SIMPLE D.C. CIRCUITS AND MEASUREMENTS-Background

This unit will discuss simple D.C. (direct current - current in only one direction) circuits: The elements in them, the simple arrangements of these elements, the variables in the circuits, the relationships between them, and the ways they are measured. The definitions given will be brief and operational. You should refer to your textbook for more careful definitions (Chapters 26 and 27) and be alert in the lecture portion of the course for a fuller conceptual treatment later.

DEFINITIONS AND UNITS

The SI (MKSA) system of units will be used. The A represents the fundamental electrical unit in this system, the ampere, which is a measure of current.

Charge is that characteristic of matter which gives rise to the electromagnetic interaction. In the SI system, the unit of charge is the coulomb, which is defined in terms of the ampere (Q = I t see below). Therefore, 1 coulomb = 1 ampere-second. The electron, which has an elementary unit of negative charge, and the proton, which has an elementary unit of positive charge, have charges of 1.6×10^{-19} coulombs.

Current - Conceptually, electric current is the flow of charge through a cross-sectional area per unit time,

$$I = \frac{\Delta Q}{\Delta t} \qquad \text{(units - amperes (A))}$$

The current is more precisely defined in terms of the force on two current carrying parallel wires. The current can be caused by the motion of either positive or negative charges. The effect of positive charges moving in one direction is the same as that of negative charges moving in the opposite direction. Conventionally, to be consistent with other definitions, the direction of the current is the direction that equivalent positive charges would move.

Electrostatic Potential Energy is the potential energy that a charged object has due to an electrostatic interaction. Changes in this potential energy can occur as the charged object is moved from one location to another. The units are energy units, i.e. joules.

Potential Difference - The potential difference between two points is the change in electrostatic potential energy a unit positive charge would undergo in moving from the one point to the other.

Potential is the potential energy per unit charge.

V = P.E./Q units - volts (V): 1 volt = 1 joule/coulomb

EMF (Electro-Motive Force, but it is not really a force) is the amount of electrical potential difference (energy per charge) generated by a device in transforming some other form of energy to electrical energy. ("Other forms" of energy includes, for instance, chemical energy, which actually is electrical energy involved in chemical bonding of molecules.) For example, a dry cell battery is a source of EMF. There is a charge separation due to the chemical interaction of the zinc and carbon. In this interaction, the chemical energy decreases by 1.5 joules for each coulomb of charge separated. This process is in equilibrium when the electrostatic forces due to the separation balance the chemical forces. Therefore, there has been a gain in electrostatic energy of 1.5 joules per coulomb in the separation process. The potential difference between the terminals is then 1.5 volts. In a realistic battery, there will be some "internal resistance" (see Ohm's Law for resistance), so that the battery can be considered as a pure source of EMF together with a pure resistor.

OHM'S LAW

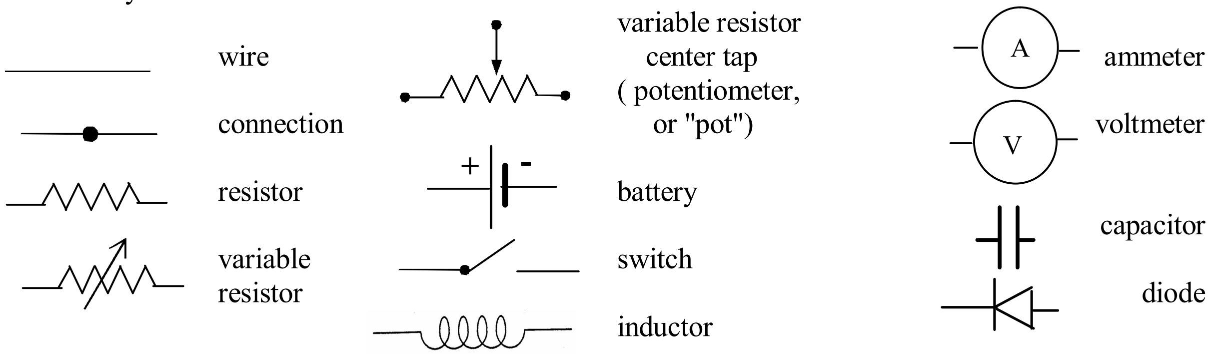
For many substances, the current between two electrodes suitably attached to the substance is directly proportional to the potential difference or applied voltage across them. The constant of proportionality is called the <u>resistance</u>, R, which is a measure of the ease with which charges can flow through this substance. (Unit of R = ohm (Ω).) Such a material is said to be an <u>Ohmic resistor</u>. For this class of materials the relationship

$$V = IR$$

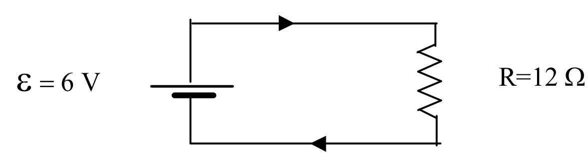
holds and is called Ohm's Law where V is the potential difference across the resistor and I is the current through it. A resistor may be of fixed or adjustable value. There are also many important "non-ohmic" materials, that is, those in which the ratio of applied voltage to current is not constant. Some devices of this type are light bulbs, diodes, transistors, vacuum tubes, and silicon controlled rectifiers.

