Laboratory 13: Millikan Oil Drop Experiment

Measurement of the Electric Charge

Introduction

The discovery of the electron as a discrete particle of electricity is generally credited to the British physicist Sir J. J. Thomson (1856-1940). His extensive studies of cathode rays culminated in the quantitative observations of the deflection of these rays in magnetic and electric fields. These researches led to methods for the measurement of the ratio of charge to mass (e/m) for the electron. In his famous oil-drop experiments, Robert A. Millikan (1868-1953) was able to measure the charge of the electron (1.60206×10^{-19} coulomb). The currently accepted value for e/m is 1.75890×10^{11} coulombs/kg, and hence the mass of the electron could be determined accurately. In this laboratory you will use the Millikan Oil Drop apparatus to measure the charge of the electron.

Each time you start a new observation with zero field and a squirt from the atomizer you will see a myriad of droplets falling through the field of view. Your problem will be to pick a droplet that is of a size such that, if it carries a charge of a few electrons (1-3 e), you will be able to pull it upward with the available electric force. To judge which droplet to pick you must estimate the terminal fall velocity and holding voltage of a suitable droplet. The velocity of a droplet is determined from a measurement of the time it takes it to fall the distance between two lines in the eyepiece.

Apparatus Function:

The instrument consists of an oil drop changer with voltage controls, a microscope, an atomizer, and a built in stopwatch. The behavior of the oil drops can be observed with an eyepiece. The oil drop chamber is made of a cylinder with two precisely parallel plates. Oil drops fall from the oil mist chamber into the oil drop chamber through a hole of 0.4 mm in diameter in the upper plate, and they are illuminated with an LED. The behavior of the oil drops can be observed with the microscope. The eyepiece has a built-in scale to measure the distance a drop travels. The full vertical scale is equal to 0.300 cm, with a minimum division of 0.050 cm.

The voltage switch controls the voltage between the two plates in the following ways:

BALANCE: applies a voltage to keep a charged drop in balance or to move it up and down (it can be varied continuously between 0 – 500 V).

DOWN: Removes the voltage between the plates so that the oil drops can fall freely.

UP: Applies roughly double the BALANCE voltage (up to ~700 V) to force charged oil drops up in the chamber.

Before each use the oil drop chamber should be leveled with the leveling screws under the unit and the bubble level beside the chamber.

Theory of Experiment:

Consider a spherical oil droplet falling through the air that has reached terminal velocity, the forces acting on the droplet are

$$\overrightarrow{F_g} + \overrightarrow{F_b} + \overrightarrow{F_d} = 0 \qquad (1)$$

Where $\overrightarrow{F_g}$ is the force of gravity on the sphere, $\overrightarrow{F_b}$ is the buoyant force from the air on the sphere, and $\overrightarrow{F_d}$is the force of drag from the air on the sphere. Gravity and buoyance depend on the drop's radius *r* and density ρ as well as the density of the air ρ_{air}

$$-\frac{4}{3}\pi r^3 \rho g + \frac{4}{3}\pi r^3 \rho_{air}\, g + \left\|\overrightarrow{F_d}\right\| = 0. \qquad (2)$$

The force of drag is more requires more care. The simplest form for the drag on a slow moving sphere is given by Stoke's drag $\left\|\overrightarrow{F_d}\right\| = 6\,\pi\,\eta\,r\,v_d$ where v_d is the terminal down and η is the air's coefficient of drag. When *r* is comparable in size to the mean free path of the air molecules a correction is needed. The experiment employs oil droplets which are light enough to be suspended or drawn upward by the electric force exerted on just a few (1 to 10) electronic charges by a field of a hundreds volts/cm. Such droplets have radii that are typically not very large compared to the mean free path of air molecules, which is 2.2 x 10^{-6} cm at normal temperature and pressure. In this case the expression for the drag force is divided by factor *(1+ a/r+O(1/r²))*, where *a* is a constant of the order of the mean free path. Thus keeping only the first term in the correction in the drag coefficient during free fall the equation representing the balance of forces is

