 \left(\frac{\delta R_2}{R_2}\right)^2}$$

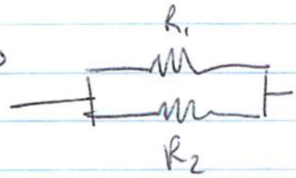
$$= \frac{200}{1000}\sqrt{(0.05)^2 + \left(\frac{1000}{250}\right)^2 (0.05)^2}$$

$$= \frac{1}{5}\, 0.05 \sqrt{1+16} = 0.04 \approx 5\%$$

so

Red black brown gold is equivalent.

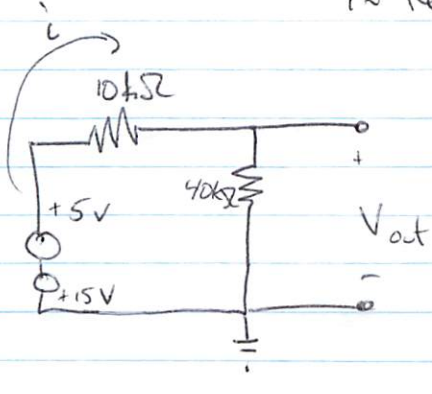
2.10



$$R_{eq} = \frac{R_1 R_2}{R_1 + R_2} \quad \text{or} \quad \frac{R_2}{1 + R_2/R_1}$$

in the limit $R_2/R_1 \to 0$ $\quad R_{eq} \Rightarrow \frac{R_2}{1+0} = R_2$

2.16



using kirchhoffs voltage law starting @ ground we have

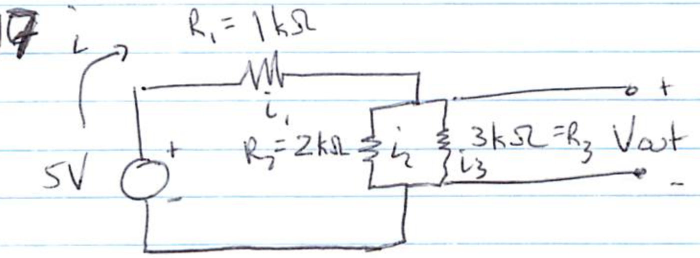
$$-15V + 5V - i\,10k\Omega - i\,40k\Omega = 0$$

$$50k\Omega\, i = -10V$$

$$i = -\frac{1}{5}\, mA$$

$$V_{out} = i\,40k\Omega = -8V$$

2.17



first the equivalent resistance for R_2 & R_3 is

$$\frac{R_2 R_3}{R_2 + R_3}$$ so the total resistance

of this resistor network is $R_1 + \dfrac{R_2 R_3}{R_2 + R_3}$

$$= 1k\Omega + \frac{2k \cdot 3k}{5k}\,\Omega$$

$$= 2.2\,k\Omega$$

$V = i R_{eq}$ or

$i = \dfrac{V}{R_{eq}} = \dfrac{5V}{2.2k\Omega} = 2.27\,mA$ out of the battery.

$i_1 = i = \boxed{2.27\,mA}$

for i_3, we can use the node rule (KCR)

☆ $i = i_2 + i_3$

and that R_2, R_3 are parallel so

$V_2 = V_3$

☆☆ $i_2 R_2 = i_3 R_3 \Rightarrow i_2 = i_3 \dfrac{R_3}{R_2}$

combining ☆ and ☆☆ gives

$i = i_3 \dfrac{R_3}{R_2} + i_3 \Rightarrow i_3 = \dfrac{i}{1 + R_3/R_2}$

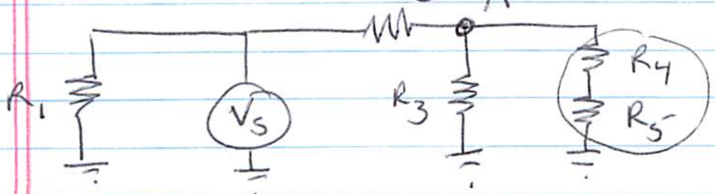
$$= \frac{2.27\,mA}{1 + 3/2}$$

$$\boxed{= 0.91\,mA}$$

and $V_2 = i_2 R_2 = (i - i_3) R_2$

$= (1.36\,mA)\,2k\Omega = \boxed{2.73V}$

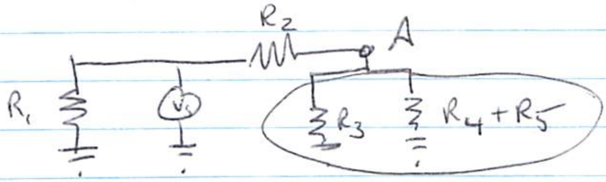
2.19 first lets clean up this diagram



series

(1)

