Laboratory 9: The Viscosity of Liquids

Introduction

The essential difference between solids and fluids lies in the nature of their response to the so-called *shearing stress*. In solids, an elastic force places a limit upon the amount of shear produced by a given shearing stress. In liquids, the deformation resulting from a constant shearing stress of any magnitude, however small, increases without limit. In other words, the shear modulus for fluids is zero, and they may be said to offer no permanent resistance to shear.

Fluids do, however, differ in their rate of yield under the influence of a shearing stress. Common experience teaches, for example, that some liquids pour more readily than others. The movement of a fluid may be thought of as the slipping of adjacent layers over one another, and the internal friction between contiguous layers is called the *viscosity*. Thus, while a fluid in motion resists a shearing stress with a frictional force which tends to retard the flow, this force disappears when the flow ceases.

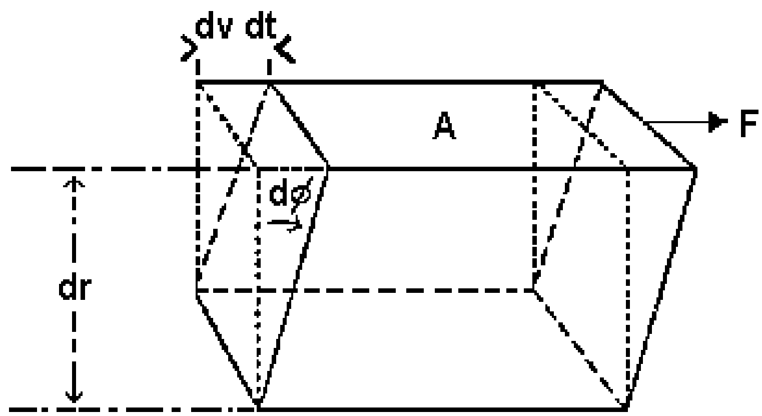

Figure 1The force F causing a fluid element to shear.

In Figure 1, let the parallelepiped represent a small element of volume in a fluid which is flowing horizontally. At equilibrium, a shearing stress *F/A* on the upper surface causes the upper surface to travel at a faster velocity *dv*, where F is the horizontal force on the top surface, *A* is the area of the horizontal cross section, and *dr* is the distance between the two surfaces. In a time *dt*, the upper surface will have slipped a distance $dv \cdot dt$. The shear angle is

$$d\varphi = \tan^{-1}\left(\frac{dvdt}{dr}\right) \cong \frac{dvdt}{dr} \qquad (1)$$

The rate of shear or velocity gradient is then

$$\frac{d\varphi}{dt} = \frac{dv}{dr} \qquad (2)$$

For streamline motion (no turbulence) the ratio between shearing stress and velocity gradient for a given fluid is found to be constant for many fluids (Newtonian Fluids). This is called the "*coefficient of viscosity*" or simply the "*viscosity*". This constant is

$$\eta = \frac{F/A}{dv/dr} \tag{3}$$

The c.g.s. unit of viscosity is called the *poise*; it is the viscosity of a substance that acquires a unit velocity gradient under the influence of a shearing stress of $1\ dyne/cm^2$.

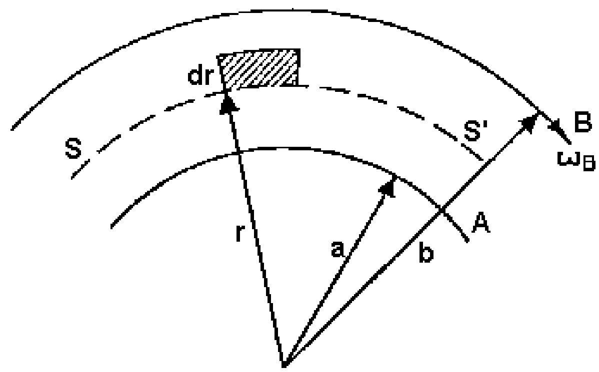

Figure 2 A fluid element better suited to this experiment. The bulk of the fluid sits between cylinder A and cylinder B.

