

MEASUREMENT OF THE CHARGE TO MASS RATIO (e/m_e) OF AN ELECTRON

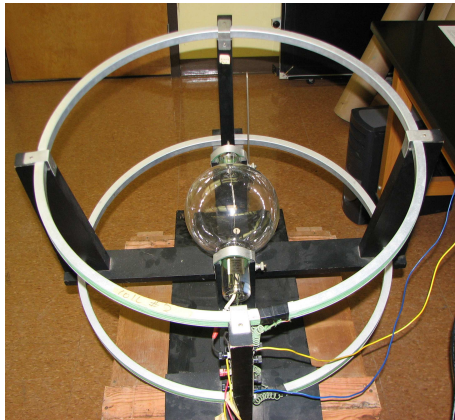
Object

This experiment will allow you to observe and understand the motion of a charged particle in a magnetic field and to measure the ratio of the fundamental charge, e , to the mass, m_e , of an electron. This measurement is one of the most historic and fundamental experiments in atomic physics. Since e had been determined in an independent experiment, (The Millikan oil drop experiment, first done in 1909); measurement of e/m_e provided the first knowledge of the mass of the electron. The same method is also used to find the masses of protons and heavier ions. In general, the technique is called mass spectrometry and was pioneered by J. J. Thomson, who is credited with the discovery of the electron.

Derivation of key relationships

If a charged body moves through a magnetic field it is acted on by a force which is given by the vector equation

$$\vec{F} = q\vec{v} \times \vec{B} \quad (1)$$



where \vec{F} is the force, q is the charge, \vec{v} is the velocity and \vec{B} is the magnetic field. When \vec{v} and \vec{B} are perpendicular then this simplifies to

$$||\vec{F}|| = |q||\vec{v}||\vec{B}|| \quad (2)$$

or simply $F = qvB$. In the experiment at hand electrons will flow in a plane perpendicular to the applied magnetic field. This causes the electrons to move in a circle. Derive a formula for e/m in terms of the speed of the electrons, the strength of the magnetic field, and the radius of the circle the electrons are moving in.

The way that the electrons are speed up is by applying an electric potential. Derive a formula that relates the velocity of the electron immediately after leaving the region with an electric field in terms of the electric potential difference, the charge on the electron, and the mass of the electron. Assume that the electrons are initially at rest.

Combine the two formulas you derived above to show that $e/m_e = 2\Delta V/(||\vec{B}||r)^2$:

One can easily measure ΔV and r but $||\vec{B}||$ is harder to measure. The \vec{B} in the above derivation is the vector sum of the earth's field, \vec{B}_{Earth} and the field due to a set of Helmholtz coils \vec{B}_{coil} . Helmholtz coils are a set of two parallel coils separated by a distance equal to their radius. For this set of coils the magnetic field strength in the region near the mid point on an axis joining the coils is given by

$$\|\vec{B}_{coil}\| = (1.96 \times 10^{-4} \frac{Tesla}{Amp}) I_c \quad (3)$$

where I_c is the current in amperes in the coils.

The plane of the Helmholtz coils is adjusted so that they are perpendicular to \vec{B}_{Earth} and \vec{B}_{coil} is anti-parallel to \vec{B}_{Earth} . The field due to the coils, \vec{B}_{coil} , can be determined by the precise determination of the current in the coils. The contribution to the total field due to the earth's field is more difficult to determine, and frequently less precise than other parts of the experiment. We will eliminate the need for determining the earth's field by the following useful trick.

The trick

Define the positive z -direction to be the direction that \vec{B}_{coil} points in, then the total field, \vec{B}_t , is $\vec{B}_t = \vec{B}_{coil} + \vec{B}_{Earth} = (\|\vec{B}_{coil}\| - \|\vec{B}_{Earth}\|)\hat{k}$ (remember \vec{B}_{coil} and \vec{B}_{Earth} are anti-parallel). Substituting this in Eq. (5) gives

$$e/m_e = \frac{2V}{(\|\vec{B}_t\|r)^2}, \quad (4)$$

and

$$\|\vec{B}_t\| = \sqrt{\frac{m_e}{e} \frac{\sqrt{2V}}{r}}, \quad (5)$$

which gives

$$\|\vec{B}_{coil}\| = \sqrt{\frac{m_e}{e} \frac{\sqrt{2V}}{r}} + \|\vec{B}_{Earth}\|. \quad (6)$$

Now instead of measuring \vec{B}_{Earth} directly we will determine it through a fit. If you graph $\|\vec{B}_{coil}\|$ vs $\sqrt{2V}/r$ then according to the above analysis a straight line should result whose slope is $\sqrt{\frac{m_e}{e}}$. The intercept of this curve will be $\|\vec{B}_{Earth}\|$. The remaining parameters (i.e., $\|\vec{B}_{coil}\|$, V , and r) can be determined with reasonable precision.

