Laboratory 9: The Viscosity of Liquids

Introduction

The essential difference between solids and fluids lies in the nature of their response to the so-called *shearing stress*. In solids, an elastic force places a limit upon the amount of shear produced by a given shearing stress. In liquids, the deformation resulting from a constant shearing stress of any magnitude, however small, increases without limit. In other words, the shear modulus for fluids is zero, and they may be said to offer no permanent resistance to shear.

Fluids do, however, differ in their rate of yield under the influence of a shearing stress. Common experience teaches, for example, that some liquids pour more readily than others. The movement of a fluid may be thought of as the slipping of adjacent layers over one another, and the internal friction between contiguous layers is called the *viscosity*. Thus, while a fluid in motion resists a shearing stress with a frictional force which tends to retard the flow, this force disappears when the flow ceases.

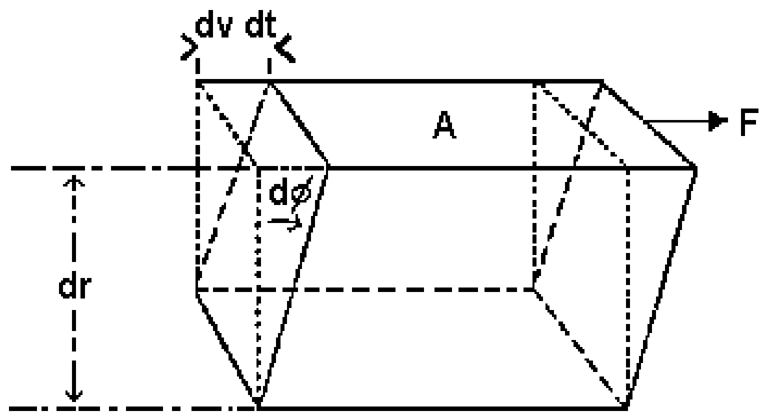

Figure 1: Shear Force on a Fluid

In Figure 1, let the parallelepiped represent a small element of volume in a fluid which is flowing horizontally. At equilibrium, a shearing stress *F/A* on the upper surface causes the upper surface to travel at a faster velocity *dv*, where F is the horizontal force on the top surface, *A* is the area of the horizontal cross section, and *dr* is the distance between the two surfaces. In a time *dt*, the upper surface will have slipped a distance $dv \cdot dt$. The shear angle is

$$d\varphi = \tan^{-1}\left(\frac{dvdt}{dr}\right) \cong \frac{dvdt}{dr} \tag{1}$$

The rate of shear or velocity gradient is then

$$\frac{d\varphi}{dt} = \frac{dv}{dr} \tag{2}$$

For streamline motion (no turbulence) the ratio between shearing stress and velocity gradient for a given fluid is found to be constant. This is called the "*coefficient of viscosity*" or simply the "*viscosity*". This constant is

$$\eta = \frac{F/A}{dv/dr} \tag{3}$$

The c.g.s. unit of viscosity is called the *poise*; it is the viscosity of a substance that acquires a unit velocity gradient under the influence of a shearing stress of $1\ dyne/cm^2$.

In the co-axial cylinder method of determining the coefficient of viscosity, it is convenient to take as an element of volume a cylindrical section instead of a parallelopiped. The movement then consists of the rotation of concentric cylindrical layers about one another. In Figure 2, let the dotted line SS' represent an imaginary cylindrical boundary lying within the liquid enclosed between the two cylinders A and B. For simplicity, consider the inner cylinder to be stationary and the outer one to rotate with an angular velocity, ω_B, the result will be the same for the reverse case. If the liquid adheres to the walls of the cylinders, a shearing takes place in which concentric cylindrical layers of the liquid slip over each other, the angular velocity increasing progressively from zero at the stationary cylinder to ω_B at the rotating one. The linear velocity of the intermediate surface SS' is $v = \omega r$, where $0 < \omega < \omega_B$. The velocity gradient at SS' is then