In the co-axial cylinder method of determining the coefficient of viscosity, it is convenient to take as an element of volume a cylindrical section instead of a parallelopiped. The movement then consists of the rotation of concentric cylindrical layers about one another. In Figure 2, let the dotted line SS' represent an imaginary cylindrical boundary lying within the liquid enclosed between the two cylinders A and B. For simplicity, consider the inner cylinder to be stationary and the outer one to rotate with an angular velocity, ω_B, the result will be the same for the reverse case. If the liquid adheres to the walls of the cylinders (doesn't slip at the wall), a shearing takes place in which concentric cylindrical layers of the liquid slip over each other, the angular velocity increasing progressively from zero at the stationary cylinder to ω_B at the rotating one. The linear velocity of the intermediate surface SS' is $v = \omega r$, where $0 < \omega < \omega_B$. The velocity gradient at SS' is then

$$\frac{dv}{dr} = \frac{d(\omega r)}{dr} = \omega + r\frac{d\omega}{dr} \tag{4}$$

The first term on the right represents the uniform rotation of a solid, thus, it can be ignored. The second term represents the effect of slippage and is the velocity gradient. Thus, Eq. 3 can be written in terms of r and ω rather than v

$$\eta = \frac{F/A}{r(d\omega/dr)} \tag{5}$$

If the torque applied to the rotating cylinder is τ, the tangential force sustained by the layer of liquid in contact with the cylinder is τ/b, and that at any boundary SS' is τ/r. Since the area of this cylindrical boundary is $2\pi rl$, the tangential force per unit area is $\tau/2\pi r^2 l$ where l is the vertical length of cylinder in contact with the fluid. Substituting yields

$$\eta = \frac{\tau/(2\,\pi\,r^2\,l)}{r\,(\frac{d\omega}{dr})} \tag{6}$$

or by separating differentials

$$\eta\,d\omega = \frac{\tau}{2\,\pi\,l}\frac{dr}{r^3} \tag{7}$$

To find the equilibrium rotational velocity of the outer cylinder we must integrate this equation between the inner and outer cylinders; i.e., between the limits $r = a$ and $r = b$. Thus,

$$\eta\,\omega_B = \frac{\tau}{2\,\pi\,l}\left(\frac{1}{a^2} - \frac{1}{b^2}\right) \tag{8}$$

Where ω_B is the rate of rotation of the outer cylinder. This simplifies to

$$\eta = \frac{b^2 - a^2}{4\,\pi\,a^2\,b^2\,l}\left(\frac{\tau}{\omega_B}\right) \tag{9}$$

The coefficient η is thus determined from the dimensional constants of the apparatus and the experimentally determined ratio τ/ω_B. The above result is equally valid if it is the inner cylinder which is rotating, since it is relative velocity that is important.

Apparatus

The viscosimeter employed in this experiment is illustrated in Figure 3. It consists of two metal cylinders A and B of radii a and b, respectively, mounted co-axially. The inner cylinder A rests on a bearing so as to rotate with very little friction inside the stationary cylinder B, the liquid under investigation being contained in the space between the cylinders. Attached to the shaft of A is a drum D around which is wrapped a fine cord that passes over a pulley W attached to a mass m. The shearing torque is given by the product of the gravitational force on the mass m and radius k of the drum; i.e., $\tau = mgk$.

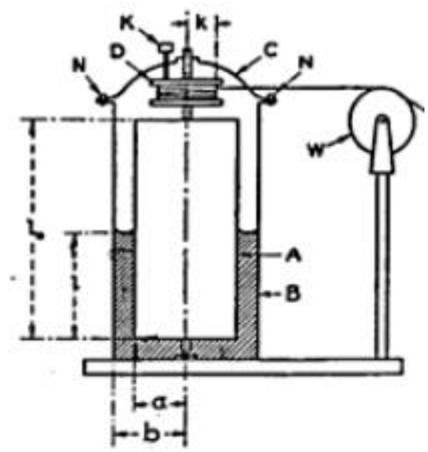

Figure 3 The viscosity apparatus with relevant dimensions defined.

A removable cover C consists of an aluminum bracket which contains an upper bearing for the rotary cylinder. Two screw clamps N hold the bracket in place, accurately centering the shaft. The cylinder A may be locked in position by means of a key K which enters a hole in the drum. The viscosimeter is equipped with an electrical heating element enclosed in a jacket surrounding the outer cylinder.

In operation, the inner cylinder will start from rest and obtain a terminal rotational velocity, ω_B, at which the viscous restraint just balances the applied torque. Note that, the angular velocity and translational velocity are related; e.g., $\omega_B = v/k$.

Measure the translational velocity of the string using a photogate-pulley combination and a Vernier computer interface.

Finally, in the foregoing discussion it was assumed that the only viscous resistance involved is that exerted by the liquid between the cylindrical surfaces. However, there is an additional torque due to the viscous drag over the ends. The magnitude of this effect depends upon the radii of the two cylinders and the distance between their closed ends. If the cylinder is totally immersed, the end effect can be treated as a correction to the length of the cylinder. To account for this add an effective length *e* to the length *1* in equation (9).

