Lab 14: The Charge Mass Ratio of the Electron

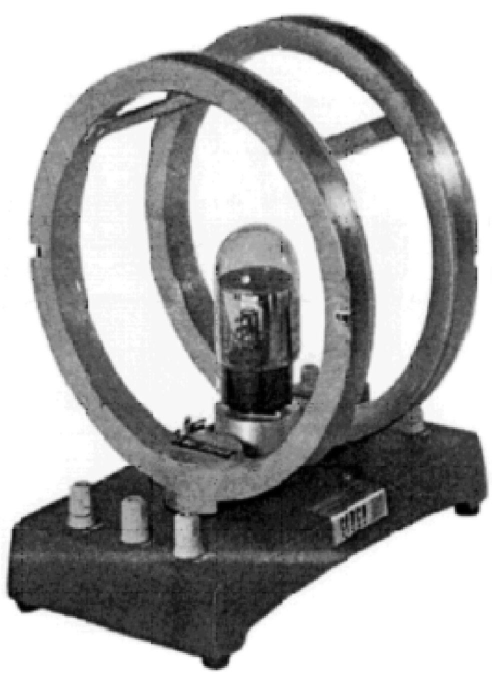
Jesse Chandler
SUNY Cortland
Physics Department

This report will examine an important parameter of the electron, the charge mass ratio. This historic experiment accelerates an electron beam through a bulb using an electric field to accelerate the electrons, and a magnetic field to force the beam into a circular path. By taking data of the relevant currents, voltages and radii, the E/M ratio can be determined. Further discussion will explore measurement error sources and magnitudes, as well as how calculated values support theoretical values.

I. Introduction

The discovery of the electron as a discrete particle by J. J. Thomson soon led to research into the charge a mass of this fundamental particle. Before both could be determined independently, their ratio must be obtained due to the extremely small value of these parameters. The classic experiment involves using an electron tube that is held between two coils and a plate. The bulb is filled with an inert gas that glows when hit by electrons, making the beam visible. The plates create an electric field which accelerates the electrons, while the coils create a magnetic field that bends the electron beam. Shown below in 1 is a picture of the apparatus used. By measuring the current in the coils, the voltage across the plates creating the electric field, and the diameter of the half circle that the beam makes, the ratio of the charge and mass of the electron can be determined using known equations available in any Electricity and Magnetism textbook [1].

Figure 1. E/M Ratio Apparatus



II. Procedure and Methods

Once the circuit for the apparatus was set up, a switch was put on the magnetic field coils so that the field could be reversed, to minimize the effect of the earth's magnetic field on data integrity. After the bulb, plate and coil were powered on, the current through the coils and voltage through across the plate was adjusted until the beam was on one of the circles inscribed inside the bulb. There bulb included inscribed circles every half centimeter for this purpose. Data was taken for these three parameters and can be seen below in I.

Table I. EM Ratio Data

Diameter (mm)	Plate Voltage (ϕ) (V)	I (amps)
10.00	79.80	4.84
20.00	80.10	2.70
20.00	64.00	2.31
15.00	63.70	3.11
10.00	63.60	4.53
20.00	44.30	1.77
15.00	44.20	2.39
10.00	44.10	3.61
20.00	100.50	2.92
15.00	100.20	3.92
20.00	88.80	2.71
15.00	88.80	3.64
20.00	70.50	2.41
15.00	70.50	3.26
10.00	70.10	4.73
20.00	55.10	2.07
15.00	55.00	2.82
10.00	54.90	4.04
20.00	75.00	2.50
15.00	74.90	3.32
10.00	74.70	4.83

Using this data, we now have a few options for calculating the charge mass ratio of the electron. We will begin by introducing the relevant equations.

$$\vec{f} = e\vec{v} \times \vec{B} \tag{1}$$

We begin by quantifying the force $\vec{f}$ exerted on a charge e moving at velocity $\vec{v}$ through a magnetic field $\vec{B}$.

$$\frac{mv^2}{r} = evB \tag{2}$$

The magnetic field will force the electron to move in a circular path, described by the centripetal force where m is the mass, v the velocity, r is the radius of the electron's path, e is the charge of the electron and B is again the magnetic field. We then assume that the electron passes through a potential gradient ϕ to attain it's velocity, which can be shown to be

$$e\phi = \frac{1}{2}mv^2 \tag{3}$$

Solving Equation 2 for v, and substituting it into Equation 3 yields

$$\frac{e}{m} = \frac{2\phi}{B^2r^2} \tag{4}$$

Which we need only calculate B, and the rest of these values are known. To calculate B, we need only consult any Electricity and Magnetism textbook to see that [1]

$$B = \frac{8\mu_o NI}{\sqrt{125}R} \tag{5}$$

Where μ_0 is the permeability of free space, N is the number of turns per coil, I is the current, and R is the radius of the coils. Using these parameters, we can calculate the E/M ratio.

