Laboratory 7: Properties of Lenses and Mirrors

Converging and Diverging Lens Focal Lengths:

A converging lens is thicker at the center than at the periphery and light from an object at infinity passes through the lens and converges to a real image at the focal point on the other side of the lens. A diverging lens is thinner at the center than at the periphery and light from an object at infinity appears to diverge from a virtual focus point on the same side of the lens as the object.

The *principal axis* of a lens is a line drawn through the center of the lens perpendicular to the face of the lens. The *principal focus* is a point on the principal axis through which incident rays *parallel* to the principal axis pass, or appear to pass, after refraction by the lens. There are principle focus points on either side of the lens equidistant from the center (See Figure 1a).
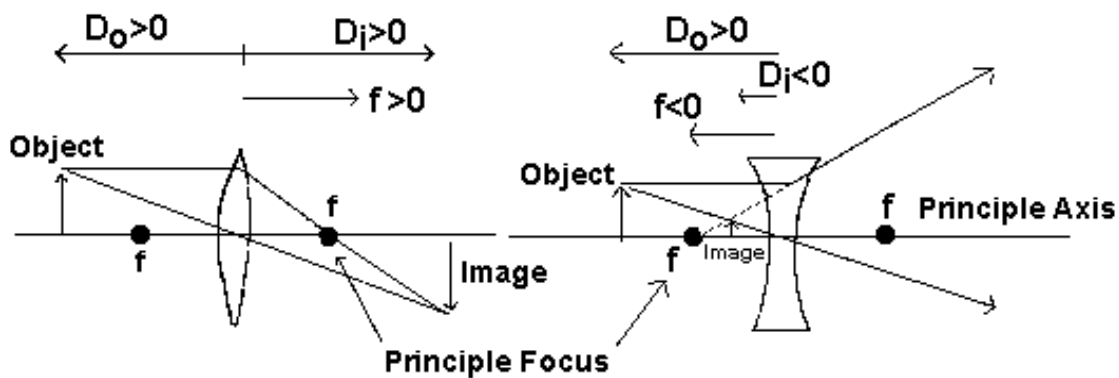

Figure 1a: Converging Lens, f>0   Figure 1b: Diverging Lens, f<0

In Fig. 1 the object and image are represented by arrows. Two rays are drawn from the top of the object. One ray is parallel to the principal axis which bends at the lens to pass through the principle focus. The second ray passes through the center of the lens and undeflected. The intersection of these two rays determines the image position.

The **focal length**, f, of a lens is the distance from the optical center of the lens to the principal focus. It is positive for a converging lens, negative for a diverging lens. The **object distance**, D_o, is the distance from the object to the optical center of the lens (Figure 1a). The distance from the **image distance**, D_i, is the distance from the image to the optical center of the lens. It can be shown that:

$$\frac{1}{f} = \frac{1}{D_o} + \frac{1}{D_i} \tag{1}$$

This is called the *thin lens equation*. The reciprocal of the focal length *in meters* is called the *power of a lens* and is expressed in *diopters*.

Note that the D_o is positive when the object is on the side of the lens from which rays come and D_i is positive when the image is on the side of the lens where rays exit. These conditions are both met for the converging lens in Figure 1a when $D_o > f$. In this case, the image is inverted and real; i.e., you can project it on a screen.

Now consider the diverging lens in Figure 1b, where the negative focal length always leads to a negative image distance. In this case, the image is upright and virtual; i.e., you can see it with your eye by looking at the exiting light rays, but you cannot project it on a screen.

Focal Length Measurement Methods:

The following are several ways of determining the focal length of lenses.

- Direct Measurement
- Method of Coincidence
- Method of Conjugate Foci
- Method of Parallax

Direct Measurement

This is a special application of the thin lens formula where the object is placed far away from the lens, so that $D_o \approx \infty$. Answer the following questions before moving on:

1. Using the method of direct measurement, what will f be equal to?
2. What are the limitations of this method? Can we use it for both types of lenses? Why?

This method can be made to work with a diverging lens with the following work around. When two thin lenses with focal lengths f_1 and f_2, respectively, are placed in contact to form a single lens, the focal length, F, of the combination is given by the equation:

$$\frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2} \tag{2}$$

Now, given a diverging lens $f_1 < 0$ and select a converging lens $f_2 > 0$ where $|f_1| > |f_2|$, then $F > 0$ for the combination. That is, the combination lens has a positive focal length. A real image forms and the direct method can be used to measure F from which F_1 can be calculated.

Method of Coincidence
In this method the object is place at the focal length.

Answer the following questions before moving on:

3. If the object is placed at the focal length then where would the image form?

4. How can you tell the position of the image?

Reflecting the light off a plane mirror on the opposite side of the lens, redirects the parallel rays back through the lense to form an image at the position of the object as shown in Figure 2.

Method of Conjugate Foci
This method uses the follow these steps:

- Solve Eq. (1) for the focal length f in terms of D_o and D_i .