APPARATUS

One or two tubes will be set up and all electrical connections already made. Study the schematic in Fig. 2 and be sure you understand the wiring of your circuit. There are three circuits:

1. supplies current to the coils to create \vec{B}_{coil}
2. supplies current to the filament to heat it and supply electrons; and
3. provides the accelerating potential between the anode ("can") and cathode (filament).

You will measure both magnet coil current and anode voltage using digital multimeters.

The current to the magnet coils, I_c , is controlled by turning the Variac knob near the magnet. The Variac varies the A. C. line voltage to the power supply and hence its D. C. output voltage. The accelerating voltage, ΔV , is provided by a variable power supply. The tube filament is heated by another power supply. It is also controlled by a Variac.

PROCEDURE

Typically, the filament current and anode voltage supplies will already be on. If not, the procedure below should be followed:

1. Set the anode voltage to an intermediate to high value (35 – 45V).
2. Gradually turn up the filament current. As you approach 4A, the anode current should start to rise. When the anode current is about 5mA, you should see a blue beam exit the slit in the anode. Be very careful adjusting the filament current. Never let the anode current exceed 8mA or let the filament current exceed 4.5A. **Once the anode current starts rising, it goes up very fast for small changes in the filament current. Use the smallest filament current that gives a good visible beam.**

Before collecting your data, take some time thinking about what you are seeing. Bring up a bar magnet and notice the effect. What is actually going on in the tube to produce the blue light? Why does the beam broaden and get wispy as it travels? What parts of the circuit would have to be changed if electrons were positive? Be sure you can explain the effect the coil's magnet field will have on the beam.

1. Set the anode voltage to approximately 35V. It does not have to be exactly that value but should stay fixed after being set.
2. Turn on the magnet power supply. Now gradually turn up the magnetic field until the outside of the beam hits the innermost pin. The ideal situation is to have the outside edge of the beam skim the outside edge of the pin.
3. Record the voltage, "pin number", and current in the coil.
4. Repeat for the other pins.
5. Set up a spreadsheet in Excel to analyze your data using the following procedure:
 - (a) Put a title and the names of your group on the sheet
 - (b) Choose a cell and put your value of ΔV (in volts) in it. Use adjacent cells to label your value.

- (c) Set up columns for Pin#, Radius(m), Current(A), $(2\Delta V)^{1/2}/r$, $\|\vec{B}_{coil}\|$ (T) (Note: To do superscripts or subscripts, either right click and choose format cells or click on Format and then cells. Then check either superscript or subscript.)
- (d) The information to get the radius for each pin is given in the lab instructions. Enter the radius values.
- (e) Enter the values for your currents in the appropriate column.
- (f) Enter a formula to calculate $(2\Delta V)^{1/2}/r$ in the first row of that column and copy it for the rest of the column. You can either use the numerical value for your voltage or refer to its cell. If you refer to its cell, put \$-signs in front of both parts of the reference. For example, if your voltage is in cell B5, enter it in the formula as \$B\$5. Then when you copy, it will continue to use the information in cell B5 and not change the reference.
- (g) Enter the formula from the lab instructions to calculate the $\|\vec{B}_{coil}\|$ from the current in the first row of that column and then copy it for the rest of the column.
- (h) Now is a good time to save your spreadsheet! Put it in the 152Lab folder. Periodically save as you go along.
- (i) Format the numbers in these last two columns so that they have 3 significant figures. Use scientific notation for the $\|\vec{B}_{coil}\|$ (since they are such small numbers).
- (j) Make a graph of B_{coil} vs. $(2V)^{1/2}/r$ and label it appropriately. (Note: You cannot do sub or superscripts for the axis titles in the dialog boxes with the chart wizard. But once the graph is completed, you can select the titles and then format them, including sub or superscripts.)
- (k) Fit a linear trendline to the data.
- (l) Right click on the line and click on "format trendline". The box should open with the "options" tab displayed. If not, click on the options tab. In the "forecast" area, put the value of (x1 - 1), where x1 is your smallest value of $(2V)^{1/2}/r$, in the "backward" box, and then click on OK. This will extend the line to the y-axis.
- (m) Click anywhere outside the graph area. Then click on the tools menu tab at the top. Select "Data Analysis"; it should be at the bottom of the list. If you do not see it, make sure that you had first put the cursor out of the graph area.
- (n) Select Regression and then click on OK.
- (o) A dialog box will open. The blinking cursor will be in the "Input Y-range" block. Move the mouse to put the "cross" in the cell with the first value for $\|\vec{B}_{coil}\|$. Click and then drag the mouse until all of the values for $\|\vec{B}_{coil}\|$ are enclosed in the "dotted line box".