III. Results

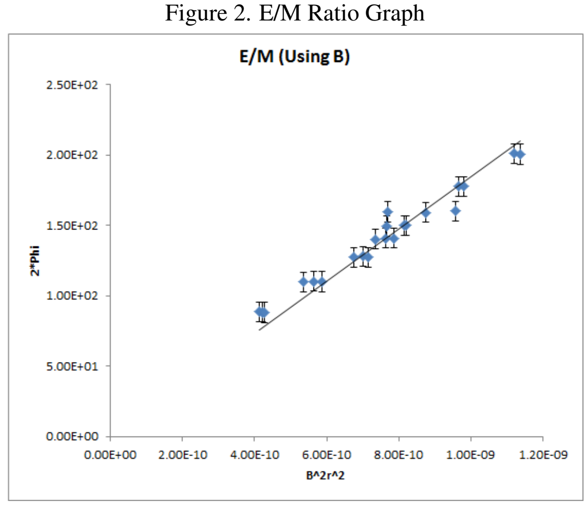
Using data from I, the magnetic field (B) can be calculated, along with the other parameters needed to calculate the E/M ratio using Equation 4. Results for these calculations can be seen below in II.

Table II. EM Ratio Calculation Data

B (T)	2ϕ (V)	B^2r^2 ($T^2 * m^2$)
5.55E-03	1.60E+02	7.70E-10
3.10E-03	1.60E+02	9.59E-10
2.65E-03	1.28E+02	7.02E-10
3.57E-03	1.27E+02	7.15E-10
5.19E-03	1.27E+02	6.75E-10
2.03E-03	8.86E+01	4.12E-10
2.74E-03	8.84E+01	4.23E-10
4.14E-03	8.82E+01	4.28E-10
3.35E-03	2.01E+02	1.12E-09
4.50E-03	2.00E+02	1.14E-09
3.11E-03	1.78E+02	9.66E-10
4.17E-03	1.78E+02	9.80E-10
2.76E-03	1.41E+02	7.64E-10
3.74E-03	1.41E+02	7.86E-10
5.42E-03	1.40E+02	7.36E-10
2.37E-03	1.10E+02	5.63E-10
3.23E-03	1.10E+02	5.88E-10
4.63E-03	1.10E+02	5.37E-10
2.87E-03	1.50E+02	8.22E-10
3.81E-03	1.50E+02	8.15E-10
5.54E-03	1.49E+02	7.67E-10

Now that these pieces are known, the E/M ratio can be determined using line regression. We will begin by graphing B^2r^2 vs 2ϕ, whose slope is the ratio in question. Shown below is the resulting graph 2.

Figure 2. E/M Ratio Graph



Using line regression yields a more accurate value for the slope and fitness. Results can be seen below in III.

Table III. EM Ratio Results

Parameter	Value
E/M Ratio	1.845E+11 C/kg
Fitness	0.997
Error Magnitude	9.57E+09
Percent Error	9.97%
% Diff Theoretical	5.29

These results are in excellent agreement with theoretical values. It is also worth noting that statistical error is small enough to be negligible, and thus measurement error alone has been assumed to be a good approximation. The fitness value shows this approximation to be a good fit, and statistical removal of outliers would be expected to increase this value to very close to 1. However, we have judged that removal of outliers is unnecessary unless the sample size is increased. Now that we have our experimental value for the E/M ratio, we can use Millikan's value for the charge of the electron $(1.6E^{-}19)$ to determine an experimental value for the mass of the electron as well. Results are shown in IV.

Table IV. Electron Mass

Parameter	Value
Mass Electron	8.67E-31 kg
Error Magnitude	4.74E-32
Percent Error	5.79
% Diff Theoretical	4.93

Here we also see our data and calculations strongly supporting theoretical values. Again, measurement error is the main contributor and statistical error is negligible.

IV. Conclusions

All of the data and calculations for this experiment fell well withing measurement error, and strongly support theoretical values. Error considerations vary, and include increasing the sample size, and the Earth's magnetic field. Earth's magnetic field ranges from 25μT to 65μT. This means a measurement error of roughly 3% for the average value calculated for B, which is propagated through further calculations. To minimize this, half of the data was taken with the B field oriented one way, and then the field was flipped for the other half of the measurement. This solution worked quite well, as shown by our calculated measurement error. Overall, calculations turned out to be quite accurate, although more data is desired to minimize error.

[1] David Jeffrey Griffiths and Reed College. *Introduction to electrodynamics*, volume 4. prentice Hall Upper Saddle River, NJ, 2013.