SIMPLE CIRCUITS

In simple circuits, a source of EMF (say a battery) is combined in a simple manner with resistors. A diagram of the arrangement is called a circuit diagram. This diagram is a schematic presentation and certain standard symbols are used to represent the various circuit elements. The following is a list of the most common symbols.



As an example of Ohm's Law, consider the simple circuit of a battery and a resistor as shown at the right



If we wished to calculate the current in this circuit we would use Ohm's Law. The current in the resistor is the voltage applied to the resistor (the EMF of the battery in this case), 6V, divided by the resistance of the resistor, 12Ω , or

$$I = \varepsilon/R = 6V/12\Omega = 0.5\ A$$

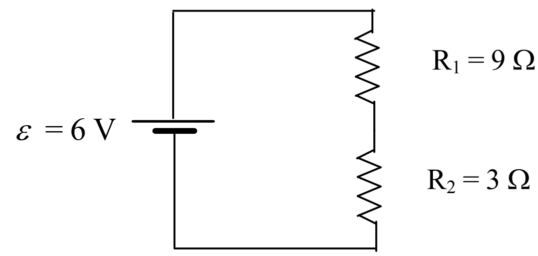
Note that the resistance and the EMF were determined by the physical characteristics of these elements and they, in turn, determined the current. In the case of this simple circuit, this current is supplied by the EMF. Suppose that the resistance had been 12 kΩ. Then the calculation would have been

$$I = \varepsilon/R = 6V/(12 \times 10^3\ \Omega) = 0.5 \times 10^{-3}\ A \text{ or } 0.5\ mA \text{ (milliamperes)}$$

In electronic circuits, it is quite common to have resistances on the order of kΩ, so that the currents are frequently in mA.

Since the direction of the current is the direction of positive charges, then the current will be from the + terminal to the - terminal as indicated on the diagram. As current passes through the resistor, the potential decreases in the direction of the conventional current, and so we often speak of the potential drop as the current passes through the resistor. There is a loss of electrical energy.

Now consider a circuit in which two resistors are connected in such a way that the same current must go through each. This type of connection is called a series connection.



How might we find the current in this circuit? There are two key principles to use here. First, we know that the current through R_1 is the same as through R_2 (this is just charge conservation), which in this case is the same as the current supplied by the source of EMF. Second, a charge must lose an amount of electrical energy in going through the resistors equal to that which it gains in going through the source of EMF. Another way of saying this is that the total change of energy when the charge makes a complete loop is zero. In terms of the potential difference, the algebraic sum of the potential differences must equal zero for a complete loop. A consequence of this is that the potential difference across the two resistors must be equal to the sum of the potential differences across each one individually. And this sum must equal the applied voltage. This is just an extension of energy conservation. Now apply these to the circuit at hand.

Look first at R_1. Ohm's Law holds for this element. The potential difference across this resistor will be equal to the current through it times the resistance of it

$$V_1 = I\,R_1 \tag{1}$$

Doing the same for R_2 gives,

$$V_2 = I\,R_2 \tag{2}$$

But the potential difference across R_1 and R_2 is just $V_1 + V_2$ so that

$$\varepsilon = V_1 + V_2 = IR_1 + IR_2 = I(R_1 + R_2). \tag{3}$$

Then the current is simply,

$$I = \varepsilon/(R_1 + R_2) = 6V/(9\Omega + 3\Omega) = 6V/12\Omega = 0.5A$$

It is interesting to look in more detail at what we have obtained. If we had started from the approach of the circuit as a whole, we would have written that the current in the circuit would be the applied voltage (ε) divided by the total (or equivalent) resistance of the circuit,

$$I = \varepsilon/R_t \quad \text{or} \quad \varepsilon = IR_t.$$

Comparison with equation (3) above shows that the total, or equivalent, resistance is

$$R_s = R_1 + R_2 \tag{4}$$

The subscript "s" is used to indicate that the resistors were connected in series. This is a general result. When any number of resistors are connected in series, their equivalent resistance is just the sum of the individual resistances. Because of this addition, the series equivalent is always larger than the largest of the individual elements.