- Define $L = D_o + D_i$ and keep it fixed.

- Find $Lf =$ in terms of D_o and D_i.
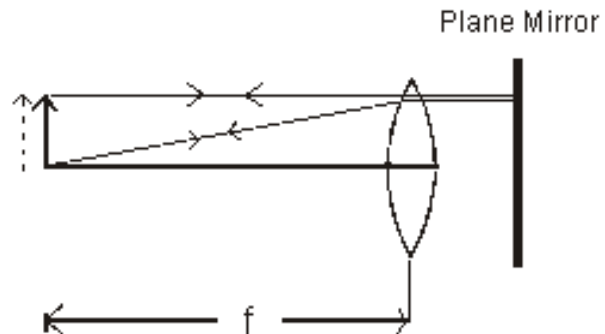
Explain your answer?

Figure 2**: Coincidence Method**

Consider the left side of Figure 3, where an object O has an image I with the lens at position **a**. If the lens is moved to a new position where $D_0^{'} = D_i$ and $D_i^{'} = D_0$, as shown in the right side of Figure 3, by moving the lens to position **b**, then $D_0^{'}D_i^{'} = D_0 D_i = Lf$ and a larger image I' will be formed. Hence, there are two positions between the fixed screen and object to place a converging lens to form a real image on the screen, namely, positions **a** and **b**. This is equivalent to swapping the object and the screen for a fixed lens. These two interchangeable positions of object and image are called *conjugate foci*.

We can remove the need to measure D_o and D_i from this method in the following way. Recall that L represents the fixed distance between the screen and the object, and define d as the distance between the two possible positions of the lens, it follows from Figure 3 that

$$D_0 = \frac{L+d}{2}$$
$$D_i = \frac{L-d}{2}$$

(3)

By substitution these values of D_o and D_i in equation (1), we get
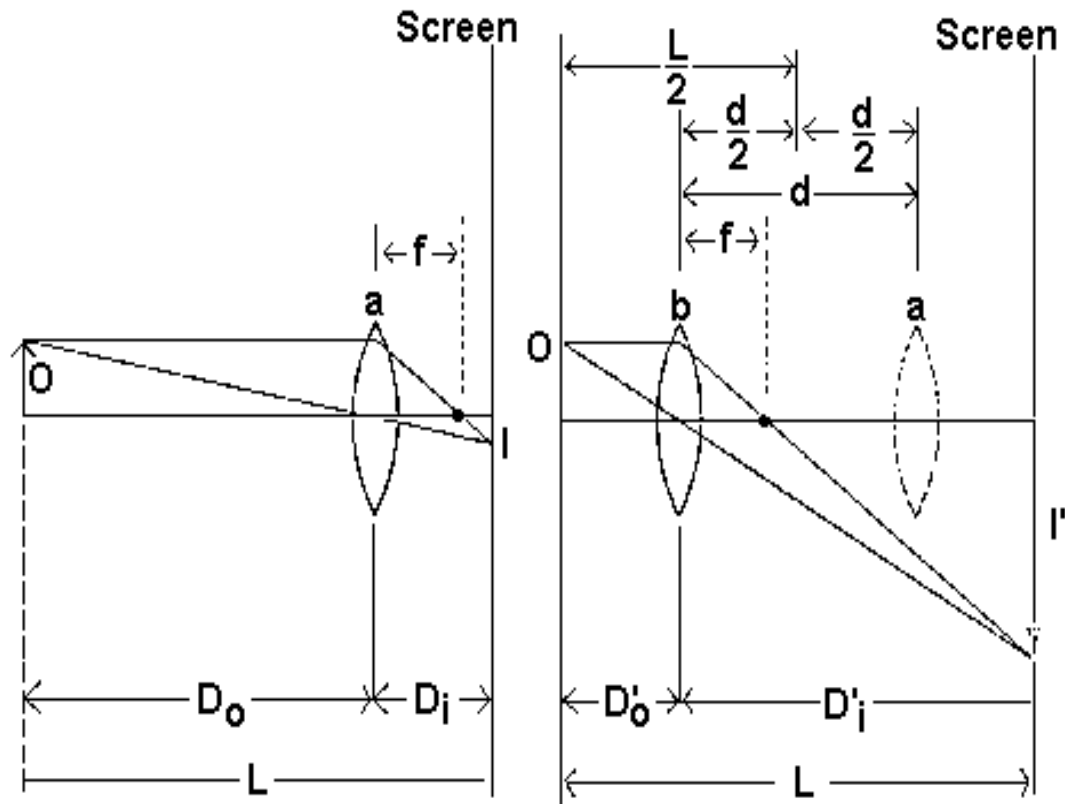
$$f = \frac{L^2 - d^2}{4L}$$

(4)

Figure 3: Focusing by the Conjugate Focus Method

This method of finding the focal length is especially valuable when using a thick lens, since the position of the optical center of the lens need not be known.

