

Laboratory 4: Wave Motion and Resonance

Part 1 – Introduction

The sound from a vibrating tuning fork is much louder if it is held over an air column whose length allows it to vibrate in sympathy with the tuning fork. Such an air column is said to be in *resonance* with the tuning fork. The waves set up in the air column are called standing waves. The shortest tube (closed at one end) that will give resonance is one-quarter of a wavelength, $\lambda/4$. Longer tubes also resonate, a second resonance occurs at $3\lambda/4$. In general, closed tubes resonances occur at $n\lambda/4$ where $n=1,3,5,\dots$

The velocity of a standing wave is given by the wave equation:

$$v = \lambda f \quad \text{or} \quad \lambda = v/f \quad (1)$$

where f is the frequency and λ is the wavelength.

A closed pipe (air column) has a node, N , (minimum vibration) at the closed end and, to first approximation, an antinode, A , (maximum vibration) at the open end, as shown in Figure 1, a, b, and c. More precisely, the antinode is a bit beyond the open end. A short distance is required for the equalization of pressure to take place. This distance of the antinode above the end of the tube is called the *end correction*, and is approximately equal to 0.6 the diameter of the pipe.

Because of the end correction, the pipe length in Figure 1a will be slightly less than $\lambda/4$. However, the distance between the first and second antinodes, as shown in Figure 1a and 1b, will give an exact value of $\lambda/2$, from which the wavelength can be found. Then, if the frequency of the source is known, the velocity of sound at room temperature can be obtained from equation (1). Finally, from the correct value of $\lambda/2$, the end correction for the pipe can be obtained as well.

If the resonating tube is open at both ends, as in Figure 1d, antinodes will appear at both ends. In this case the tube length is one wavelength. The shortest open tube that would resonate with a node at the center and antinodes at each end would have a length of $\lambda/2$.

The velocity of sound in air at $T=0^\circ\text{C}$, v_0 , is 331.5 m/sec and as the temperature, T , rises, increasing at the rate of about $60 \frac{\text{cm}}{\text{s } ^\circ\text{C}}$. Hence, the velocity v_T at temperature T is obtained from the relation:

$$v_T = v_0 + 0.6 T \quad \text{m/sec} \quad (2)$$

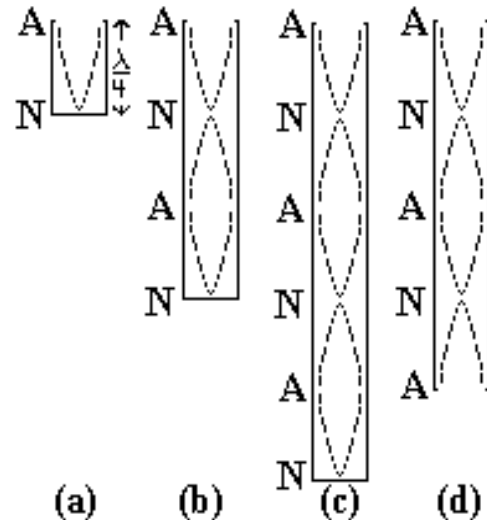


Figure 1: Resonance Air Columns

Procedure for Sound Waves

In this lab, you are required to come up with an experimental design to calculate the speed of sound at room temperature in the laboratory and compare it to the speed you would calculate from equation (2). During this process, you have to fill out Table 1, and make any additional tables, calculations and formula derivations as necessary. In your laboratory report, you need to include a detailed step by step experimental procedure and error analysis as well as *make use of the data analysis you studied in the previous three labs*.

R tip: `mean()` and `sd()` can sort by a variable, to calculate the mean of all values of `data_frame$x` with the same value of `data_frame$y` use

- `mean(~x|y data=data_frame)`

Include plots for you data as necessary. Remember that your laboratory description must be useful to someone who wants to do this lab with your experimental design; it must include drawings of the lab setup as well.

The equipment you have on hand are listed below:

- A closed tube which has length marks on it on a stand.
- A water reservoir, with a stand.
- A speaker connected to a signal generator (when using the signal generator at high frequencies you need to turn the gain down).
- A microphone that works with the Verneir Logger Pro software through the LabPro2 interface. Note that to display a graph of the data live you must; select Experiment, Data, check the Repeat box.
- A thermometer to read the room temperature.