$$\frac{4}{3}\pi r^3(-\rho + \rho_{air})g + \frac{6\,\pi\,\eta\,r\,v_d}{1+\frac{a}{r}} = 0 \qquad (3)$$

where *g* is the acceleration of gravity. The term in brackets on the right hand side of equation (3) approaches unity as the radius becomes large compared to *a*. It turns out that the radii of the typical droplets used in this experiment are large enough so that *a/r <<1*. To zeroth order in a/r, but still to high accuracy, we find the radius of the drop to be

$$r_o = \sqrt{\frac{9\,\eta\,v_d}{2\,(\rho-\rho_{air})g}} \qquad (4)$$

Then rewriting Eq. 3, which is first order in a/r, in terms of r_o

$$r_1^3 = \frac{r_o^2 r_1}{1+a/r_1}. \qquad (5)$$

Solving for r_1

$$r_1 = r_o \left(\sqrt{1 + \left(\frac{a}{2r_o}\right)^2} - \frac{a}{2r_o} \right) \tag{6}$$

But since we kept only first order terms in our force balance equation we should still only keep first order terms and Equation 6 becomes

$$r_1 = r_o \left(1 - \frac{a}{2r_o}\right) \tag{7}$$

A charged droplet in the presence of an electric field will have a new force on it and Eq. (1) becomes

$$\overrightarrow{F_g} + \overrightarrow{F_b} + \overrightarrow{F_d} + \overrightarrow{F_e} = 0 \tag{8}$$

Droplet carrying a charge *ne* and move upward with terminal velocity v_{up} under the influence of an electric field E= *V/s* between two parallel plates separated by the distance *s* and a potential difference *V*, the equation of motion is

$$\frac{4}{3}\pi r_1^3(-\rho + \rho_{air})g - \frac{6\,\pi\,\eta\,r_1\,v_{up}}{1+\frac{a}{r_1}} + \frac{V\,ne_1}{s}=0 \tag{9}$$

Subtracting equation (3) to equation (9) and solving for *ne*, we obtain

$$ne_1 = \frac{6\,\pi\,\eta\,r_1\,s}{V}\,\frac{(v_d + v_{up})}{1 + a/r_1}$$

Which is, to the same order in a/r, equivalent to

$$ne_1 = \frac{6\,\pi\,\eta\,s\,(r_1 - a)}{V}\,(v_d + v_{up}) \tag{10}$$

If we knew the value of *a*, the factor in Stoke's law that corrects for the effects of the granularity of air, then for each droplet we could find the value of *ne* and seek the greatest common divisor of the set of values, which would be the likely value of e_1. The problem is how to determine *a*. For this purpose we define e_o to be the zeroth order approximation to the charge on the electron

$$e_o = \frac{6\,\pi\,\eta\,s\,(r_o)}{n\,V}\,(v_d + v_{up}) \tag{11}$$

which we can calculate explicitly

$$e_o = \frac{6\,\pi\,\eta\,s}{n\,V}\,\sqrt{\frac{9\,\eta\,v_d}{2\,(\rho - \rho_{air})g}}\left(v_d + v_{up}\right). \tag{12}$$

Substituting Equation (11) into equation (10) and rearranging we obtain

$$ne_1 = ne_o\,\frac{r_1 - a}{r_o} \tag{13}$$

Or, again to the same order in a/r,

$$e_o = e_1\left(1 + \frac{3a}{2r_o}\right) \tag{14}$$

Many measurements of ne_o for drops of various radii will yield a collection of values; presumably, each is close to an integer multiple of the fundamental unit of charge. When the numbers of unit charges involved in each of the measurements has been figured out, then each measurement, divided by the proper integer number, yields a value of e_o. A plot of e_o against $1/r_o$ should show data points clustered around a line with a slope equal to 3 a $e_1/2$ and an intercept of e_1 on the $1/r_o = 0$ axis (corresponding to $r \rightarrow \infty$). Note that e_1 is the best estimate of *e* we can determine.

Note: You can obtain results of impressive accuracy in this experiment provided you take care in reducing random errors of measurement. The most important thing of all is to select appropriately sized droplets carrying very few elementary charges: 1e to 3e or 4e. Make a preliminary analysis of the data for each droplet immediately after you obtain it so that you can perfect your judgment as to which droplets to select and what voltages to use. The timing measurements are like a video game in which practice makes perfect. *N* repetitions of any given measurement will reduce the random error of the mean in proportion to $N^{1/2}$.