Release and this range of cells should now be in the Input Y-range" block. Now click the mouse in the "Input x-range block", highlight your $(2V)^{1/2}/r$ values and that range of cells should be place in that block.

- (p) Still in the dialog box, in "output options" select "output range". Click the mouse in the adjacent block and then move the "cross" pointer to a cell on the left side of the sheet, below your graph. Click and that cell should be entered in the output range block. Now click on OK to close the dialog box.
 - (q) A mass of statistics will be displayed below the graph. Highlight the entire tables for "Regression Statistics" and "ANOVA". Click on the Edit menu tab, select "Clear" and "All" which deletes all of this including the borders. Now highlight the last table and move it up, so that it is just under the title "Summary Output".
 - (r) Delete the last six columns (beginning with "t stat").
 - (s) Look at what you have left. In the first column (headed Coefficient) is the intercept and slope (X variable 1), and in the second column (headed Standard Error) are the uncertainties of those two values.
6. Determine e/m_e and $||\vec{B}_{Earth}||$ (with their uncertainties!) using the data for your best fit straight line.
 7. Compare these values with the accepted value for e/m_e and a handbook value for $||\vec{B}_{Earth}||$. You can find accepted values for e and m_e from your textbook. From these values determine the accepted value for e/m_e . An accepted value for \vec{B}_{Earth} can be inferred from an older version of the CRC Handbook. The accepted values quoted for \vec{B}_{Earth} in Pennsylvania are: Dip angle = 71° ; horizontal component of the field = 0.18 gauss. Note that the dip angle is the angle the field makes with respect to the horizontal. The lab equipment should be aligned so the $||\vec{B}_{Earth}||$ is the magnitude of the total Earth's field.
 8. Print one copy of your spreadsheet for the group to include with your records.

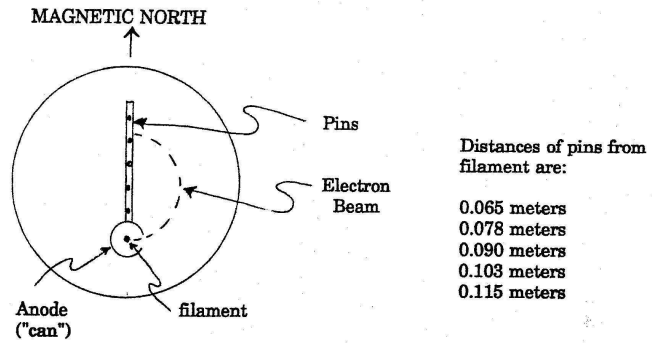


FIGURE 1

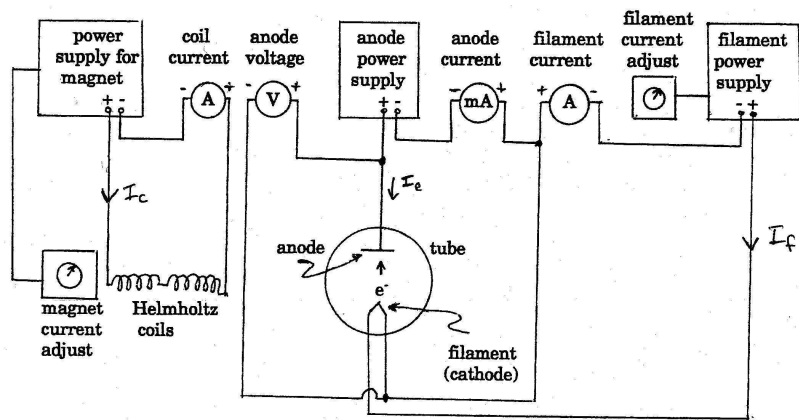


FIGURE 2