Table 1

Temperature of room:		Diameter of tube:		
Frequency (Hz)	1 st Resonance (m)	2 nd Resonance (m)	$\langle \lambda \rangle$ (m)	$\sigma_{\langle \lambda \rangle}$ (m)
400				
500				
600				
700				
800				
900				
1000				

Questions for Sound Waves:

1. Compare the shortest tube lengths in the case of a closed and open tube for the same frequencies. Comment on your results.
2. How does your measured velocity compare with your experimental velocity?
3. Does it appear from your results that the velocity of sound in air depends on either the frequency or the wavelength of the waves produced by the speaker? Why?
4. What are the main sources of error in your experiments? How can you reduce error?

NOTE: “Human error” is not an acceptable answer to any question in this or any other lab.

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Part 2 – Waves on a String

When two equal wave trains traveling in opposite directions interact they can produce a *stationary wave*. This type of wave motion may be produced by either longitudinal (compressive) or transverse (lateral) waves. You can observe stationary waves in a light, flexible string with one end attached to a vibrator and the other end stationary. The waves travel from the vibrator down the string to the stationary end and are reflected back to the vibrator. If the velocity of the wave and the length of the string are properly adjusted, these two oppositely directed wave trains superimpose on the string in such a way as to give alternate regions of no vibration, N , and regions of maximum vibration, A . These regions N and A are called *nodes* and *antinodes*, respectively, and the segment between two nodes is called a loop. A change in the number of loops means a change in the length of a loop, thus, a change in wavelength. This is the same phenomenon observed with sound waves in a tube, with the only difference being that sound waves are longitudinal waves and waves on a string are transverse waves.

Assume you start with a resonance condition. How can you change the wave pattern, and the number of antinodes in the string?

NOTE: The end *not* connected to the vibrator must be a TRUE stationary point.

Procedure for Waves on a String:

In this lab, you are required to come up with an experimental design to calculate the speed of wave propagation in the string as a function of the parameters of the system. During this process, you have to fill out tables 2 and 3, and make any additional tables, calculations and formula derivations as necessary. In your laboratory report, you need to include a detailed step by step experimental procedure and error analysis, make use of the data analysis you studied in the previous three labs. Include plots for you data as necessary. Remember that your laboratory description must be useful to someone who wants to do this lab with your experimental design; it must include drawings of the lab setup as well.

The equipment you have on hand is listed below:

- Three strings with different mass densities, $\mu=m/L$.
- An analytical balance which gives you the mass in the nearest milligram unit.
- A vibrator which can be plugged into a variable voltage DC power supply and a clamp to attach it to the table. (**NOTE:** Set the voltage on the DC supply to 3.0 volts using the digital volt meter monitor it but do *not* change it throughout the experiment.)
- A pulley with attached table clamp.
- A weight hanger with a set of weights of different masses.
- A microphone that works with the Verneir Logger Pro software through the LabPro2 interface. Note that to display a live graph of the data you must; select Experiment, Data, check the Repeat box.

NOTE: For strings 1, 2, and 3 you will need to make at least one measurement using the same load mass. You will need this data to investigate the relation of wavelength to the mass per unit length. You should choose the common mass to be near the center of the range you used for string 1. Once you have the measurements for string 1 and selected a common mass hang it from string 2 or 3, adjust the length of the string to find one of the desired resonances ($N = 1, 2, 3, 4, 5, \text{ or } 6$). Record this data for load mass, length, and number of loops in the final lines of Table 3 for strings 2 and 3. Use these three data points taken at the same load mass in the derivation of m in equation 4.

Be sure to be clear about what the independent parameters of your system are as you take the measurements to fill out Table 3.

Table 2

	Mass (kg)	Length (m)	$\mu = \text{mass} / \text{length} \text{ (kg/m)}$
String 1			
String 2			
String 3			

Table 3

String	Load Mass (kg)	Force, F (N)	Length measured L (m)	Number of loops N	Wavelength λ (m)
1				6	
				5	
				4	
				3	
				2	
				1	
2				6	
				5	
				4	
				3	
				2	
				1	
String	Load Mass (kg)	Force, F (N)	Length measured L (m)	Number of loops N	Wavelength λ (m)
3				6	
				5	
				4	
				3	
				2	
				1	

Now ask yourselves: how does the tension affect the wavelength?