Substituting all of these relationships into equation (9) gives

$$\eta = \frac{(b^2 - a^2)\, k^2}{4\, \pi\, a^2\, b^2} \frac{m\, g}{v\, (l + e)}$$

or more succinctly

$$\eta = G \frac{F_g}{v\, (l+e)} \tag{10}$$

where $G = \frac{(b^2-a^2)\, k^2}{4\, \pi\, a^2\, b^2}$ is a dimensionless constant determined by the geometry of the problem and $F_g = mg$ is the force of gravity.

Procedure

You will perform three distinct experiments with this apparatus on SAE60 motor oil. From the first you will determine both the viscosity of the fluid at room temperature and the size of the end correction. In the second experiment you will test assumption in Eq. (3) that the fluid is a Newtonian fluid (as opposed to shear thinning or shear thickening). In the third experiment you will measure how the viscosity changes with temperature.

End Correction Determination

At constant temperature we expect the viscosity to be constant. If the mass and physical dimensions of the apparatus remain fixed, then equation (10) implies 1/v should be inversely proportional to the corrected height of the fluid; i.e., $1/v \propto (l + e)$. By making a series of measurements for different fluid levels, one should be able to plot 1/v vs *l* and determine the end correction *e* from the x-intercept and the viscosity, η, from the slope.

1. Locate the apparatus shown in Fig. 3 and carefully measure and record the dimensions *a, b, k,* and l_o(use a caliper). Reassemble the apparatus, taking care that the cylinder turns freely on its pivot points. Thread the string over the pulley as shown the photogate pulley and lock the drum. Use a mass hook to apply weight to the string.
2. With a dropper, add liquid until the level is about 1.5 cm above the lower end of the cylinder

and measure the immersed length, *l*. This requires care. One method is to use the inner cylinder as a dipstick, withdraw the inner cylinder and, holding it so as to drain fluid back into the outer cylinder, make a measurement of the fluid level on the cylinder.

3. If you have removed it, replace the cylinder and lock in place. Hang 25 gm on the string.
4. Release the cylinder and observe velocity of the falling mass. You should wind the string up so that the mass, in falling, has a chance to reach its terminal velocity. You will also find it convenient to stop the drum after it has reached terminal velocity but before the string comes loose from the drum, thus saving yourself the trouble of rethreading the string.
5. Record the average velocity and uncertainty restricting your attention to the period of time that the weight/string are moving at a constant speed.
6. Make a series of at least 10 observations, keeping the mass constant, but increasing the effective length of the cylinder by adding fluid. The maximum oil depth should be 1mm shy of the inner cylinder's top.
7. Make a plot of 1/v vs *l* and apply a linear fit. From the fit determine the end correction and the viscosity in cgs units.

Variation with Mass

1. With the level of the liquid at the top of the inner cylinder, take a series of ten observations, varying the mass from 25 to 150 gm.
2. Use Eq. 10 and the end correction from the first experiment to determine the viscosity for each data point.
3. Plot viscosity vs applied torque.
4. Does the viscosity appear to depend on the torque? How can you test this rigorously?
5. Measure the temperature of the fluid.
6. Calculate your best value for viscosity at that temperature.

Variation with Temperature

From practical experience you know that fluids become less viscous with an increase in temperature (e.g., warm liquids flow more easily than cold ones). This follows from the fact that with temperature increase the fluid molecules become more agitated and therefore have a larger equilibrium separation, thus, the shear forces are less.

1. Attach a 50 gm mass and keep this constant throughout the experiment.
2. Measure the temperature and take a set of time readings.
3. Plug the viscosimeter into the variac and set the variac to 100 VAC. Surround the cylinder

with a number of foam insulating blocks to help stabilize the temperature of the system. Leave the thermometer in the fluid and observe the temperature. When the temperature reaches just over 70^{o}C, unplug the variac. Adjust the fluid level to nearly cover the cylinder.

4. Take a set of time readings when the temperature stabilizes at 70^{o}C.
5. As the temperature falls, take new sets of time readings at 5^{o}C intervals down to within 5^{o}C of your starting temperature.
6. Calculate the viscosity of the fluid as a function of temperature. Plot your results.
7. The fluid you are using is Pennzoil SAE 60 racing motor oil. The book values for the viscosity of the oil is 207 centipoise (cp) at 40^{o}C and 15.1 centipoise (cp) at 100^{o}C. Plot and compare the book values with your results.

DRAIN THE FLUID. WIPE OFF THE CYLINDER. LEAVE THINGS AS YOU FOUND THEM.