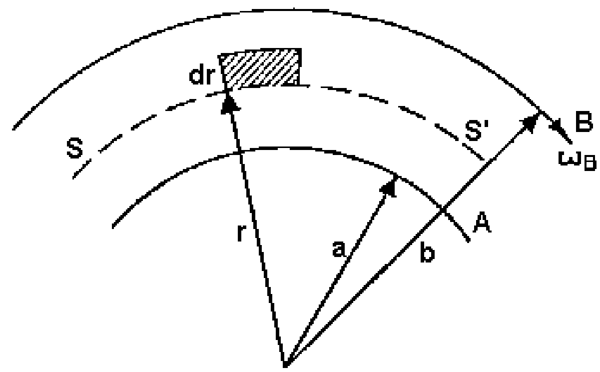

Figure2: Cylindrical Surface in Motion

$$\frac{dv}{dr} = \frac{d(\omega r)}{dr} = \omega + r\frac{d\omega}{dr} \tag{4}$$

The first term on the right represents uniform rotation or no slippage, thus, it can be ignored. The second term represents the effect of slippage and is the velocity gradient. Thus, the viscosity can be written

$$\eta = \frac{F/A}{r(d\omega/dr)} \tag{5}$$

If the torque applied to the rotating cylinder is τ, the tangential force sustained by the layer of liquid in contact with the cylinder is τ/b, and that at any boundary SS' is τ/r. Since the area of this cylindrical boundary is $2\pi rl$, the tangential force per unit area is $\tau/2\pi r^2 l$ where l is the length of cylinder in contact with the fluid. Substituting yields

$$\eta = \frac{\tau/\left(2\pi r^2 l\right)}{r(d\omega/dr)} \tag{6}$$

or by separating differentials

$$\eta d\omega = \left(\frac{\tau}{2\pi l}\right)\frac{dr}{r^3} \tag{7}$$

To find the equilibrium rotational velocity of the outer cylinder we must integrate this equation between the inner and outer cylinders; i.e., between the limits $r = a$ and $r = b$. Thus,

$$\eta\omega_B = \left(\frac{\tau}{4\pi l}\right)\left(\frac{1}{a^2} - \frac{1}{b^2}\right) \tag{8}$$

or

$$\eta = \frac{\left(b^2 - a^2\right)}{4\pi a^2 b^2 l}\left(\frac{\tau}{\omega_B}\right) \tag{9}$$

The coefficient η is thus determined from the dimensional constants of the apparatus and the experimentally determined ratio τ/ω_B. The above result is equally valid if it is the inner cylinder which is rotating, since it is relative velocity that is important.

Apparatus

The viscosimeter employed in this experiment is illustrated in Figure 3. It consists of two metal cylinders A and B of radii a and b, respectively, mounted co-axially. The inner cylinder A rests on