Your data should indicate that as the tension F increases, the wavelength λ also increases, but it is not a linear change. Plot your data for each string using R and find the relation for the best fit to your data. **NOTE:** Your data is not linear so you might need to use the log of your data to graph $\log(Y)\sim\log(X)$ or $\log(Y)\sim X$, try them all. Just as you can access variables in a `data_frame`, you can also define new variables and assign values to them. For example a `data_frame` called `string1` with a variable called `velocity` is accessed by

- `string1$velocity`

If you want a new variable called `logVelocity` with the value `log(velocity)` you can construct that using:

- `string$logVelocity<-log(string1$velocity)`

You should be able to find n if we were to write a relation between wavelength and tension in the following form:

$$\lambda = kF^n \quad (3)$$

where k is a proportionality constant, and n is an exponent of unknown value. Presumably the other variables affect the velocity through the constant k . Rewrite the final form of equation (3) with the obtained value of n , before you move on check your work with the lab instructor, it is crucial that you find the correct value for n .

String 1:	n =	uncertainty in n =
String 2:	n =	uncertainty in n =
String 3:	n =	uncertainty in n =
Reported Value:	n =	overall uncertainty in n =

Examine your recorded values of F and λ and answer the following questions:

1. When λ is doubled, how does the tension F change?
2. Does F change faster or slower than λ ?
3. Do you expect n to be smaller, larger or equal to 1?

Now investigate the relation of wavelength to the mass per unit length, μ , with constant tension.

Compare the data you have for the three strings of different mass densities from tables 2 and 3 where you have taken data at the same tension. How does k depend on μ ? Use R to fit the dependence of λ on both F and μ simultaneously. This is possible whenever you have equations of the form $y=ax+bz+c$ where x and z are independent variables, y is the dependent variable, and a , b , and c are fit parameters. Your data is not linear but can be made so with an appropriate scaling. You should be able to find n , m , and k' if we were to write a relation between wavelength and mass per unit length in the following form:

$$\lambda = k'F^n\mu^m \quad (4)$$

where k' is a proportionality constant, and m is an exponent of unknown value. Presumably the other variables affect the wavelength through the constant k' . Rewrite the final form of equation (4) with the obtained value of m , before you move on check your work with the lab instructor, it is crucial that you find the correct value for m .

$$m = \quad \quad \quad \text{uncertainty in } m =$$

Examine your recorded values of μ and λ and answer the following questions:

1. How does λ change for the same tension in the two strings?
2. Do you expect m to be smaller, larger or equal to 1?

Note: it is possible to fit for the dependence of λ on μ and F at the same time, ask me if you wish to find out how.

Questions for Waves on a String:

1. What effect does increasing the tension in a vibrating string have on the wavelength?
2. What effect does increasing the tension (stretching force) have on the frequency of the wave? Give evidence from your data in support of your answer. What determines the frequency of the wave?
3. If the pulley, over which the masses are suspended, were moved slowly toward the vibrator for a considerable distance, what effects would you likely observe? If not sure, try it and see.
4. All strings on a violin are the same length. What characteristic differences do they have that gives them different frequencies (different pitch)? What other way can the frequency be changed?
5. On the basis of Newton's second law of motion and the inertia of the heavier string, make a qualitative statement about the comparative velocities of the waves in the two strings under equal tensions.
6. Using the known solution from Newton's second law for the velocity of the waves on string and the relationship $f = \lambda/v$, find the frequency at which the wave source oscillates as well as the uncertainty for f using data from all three strings by use of graphical analysis.
7. If you have not already done so, connect the microphone to the LabPro data capture system. Collect pressure versus time measurements for the sound coming off of the oscillator/string using a sample rate of 5,000 data points per second and duration of 0.4 seconds. Insert a FFT graph (insert, other graphs, FFT graph) and identify the most prominent frequency in your

data.

- Repeat the measurements in step 7 for the six forces and lengths used for string 3 and recorded in Table 3. Use this set of 6 measurements to find the average frequency at which the wave source oscillates as well as the standard error in the mean for f . Comment on how these results compare with your estimate for f determined from the measurements of λ and v from part 6.

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