is equivalent to

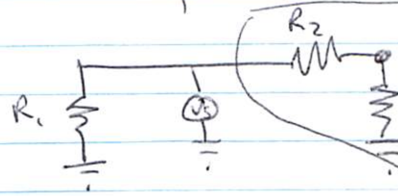


parallel

(2)

is equivalent to

series

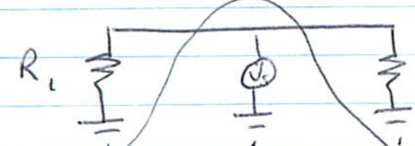


$$\frac{1}{\frac{1}{R_3}+\frac{1}{R_4+R_5}} = \frac{R_3(R_4+R_5)}{R_3+R_4+R_5}$$

(3)

is equivalent to

parallel



$$R_2 + \frac{R_3(R_4+R_5)}{R_3+R_4+R_5}$$

(4)

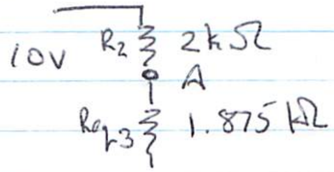
is equivalent to

$$\frac{R_1\left(R_2+\frac{R_3(R_4+R_5)}{R_3+R_4+R_5}\right)}{R_1+R_2+\frac{R_3(R_4+R_5)}{R_3+R_4+R_5}}$$

and with $R_1 = 1k\Omega$, $R_2 = 2k\Omega$, $R_3 = 3k\Omega$, $R_4 = 4k\Omega$, $R_5 = R_1$

$$\boxed{R_{eq} = \frac{1k\left(2k + 3k\frac{(5k)}{8k}\right)}{1k+2k+3k\frac{(5k)}{8k}} = \frac{3.875k}{4.875} = 795\Omega}$$

looking @ circuit 3 Node A is in a voltage divider



so by eq. 2.24 $V_A = \frac{R_2}{R_2+R_{eq3}} V_s$

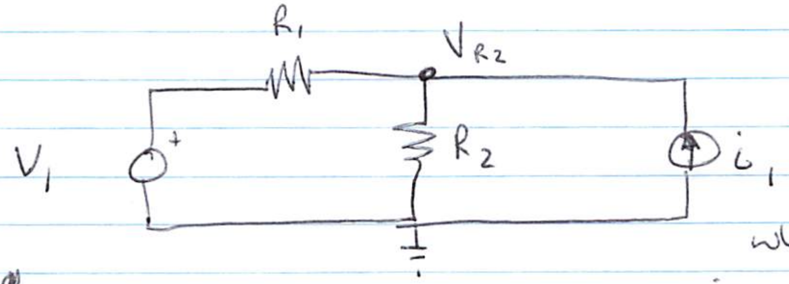
$$\boxed{V_A = \frac{1.875k\Omega}{3.875k\Omega} 10V = 4.8V}$$

the voltage across R_5 can be found the same way

$$V_5 = \frac{R_5}{R_4 + R_5} V_A = \frac{1k\Omega}{5k\Omega} 4.8V = 0.97V$$

and $V_5 = i_5 R_5$ so $i_5 = \frac{V_5}{R_5} = \frac{0.97V}{1k\Omega} = 0.97mA$

2.21



where

$i_1 = 1A$
$V_1 = 1V$
$R_1 = 10\Omega$
$R_2 = 100\Omega.$

By KVL

$$V_1 - i_{R_1} R_1 - i_{R_2} R_2 = 0$$

and by the node rule (KCL)

$$i_{R_1} + i_1 = i_{R_2}$$

so

$$V_1 - i_{R_1} R_1 - (i_{R_1} + i_1) R_2 = 0$$

$$i_{R_1}(R_1 + R_2) = V_1 - i_1 R_2$$

$$i_{R_1} = \frac{V_1 - i_1 R_2}{R_1 + R_2}$$

and w/ $i_{R_2} = i_{R_1} + i_1$

we have $$i_{R_2} = \frac{i_1(R_1+R_2) + V - i_1 R_2}{R_1+R_2}$$

$$= \frac{i_1 R_1 + V}{R_1 + R_2} = \frac{1A\ 10\Omega + 1V}{110\Omega}$$

$$= \frac{11V}{110\Omega} = 0.1 \text{ Amps}$$

(the rest is going backwards through the battery, charging it.)

and $V_{RZ} = i_{R_2} R_2$

$$= 0.1\text{Amp} \cdot 100\Omega$$
$$= 10V$$