Experimental Procedure:

Determine when the microscope is focused at the center of the chamber. This can be done by placing the needle tip at the center of the chamber (the needle for loading the bulb fits through the top hole in the chamber) and then focusing on the tip. The tip will be big and bright.

Remove the needle and adjust the eyepiece (it rotates) so that the grid is in sharp focus for your eye. Apply an intermediate electric potential (~200V) with the switch in the BALANCE position. Spray droplets of oil from the atomizer through the microscope port into the chamber (take it easy, one small squirt is generally sufficient after you have primed the atomizer with a few squirts into a towel). As the droplets drift in the chamber they are illuminated by the LED light.

When you see a droplet through the microscope as a slowly drifting unresolved point of diffracted light, release the droplet and watch it fall by toggling the switch to DOWN. Toggling the switch to UP will cause the drop to rise. If the droplet's motion is influenced by the electric force, then clearly the droplet is charged and an increase in the BALANCE voltage may arrest or reverse its motion. If the droplet accelerates downward under the action of electric force, the voltage should be reversed. If the electric field has no effect, then the droplet you are following is not charged at all. Try another droplet. When you catch one that works, i.e. it drifts slowly downward (~15 s fall time indicates the drop has about the optimum weight) in the switch in the DOWN position and can be lifted by the voltage applied with in the switch in the UP position your drop will have only a few excess charges of one sign or the other.

Using as your race track the gap between two horizontal lines of the eyepiece, measure the free-fall times with the voltage off and the rise times with the voltage on for one droplet. Repeat as many times as possible or as long as you have patience. When you think you have your first really good set of repeatable data for one droplet, stop and analyze it, and derive the value of the charge on the droplet. If everything seems reasonable and your value is close to a small (~1-5) integer multiple of the known value of e, proceed to get data on more drops at different BALANCE voltages, working with each one as long as possible, and analyzing the data after each droplet is finished. Try to observe several with the shortest free-fall time (largest radius) you can measure accurately in order to have a good basis for extrapolating your values of e_o to $' \to \infty$.

Air currents can be a problem in this experiment. Take care that the chamber is well sealed, and reduce as much as possible any movement of air in the room.

You may find the following procedure convenient:

- Pull the droplet above the top line by toggling the switch to UP.
- Toggle the switch to DOWN (V=0V) and measure the time to fall from the top to the bottom line of the eyepiece. Keep your left hand on the timer controls and your right hand on the voltage controls
- After the droplet has passed the bottom line arrest the downward motion by toggling the switch to BALANCE.
- Read and record the fall time, taking care that you will be able to identify the droplet after you have looked away for a moment to read and record the clock.
- Measure and record the BALANCE voltage that renders the droplet exactly stationary ($v_{up} = 0$).
- Toggle the switch to UP to pull the droplet up measuring the time to cross the top line and the voltage applied.

Repeat the sequence many times to reduce the random errors of the time and voltage measurements.

Tabulate your data in a format that will allow you to reduce it in an orderly fashion in adjacent columns. You will find that it takes a considerable amount of practice to achieve high accuracy in this experiment. All members of a team should perfect their skill at making all the various measurements. The more droplets you measure and the more data you accumulate on each droplet, the more accurate will be your final result.

For each droplet find the value of r_o and e_o and the errors. Plot $e_o$against $1/r_o$. Determine the slope ($3\ a\ e_1/2$), intercept (e_1) and errors by linear regression. From these results compute your best estimate for *a* and *e* and their errors. Take special care in understanding and evaluating the random and systematic errors.

The values of the constants are given below, but don't trust the oil density:

Oil density	ρ=981 kg/m^3
Air viscosity	$\eta=1.83\times10^{-5}$kg/(m sec)
Atmospheric pressure	P=76.0 cmHg
Plate separation	d=5×10^{-3}m

WHEN YOU FINISH: LEAVE THINGS AS YOU FOUND THEM!