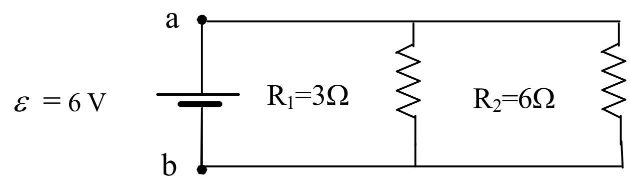
Let's go back and look at each element of the circuit to make sure that our result is consistent. The potential difference across R_1 is

$$V_1 = I\,R_1 = (0.5A)\,(9\Omega) = 4.5\ V$$

Similarly, the potential drop across R_2 is

$$V_2 = I\,R_2 = (0.5A)\ (3\Omega) = 1.5\ V.$$

Notice that indeed the sum of V_1 and V_2 is 6V as expected. Also notice that the ratio of the potential drops across the resistors is the same as the ratio of the resistors. R_1 is 3 times larger than R_2 and V_1 is 3 times larger than V_2. This is true because the same current flows through each.



Now look at a combination in which two resistors are connected in such a way that the current splits between them. Such a connection is known as a parallel connection. In order to analyze this, we need to be aware of another rule. This states that the algebraic sum of the currents at a junction must equal zero. In other words, the current into a junction must equal the current out. This is just a consequence of charge conservation; we are not creating or destroying charge at the junction. Consequently, the current through R_1 plus that through R_2 must equal the current in the rest of the circuit. Now for R_1, the current would be

$$I_1 = \frac{V_1}{R_1} \tag{5}$$

and for R_2, it would be

$$I_2 = \frac{V_2}{R_2} \tag{6}$$

But $V_1 = V_2$, since the potential difference between points a and b must be the same, independent of the path taken. In this case, $V_1 = V_2 = 6V$, because those two points are also across the battery. It is then possible to calculate I_1 and I_2 directly.

$$I_1 = 6V/3\Omega = 2A$$

$$I_2 = 6V/6\Omega = 1A$$

Then the current supplied by the battery would be

$$I = I_1 + I_2 = 2A + 1A = 3A$$

Notice in this case that the current is inversely proportional to the resistance. That is, the ratio of the currents in inverse to the ratio of the resistances. The resistor with the lowest resistance has the highest current. R_2 is 2 times larger than R_1 and the current through R_2 is 1/2 that through R_1.

Again, let's look at the situation from another view point. Adding equations (5) and (6) gives the total current through the combination.

$$I = I_1 + I_2 = \frac{V}{R_1} + \frac{V}{R_2} = V(\frac{1}{R_1} + \frac{1}{R_2}) \tag{7}$$

where V is the potential difference across the combination. If we were to write Ohm's Law for the combination, we would have

$$V = IR_t$$

where R_t is the total or equivalent resistance. Rewriting this, we have, $I = V\,(\frac{1}{R_t})$

Comparing this with (7), we see that

$$\frac{1}{R_p} = \frac{1}{R_1} + \frac{1}{R_2}$$

where p is used to indicate the equivalent resistance when the resistors are combined in parallel. For this example, we would have

$$\frac{1}{R_p} = \frac{1}{3\Omega} + \frac{1}{6\Omega} = \frac{3}{6}\,\Omega^{-1} = 0.5\,\Omega^{-1}$$

$$R_p = 2\Omega$$

Using this we get

$$I = \frac{V}{R_p} = \frac{6V}{2\Omega} = 3A$$

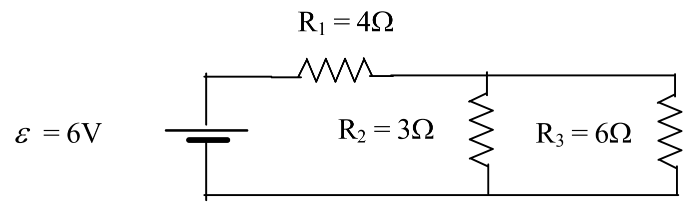
which agrees with the previous result.

This method of obtaining the equivalent resistance for a parallel combination of resistors is a general result. If n resistors are connected in parallel, then the equivalent resistance of the combination, R_p, is given by

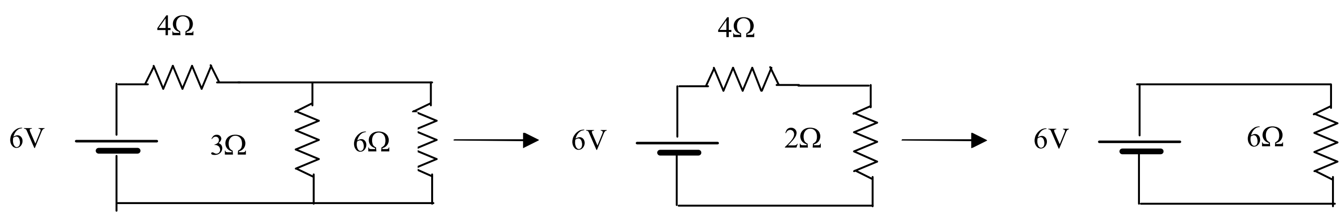
$$\frac{1}{R_p} = \frac{1}{R_1} + \frac{1}{R_2} + \bullet\bullet\bullet + \frac{1}{R_n}$$

Notice that because of the way the addition is accomplished, the equivalent resistance is always less than the smallest of the individual resistances.