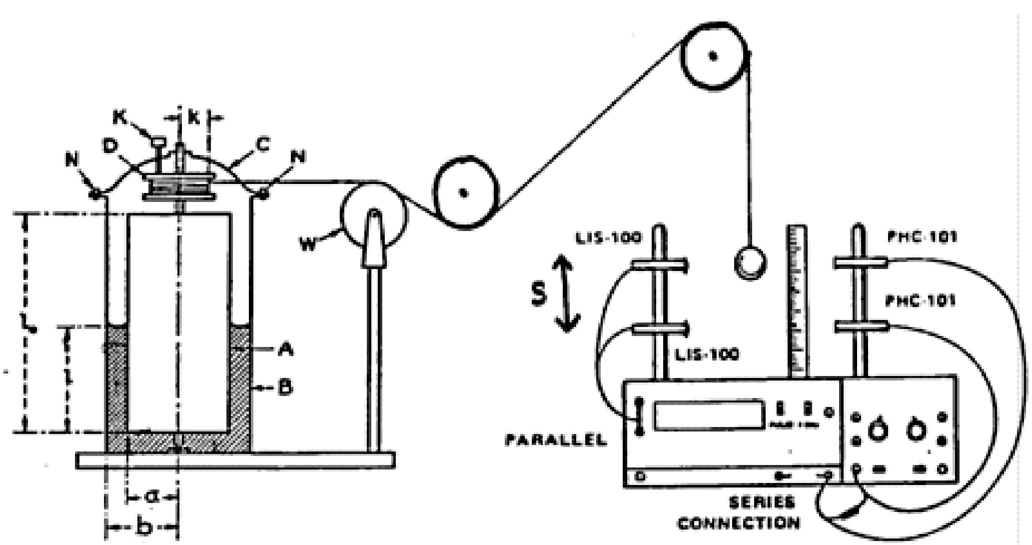

Figure 3: Viscosity Apparatus

a bearing so as to rotate with very little friction inside the stationary cylinder B, the liquid under investigation being contained in the space between the cylinders. Attached to the shaft of A is a drum D around which is wrapped a fine cord that passes over a pulley W attached to a mass *m*. The shearing torque is given by the product of the gravitational force on the mass *m* and radius *k* of the drum; i.e., $\tau = mgk$.

A removable cover C consists of an aluminum bracket which contains an upper bearing for the rotary cylinder. Two screw clamps N hold the bracket in place, accurately centering the shaft. The cylinder A may be locked in position by means of a key K which enters a hole in the drum. The viscosimeter is equipped with an electrical heating element enclosed in a jacket surrounding the outer cylinder.

In operation, the inner cylinder will start from rest and obtain a rotational velocity, ω_B, at which the viscous restraint just balances the applied torque. At that point, the angular velocity just equals the linear velocity of the string divided by the radius of the drum; i.e., $\omega_B = v/k$.

The linear velocity of the string is found by directing the string over a system of pulleys past a pair of photodetectors. The signal from the photodetectors is amplified and used to gate a scalar counting at a rate of 1 kHz. If the distance *s* between photodetectors is measured and the time *t* determined for passage of the mass between them, then $v = s/t$ and $\omega_B = s/kt$.

Finally, in the foregoing discussion it was assumed that the only viscous resistance involved is that exerted by the liquid between the cylindrical surfaces. However, there is an additional torque due to the viscous drag over the ends. The magnitude of this effect depends upon the radii of the two cylinders and the distance between their closed ends. If the cylinder is totally immersed, the end effect can be treated as a correction to the length of the cylinder. Thus an effective length *e* must be added to the length *1* in equation (9).

Substituting all of these relationships into equation (9) gives

$$\eta = \frac{(b^2 - a^2)k^2 g}{4\pi a^2 b^2 s(l+e)} mt = cmt \qquad \text{where} \qquad c = \frac{(b^2 - a^2)k^2 g}{4\pi a^2 b^2 s(l+e)} \tag{10}$$

c is a constant determined by the dimensions of the problem and the end correction factor which can be determined by experiment.

Procedure

End Correction Determination

At constant temperature the viscosity is a constant. If the mass and physical dimensions of the apparatus remain fixed, then equation (10) implies that the travel time t should vary directly as the corrected height of the fluid l; i.e., $t \approx (l + e)$. By making a series of measurements for different fluid levels, one should be able to plot a graph similar to that shown in Figure 4 and determine the end correction.