Method of Parallax

The final technique for determining focal length applies to situations where the image is virtual and is called the *method of parallax*. Images are always virtual for a diverging lens as $f < 0$, but also occurs for a converging lens, $f > 0$, when $D_o < f$. Consider the arrangement shown in Fig. 1b. Use pointer 1 as the object, see Fig. 4. Place a second pointer at a place where the image should be, as shown in Figure 4.
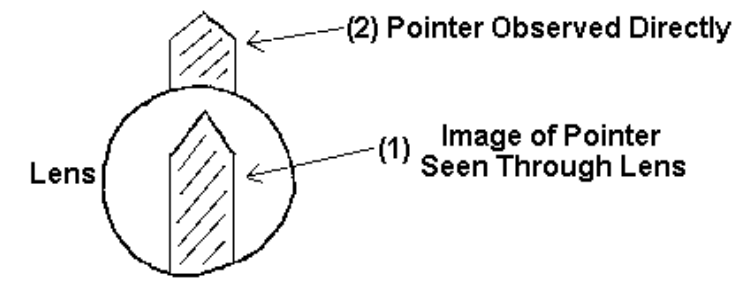
Observe the image of pointer 1 by looking *through* the lens while simultaneously observing pointer 2 *directly*

Figure 4: Virtual Image

over the lens. The image of pointer 1 should be superimposed on the real pointer 2. To confirm your choice of position for pointer 2, move your head back and forth, side to side. When properly placed, the real pointer 2 and the image of pointer 1 will both be in the same position then the two will appear to move together (no parallax). Otherwise the image of pointer 1 and real pointer 2 will not track together. You can get a feel for this situation by placing your thumb and pinky finger at different distances from your face. When you move your head side to side the thumb moves more rapidly than the farther finger.

We are going to use all of these methods to determine the focal length of four lenses, and then use these lenses to build various optical instruments. As the term is generally used, an optical instrument is an instrument designed to form an enlarged image of the object on the retina of the eye. The magnifying power is defined as the ratio of the angle subtended at the eye by the enlarged image of the object when viewed through the instrument to the angle subtended at the eye by the object when viewed directly. The instruments we will study are the magnifying glass, the microscope, the telescope, and the opera glass.

Experimental Procedure:

Focal Length by Direct Measurement

1. Mount the thicker convex lens on the optical bench together with a cardboard screen. Point the optical bench at a window or distant illuminated object. Make three independent measurements of the real images formed on the cardboard screen and record them in Table 1. Use <D_i> and assuming $D_o = \infty$ to determine the focal length of lens 1.

2. Repeat the above procedure for the other converging lenses.

3. Mount the two thin lenses used in step 2 in a single lens holder. A strip of paper about 2 cm wide placed around the periphery of the pair of lenses will help hold them together in the lens holder. Handle the lens combinations very carefully to avoid dropping the lenses and breaking them. Follow the procedure in step 1 to determine f for the lens combination. Record this value in Table 1. Calculate the expected focal length from the separate measurements of f from each lens.

4. Determine the focal length of the diverging lens by repeating the above procedure pairing the diverging lens with the thicker converging lens. Use the thick converging lens' f that you already have measured. Record the focal length of the diverging lens in Table 1.

Focal length by the Method of Coincidence

1. Use lens 2 for this part. Mount a *plane* mirror as close to lens 2 as possible. Mount the illuminated object on the opposite site of the lens from the mirror and adjust its position until a distinct image of the cross wire shows on an object screen held just to the side of the illuminated object. This happens if the illuminated light source is positioned at a principal focus of the lens as described above. Make three independent trials, record the image distances in Table 1, and determine the mean value of the focal length of lens 2.

Table 1: Focal Length by Direct Measurement and Coincidence Method

Trial	Image Distance (step 1)	Image Distance (step 2)	Image Distance (step 2)	Image Distance (step 3)	Image Distance (step 4)	Image Distance by Coincidence Lens No. 2
1						
2						
3						
Mean	$f_1 =$	$f_2 =$	$f_3 =$	$f_{MEAS} =$ $f_{CALC} =$	$f_4 =$	$f_2 =$

Focal Length by the Conjugate Foci Method

1. This method is illustrated in Figure 3. Use lens 2 again. Mount the illuminated object near one end of the optical bench, and place the screen at some convenient distance toward the other end so that the distance between them is about 10 cm more than $4f_2$. Adjust the lens to find the two positions for image formation, and determine whether both the enlarged and the diminished images are clear and distinct. If not, change the distance between the screen and the object until they are distinct, and then record their positions (meter-stick readings).

2. Measure L and record your value in Table 2.

3. Now make three independent trial measurements of $d = D_o - D_o'$ and determine $<d>$.

4. Compute the focal length of lens 2 and record in Table 2.

Focal Length by Method of Parallax

We will now find the focal length when the image is virtual. Remove the illuminated object and screen as you will not need them.

1. Mount a pin (the object) at a distance $\frac{2}{3} f_2$ from the lens. Look at the marker through the lens in the same manner as you would use a magnifying lens. The virtual image appears to be at some point behind the lens.