A slightly more complex circuit might have a combination of series and parallel resistors, such as shown at the right.



Suppose we wished to find the current in R_3. One approach would be to find the equivalent resistance of the circuit, obtain the current supplied by the battery, and then work backwards to get the desired current. The circuit can successively be reduced as follows:



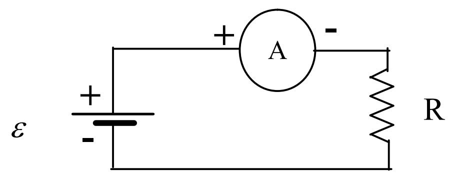
Now, the current supplied by the battery is $I = \frac{6V}{6\Omega} = 1.0A$. The potential difference across the 2Ω equivalent resistor is, from Ohm's Law, $V = IR = (1A)(2\Omega) = 2V$

But this same potential difference is across the 6Ω resistor which was part of that parallel combination, so that the current through R_3 (the 6Ω resistor) is just, $I_3 = \frac{2V}{6\Omega} = .33A$

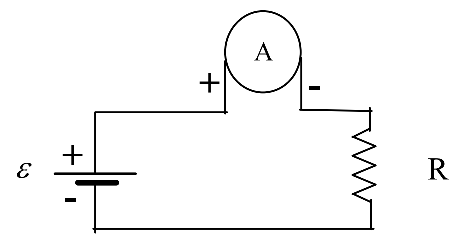
As a check, the current through R_2 is $$I_2 = \frac{2V}{3\Omega} = .67A,$$
so that the sum of the current through R_2 and R_3 is 1.0 A, as we saw before.

METERS

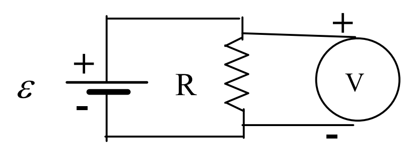
An ammeter is a meter used to measure current. The ammeter is placed in the circuit so that the current goes through the meter (in series). If the insertion of the ammeter is to have little effect on the circuit, then the resistance of the ammeter must be very small. An ideal ammeter would have zero resistance. With some meters, this is accomplished by placing a very small internal resistor in parallel with the basic meter movement. Different parallel resistors would determine different current scales. Terminals on an ammeter are typically marked with "+" and "-". The current should go through the meter from "+" to "-" (in at the "+" and out at the "-"). This means that in a circuit with a single EMF source, the "+" terminal should be closest to the "+" side of the EMf. The figure below shows the placement of an ammeter in a simple circuit with the polarity marked. Note that you must open (break) the circuit at the point you wish to measure the current and insert the meter at that point. The meter is placed in series.



or



The sensitivity of a meter is the smallest current that will cause a noticeable effect. This does not necessarily mean that the meter is more accurate just because it is more sensitive. By the use of amplifying circuits, the sensitivity of some meters is increased greatly. An electrometer, for instance, can detect currents as small as 10^{-12} A.



A voltmeter measures the potential difference between two points. The leads of the voltmeter are placed at these two points without changing anything else in the circuit. For instance, if the potential difference across a resistor is desired, the voltmeter would be placed across the resistor (in parallel with it). The "+" terminal of the voltmeter should be at the point of highest potential, which for a circuit with a single source of EMF would be the point closest to the "+" side of the source of EMF. See the figure above for the placement of a voltmeter in a circuit (measuring the potential drop across the resistor) with the polarity indicated.

If the placement of the voltmeter is to have little effect on the circuit then the resistance should be very large. In some meters, this is accomplished by a large internal resistance placed in series with the basic meter movement. Different series resistors would determine different voltage scales. Special circuits are sometimes used to make the resistance extremely large (on the order of 10^{10} Ω). An ideal voltmeter would have an infinite resistance.

An ohmmeter is used to measure resistance. Usually an internal battery supplies a current through the unknown resistance. The magnitude of the current is related to the resistance. The meter measures the current on a scale which is calibrated in ohms. Because it functions this way, it is important that other sources of current (EMF's) are disconnected from the resistance. In our applications, this typically means removing the resistor or combination of resistors from the rest of the circuit and then placing the leads of the ohmmeter across the resistance.

In a multimeter a switch or buttons select different functions and different sensitivities (scales). Typically, the multimeter will include voltmeter, ammeter and ohmmeter functions. Some meters will require the leads to plug into different jacks for different functions. Also, the meters may have and AC - DC option as well as some other functions.