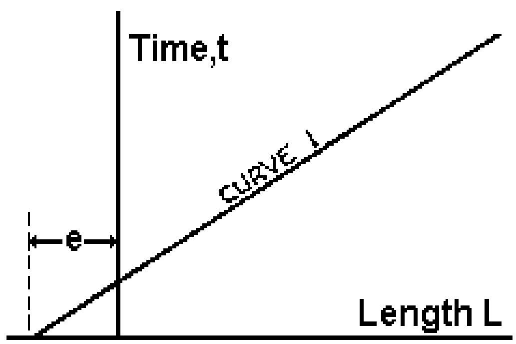

Figure 4: Time versus Length

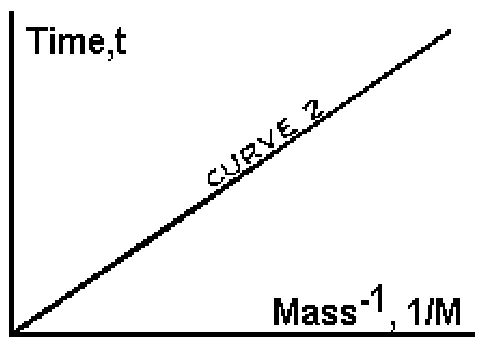

Figure 5: Time versus 1/mass

1. Locate the apparatus as shown in Figure 3. With the vernier caliper, carefully measure and record the dimensions a, b, k, and l_o. Reassemble the apparatus, taking care that the cylinder turns freely on its pivot points. Thread the string over the pulleys as shown and lock the drum. The black can at the end of the string is used to attach weights and insure that the photodetectors fire as the can passes them.
2. With a dropper, add liquid until the level is about 1.5 cm above the lower end of the cylinder and measure the immersed length, l. This is a difficult measurement to make and must be done carefully. Withdraw the inner cylinder and, holding it so as to drain fluid back into the outer cylinder, make a measurement of the fluid level on the cylinder.
3. Replace the cylinder and lock in place. Add a 5 gm weight to the can.
4. Release the cylinder and determine the time of fall between the photodetectors. You should wind the string up so that the mass, in falling, has a chance to reach its terminal velocity. You will also find it convenient to stop the drum after the weight has triggered the second photodetector but before the string comes loose from the drum, thus saving yourself the trouble of rethreading the string through the pulleys.

5. Repeat this measurement several times for average.
6. In order to check your measurements and ensure the terminal velocity has been reached, set up one of the sonic range finders at the floor aiming up at the hanger. Take three sets of measurements using the range finder and calculate both the average velocity and the average acceleration. Comment on how these values compare to your measurements using the photodetectors.
7. Make a series of at least 10 determinations using the photodetectors, keeping the mass constant, but increasing the effective length of the cylinder by adding fluid. The last observation should be made with the apparatus filled 1/8" over the top of the inner cylinder.
8. Make a plot similar to Figure 4 and determine the end correction.
9. Calculate the constant c using the parameters you have measured. Use c.g.s. units.

Variation with Mass

10. With the level of the liquid at the top of the inner cylinder, take a series of ten observations, varying the mass from 5 to 50 gm.
11. Explain why if the physical parameters are held constant, the time of fall should be inversely proportional to the mass?
12. Make a plot similar to Figure 5 and convince yourself of this fact. Explain any deviations from straight line behavior in terms of bearing friction.
13. From the slope of this curve extract the best value of $m \cdot t$
14. Measure the temperature of the fluid.
15. Calculate your best value for viscosity at that temperature.

Variation with Temperature

From practical experience you know that fluids become less viscous with an increase in temperature. This follows from the fact that with temperature increase the fluid molecules become more agitated and therefore have a larger equilibrium separation, thus, the shear forces are less.

16. Attach a 5 gm mass and keep this constant throughout the experiment.
17. Measure the temperature and take a set of time readings.
18. Plug in the viscosimeter to the variac and set the variac to 100 VAC. Surround the cylinder with a sytofoam insulating block to help stabilize the temperature of the system. Leave the

thermometer in the fluid and observe the temperature. When the temperature reaches just over 70^{o}C, unplug the variac. Adjust the fluid level to cover the cylinder.

19. Take a set of time readings when the temperature stabilizes at 70^{o}C.
20. As the temperature falls, take new sets of time readings at 5^{o}C intervals down to within 5^{o}C of your starting temperature.
21. Calculate the viscosity of the fluid as a function of temperature. Plot your results.
22. The fluid you are using is Pennzoil SAE 60 racing motor oil. The viscosity of the oil is 207 centipoise (cp) at 40^{o}C and 15.1 centipoise (cp) at 100^{o}C. Plot and compare the book values with your results.

DRAIN THE FLUID . WIPE OFF THE CYLINDER. LEAVE THINGS AS YOU FOUND THEM.