2. Now peer over the lens and place a second tall pin (the image marker) at the *apparent* position of the object's vertical image.

3. Now to refine the image marker's location using the parallax method. When not set correctly the image and image marker do not track the same way. This apparent change in relative position is called parallax. With the object's position remaining fixed, tweak the image pin's location until, while viewing image (object through the lens) and the image marker (over the lens), all *parallax* is eliminated.

4. Make two or three independent trials for the image position. Record the object and image distances, and find the mean of D_i.

5. Compute the focal length and record in Table 2.

6. Repeat the steps 1-5 with the diverging lens, using $D_o = 20$ cm.

Table 2: Focal Length by Conjugate-Foci and Parallax Methods

	Conjugate-Foci Method			Parallax Method	
	Lens 2 Position of object _____ Position of screen _____		L =	Lens 2 $D_0 =$	Diverging Lens $D_0 = 20cm$
Trial	D_o	D_o'	D	D_i	D_i
1					
2					
3					
Mean					
f, Calculated			$f_2 =$	$f_2 =$	$f_3 =$

Questions for Geometric Optics

1. Compute the power of the converging lens 2 and 3 in diopters from the experimental focal lengths. What relation between change in focal length and diopters do you notice?

2. In which parts of the experiment did you find the image inverted? Erect? Real? Virtual?

Enlarged? Reduced? Make a general statement which will embody all of these answers which includes the lens type and object distances for producing these results.

3. In the conjugate foci procedure you were told to make L, the distance between object and screen, about 10 cm more than *4f*. If you had used $L = 4f$, what changes, if any, would this have made in your experiment?

4. When locating the image by the method of parallax, how do you tell when the image marker is too near or too far away?

Optical Instruments

Simple Magnifier

The normal, unaided, human eye can focus a sharp image of an object on the retina if the object O is located anywhere from infinity inward to a certain point called the near point P_n or the point of most distinct vision. If you move the object closer than the near point, the retinal image becomes fuzzy because the muscles which thicken the lens in your eye cannot be adjusted to produce a smaller focal point. The location of the near point normally varies with age. Find your own near point by moving this page closer to your eyes until you reach a position at which the image begins to become indistinct. In what follows, we take the near point to be 15 cm from the eye, a value typical for a 20-year-old.

Figure 5: Simple Magnifier

Figure 5a shows an object O placed at the near point P_n. The size of the perceived image on the retina is determined by the angle θ. One way to make the object seem larger is to move it closer to your eye, as in Figure 5b. The image on your retina is now larger, but the object is now so close that your eye lens cannot bring it into focus. We can give the eye some help by inserting a converging lens (of focal length f) just in front of the eye as in Figure 5c. The eye now perceives an image at infinity, rays from which should be comfortably focused by the eye.

The angle of the image rays is now θ', where $\theta' > \theta$. The *angular magnification, m_θ, or magnifying power* of a lens (not to be confused with the lateral magnification M) can be found from

$$m_\theta = \frac{\theta'}{\theta}$$

(5)

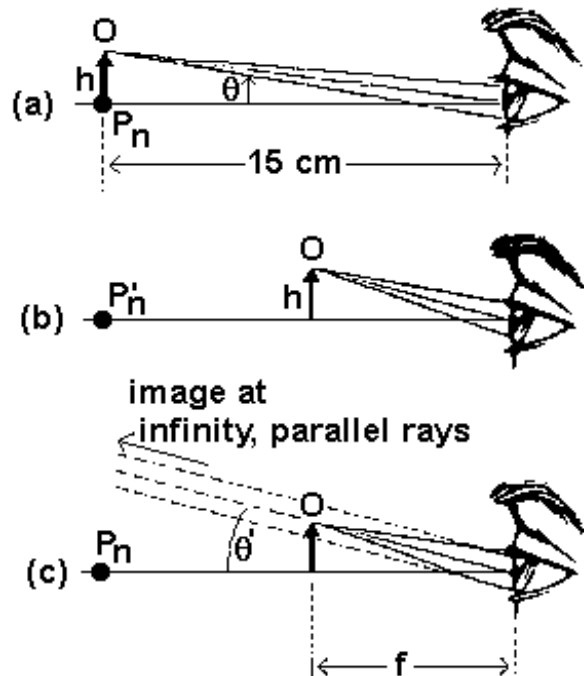This is just the ratio of the sizes of the two images on the eye's retina. From Figure 4, $\theta \approx h/15cm$

and $\theta' = h/f$, so that the *angular magnification of a simple magnifier* is $m_\theta = 15cm/f$.

Experimental Procedure for Simple Magnifier:

1. Mount lens 1 (the thickest convex lens) a few centimeters from the zero end of the optical bench and the ruler side of the white card about 5 cm away. View the centimeter scale through the lens with one eye and *directly* with the other eye. Adjust the position of the card for the most distinct image. By overlapping the two images in your mind, estimate the angular magnification of the lens and record your results in Table 3.

$$m_\theta = \frac{length \quad measured \quad directly}{length \quad measured \quad through \quad lens} \tag{6}$$

2. Repeat step 1 for the two other convex lenses (lens 2 and 3).
3. Calculate the expected magnification using previous data for the focal lengths and compare your results.

TABLE 3: Simple Magnifier

Angular Magnification	Lens 1	Lens 2	Lens 3
m_θ (measured)			
m_θ (calculated)			

Compound Microscope

The compound microscope consists of a short-focal-length objective lens and a longer-focal-length eyepiece lens or magnifier (Figure 6), both lenses being convex. The object, S, is placed just outside the principal focus, f_o, of the objective lens and produces a real enlarged image, S_1, just inside the principal focus, f_e, of the eyepiece. This real image serves as an object for the eyepiece which produces a magnified virtual image, S_2, for the eye at the distance of most distinct vision.

The *lateral magnification M_o of the objective lens* is the ratio, S_1/S. From the figure you should note that S and S_1 form the bases of similar triangles whose identical apex angles meet at the objective lens, thus
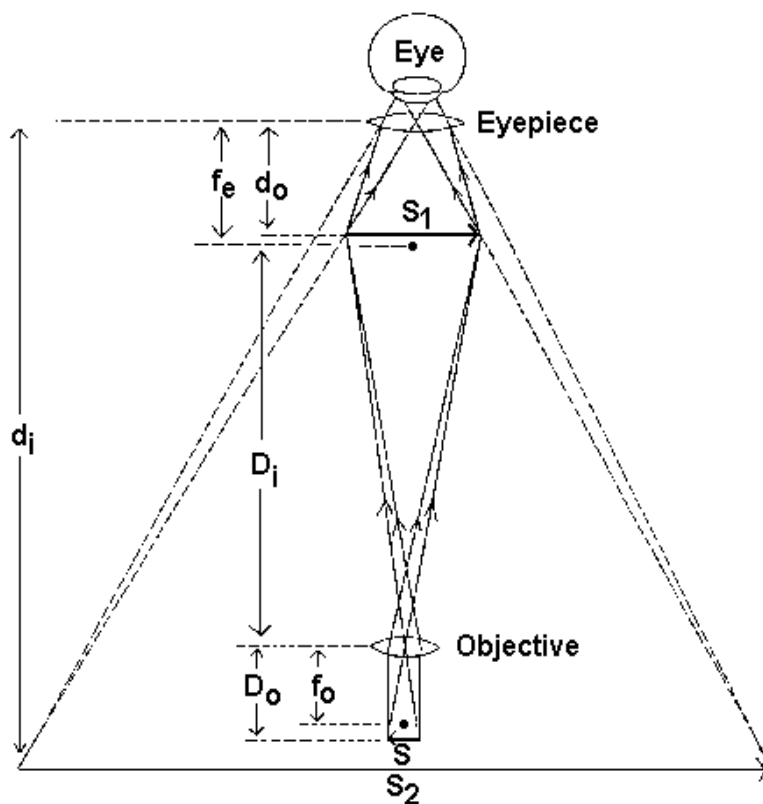

Figure 6: Compound Microscope

$$M_0 = \frac{S_1}{S_2} = \frac{D_i}{D_o} \tag{7}$$

where D_i and D_o are the distances of the lens from S_1 and S, respectively.

The total magnifying power of the microscope is the product of the lateral magnification of the objective lens times the angular magnification of the eyepiece, i.e.

$$M = M_o m_\theta \tag{8}$$

Experimental Procedure for Compound Microscope:

Use lens 1 for the objective and lens 3 for the eyepiece. For your convenience the focal lengths you measured earlier for these lenses in Table 4.

1. Mount the eyepiece a few cm from the zero end of the optical bench and the white ruled card at the 50 cm point. Mount the objective lens at the 43 cm point. You now have the arrangement for a compound microscope as illustrated in Figure 6. Adjust the position of

both lenses for the most distinct image of the ruler scale. Now select some number of scale divisions on the card within the field of view of your microscope and, by observation through the microscope and directly, estimate the total magnifying power, M, of the microscope.

2. Record the angular magnification, m_θ, of the eyepiece previously measured.

3. Without changing the positions of the lenses, replace the card with the illuminated object. By use of a cardboard screen placed between the lenses, adjust its position until you get a distinct real image of the illuminated object. Record the object and image distances, D_o and D_i, respectively, and compute the lateral magnifying power of your objective lens, M_o.

4. Finally, compute and record the magnifying power of the entire microscope. Compare the calculated and measured total magnifying powers of the eyepiece and microscope.

Simple Telescope

The objective of a simple (or astronomical) telescope gathers light from a distant object and gives a real, inverted image S_1 (Figure 7) very near the principal focus of the objective. This real image (just slightly inside the principal focus of the eyepiece) acts as the object for the eyepiece, which gives an enlarged *virtual* image S_2. The angle α is half of the angle subtended at the eye by the image S_2, and the angle β is half of that subtended by the

Figure 7: Simple Telescope

original object, S. In practice, the angles are quite small so that the magnifying power is

$$m = \frac{\alpha}{\beta} \cong \frac{\tan \alpha}{\tan \beta} \tag{9}$$

When using a simple telescope, the image size is not actually larger than the object. The image is merely brought close to the eye and is appears larger than the object does at a great distance.
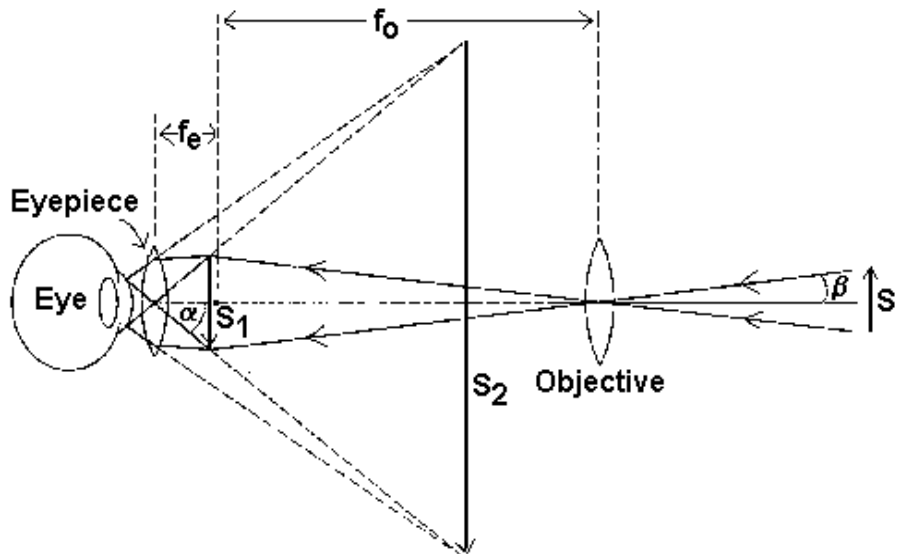
Experimental Procedure for Simple Telescope:

In constructing a simple telescope, use lens 2 for the eyepiece and lens 1 for the objective.

1. Mount the two on the objective and eyepiece on optical bench separated by $f_o + f_e$. Viewing some distant object (say a ruled piece of paper taped to the wall furthest from you) and adjust the relative positions of the lenses until a distinct image is seen.

2. Measure the magnifying power. To do this make two horizontal pencil marks on the piece of paper about 10 cm apart and focus your telescope on them from across the room. Now, while looking at the marks through the telescope with one eye and at the marks directly with the other eye, direct your partner to make two marks on the paper which appear to coincide with the apparent positions of the image marks as seen through the telescope. Measure and record the distances between both sets of marks. Exchange places with your partner and make another trial. Compute the observed magnifying power and compare with the calculated value.

Opera Glass

The opera glass (or Galilean telescope) (Figure 8) employs a concave (diverging) lens as the eyepiece and gives an erect virtual image, S_1, of the object S. This instrument is designed to give an enlarged image of a not-too-distant terrestrial object. Its magnifying power is also given by equation (9), the same form as for the simple telescope.
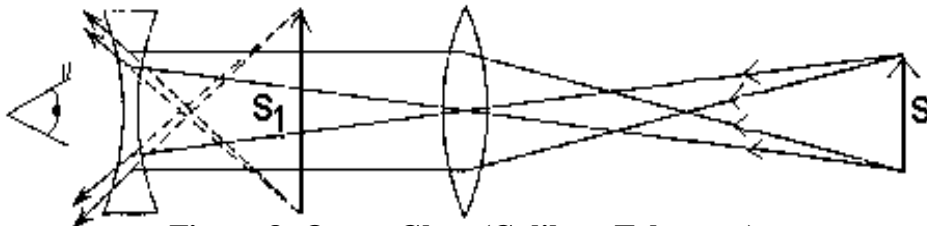

Figure 8: Opera Glass (Galilean Telescope)

Experimental Procedure for Opera Glass or Galilean Telescope:

1. Use lens 3 as the objective and the concave lens (lens 4) as the eyepiece. Repeat the above procedures for the simple telescope.

TABLE 4: Microscope, Telescope, Opera Glass

Observations and measurements	Compound microscope	Simple telescope	Opera glass (Galilean telescope)
Focal length of eyepiece (from Table 1)	lens 3 $f_e =$	lens 2 $f_e =$	lens 4 $f_e =$
Focal length of objective (from Table 1)	lens 1 $f_o =$	lens 1 $f_o =$	lens 3 $f_o =$
Magnifying power of complete instrument - observed	$M = S_2/S$ $=$	$M = S_2/S$ $=$	$M = S_1/S$ $=$
Angular magnification of eyepiece — observed, Table 3	$m_\theta =$		
Distance of object from objective lens - observed	$D_o =$		
Distance of image from objective - observed	$D_i =$		
Magnifying power of objective (calculated)	$M_o =$		
Magnifying power of complete instrument (calculated)	$M = M_o m_\theta$ $=$	$M = f_o/f_e$ $=$	$M = f_o/f_e$ $=$
Percentage difference observed vs. calculated			

Questions for Optical Instruments:

The Magnifier
1. In using each lens as a magnifier, what relationship do you observe between focal length and magnifying power?

The Compound Microscope
2. What changes could be made to construct an instrument with greater magnifying power?

The Simple Telescope or Astronomical Telescope
3. Did you see any chromatic aberration in the telescope?   Try putting a colored object in view. Where does the aberration come from?
4.   How does the distance between the two lenses compare with the sum of their focal lengths?
5. What is the advantage of making the objective lens of an astronomical telescope a large diameter lens?

The Opera Glass or Galilean Telescope
6. How does the magnifying power compare with that of the astronomical telescope?
7. What advantage does the Galilean telescope have over the astronomical telescope?

Image Formation with Mirrors

Concave Mirror

As you have demonstrated for lens, concave mirrors can also be used to create real and virtual images. In this part of the experiment you will use three different methods to determine the focal length of a large concave (i.e. converging) mirror. The first method is by direct measurement of the convergence of parallel beams. As shown in Figure 9, for a spherical mirror, a set of parallel beams will all converge at the focal point of the mirror. For an ideal spherical mirror the focal length should be related to the radius of curvature by Equation 10.
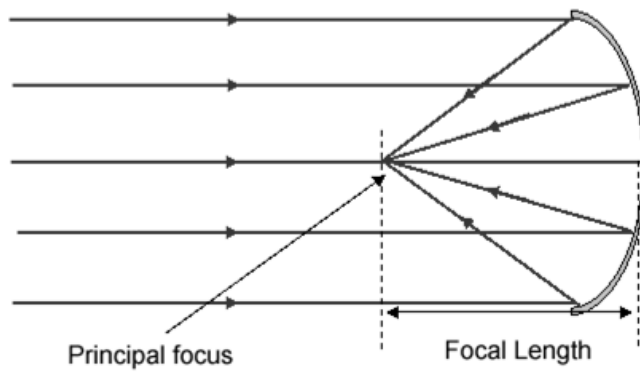
$$f = \frac{R}{2} \tag{10}$$

Figure 9: The focal point of a converging spherical mirror.

The second method for finding the focal length of the concave mirror will use the image formation of a small upright object. Figure 10 shows the formation of real images from a concave mirror for a variety of object distances greater than or equal to the focal length.
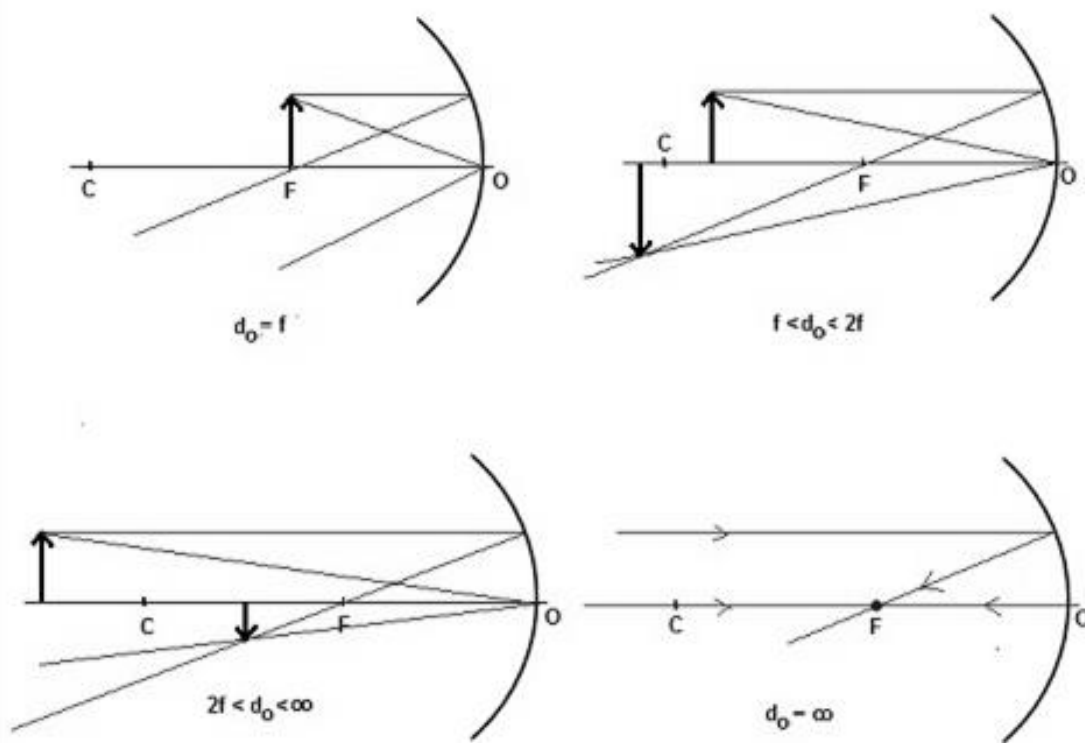

$d_o = f$

$f < d_o < 2f$

$2f < d_o < \infty$

$d_o \to \infty$

Figure 10: Ray diagrams for image formation from a concave mirror for various object distances (d_o) compared to the focal length (f).

As with the thin lenses you explored previously, the location of the image formed by the converging mirror (d_i) can be related to the location of the object (d_o) and the focal length (f) as

shown in Equation 11.

$$\frac{1}{f} = \frac{1}{d_0} + \frac{1}{d_i}$$

(11)

Thus, measuring both the image and object distances will allow a determination of the focal length of the mirror. In addition, the magnification of the image, i.e. the ratio of the image height (h_i) to the object height (h_o), can also be found from the image and object locations as shown in Equation 12.

$$M_0 = \frac{h_i}{h_o} = -\frac{d_i}{d_o}$$

(12)

Finally, the third method for measuring the focal length of the concave mirror is by use of the Spherometer Equation. By measuring two physical parameters of the concave mirror including the width of the mirror (A) and its depth (D), it is possible to use geometry to determine its radius of curvature, and thus its focal length.

$$R = \frac{\left(A^2 + 4D^2\right)}{8D}$$

(13)
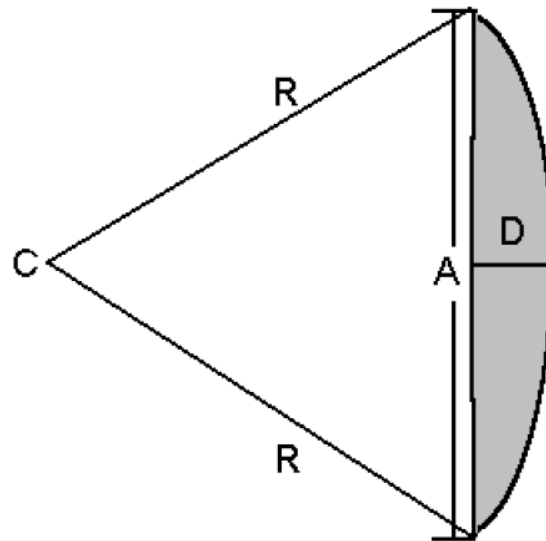
Experimental Procedure:

Focal Length by Direct Measurement

1. Mount the large concave mirror on the stand and align it with an optical bench about 30 to 50 centimeters from the mirror's center to be used for distance measurements. Mount three lasers at the far end of the table each pointing at a different location on the top half of the mirror. Use the level provided to ensure that the mirror and the lasers are aligned as shown in Figure 9.

2. Using a piece of card-stock paper attached to a stand on the optical bench, locate the point at which the three laser beams converge. Measure the distance to this point from the center of the mirror. This is the focal length of the mirror.

3. Relocate the lasers in slightly different locations and repeat steps 1 and 2 for a total of at least five measurements of the focal length.

4. Determine the average and standard error in the mean for these five measurements of f.

Focal Length by Image Formation

1. Replace the lasers with a crossed-arrow light source and a piece of card-stock on the optical bench.

2. Measure the height of the object on the light source for use in calculating the mirror's magnification.

   a. Start with the light source 10 centimeters beyond your estimate for the focal length from the previous part of the experiment.

   b. Locate the image formed by the object and record the image distance, the object distance, and the height of the image.

   c. Repeat this process for at least seven measurements with object distances out to at least 2.5 times your initial estimate for the focal length of the mirror. Figure 10 shows the expected image formation processes for this range of object distances.

3. From a plot of $d_o d_i$ versus $d_o + d_i$ (see Equation 14) determine the focal length of the mirror including its uncertainty.

$$\frac{1}{f} = \frac{1}{d_0} + \frac{1}{d_i} \longrightarrow f = \frac{d_o d_i}{d_o + d_i} \tag{14}$$

4. For each of the data points collected in step 2, calculate the image magnification directly by taking the ratio of the measured image and object heights. Discuss whether your results are consistent with those that would be calculated by use of Equation 12.

Focal Length by Spherometer Equation

1. Take the mirror off of its stand and lay it flat on the table facing up.

2. Make a series of at least five measurements of the width of the mirror (A) and its depth (D) using a ruler and meter stick.

3. Using equations 10 and 13, calculate the focal length of the mirror including its uncertainty using error propagation.

4. Compare your results for f from the three measurement techniques and comment on whether they are consistent with each other. If not identify likely reasons to explain the difference and how you would seek to test your hypothesis.

WHEN YOU FINISH: LEAVE THINGS AS YOU FOUND THEM!