Gravity

February 3, 2011

Purpose:
To study the effect of gravity on an object near the surface of the earth.
To use this information to determine the mass of the earth.

1 Theory

Galileo Galilei studied the motion of a rolling ball down an inclined plane and discovered that objects with different masses fall at the same rate, that the distance an object falls depends on the square of the time, and that unless acted upon by an outside force an object will move in a straight line at a constant speed. Today we will do a variation on Galileo's experiment using carts on a track instead of a rolling ball.

Later Issac Newton realized that how quickly an object speeds up, which is called its acceleration, depends on the force on the object and the mass of the object. The equation that relates these three quantities for an object (in our case a cart) is

$$a = \frac{F}{m_{cart}} \tag{1}$$

where a is the acceleration of the cart, F is the force on the cart, and m_{cart} is the mass of the cart.

Newton also realized that the force of gravity that holds us on the surface of the earth is the same force that holds the solar system together. Newton's universal law of gravitation gives the force from one object (such as the earth) on a second object (such as a cart) and has the equation

$$F = \frac{G m_{cart} m_{earth}}{r^2} \tag{2}$$

where m_{earth} is the mass of the earth, m_{cart} is the mass of the cart, r is the distance from the middle of one object to the other, and G is the universal gravitational constant which has the value $G = 6.67x10^{-11} m^3/(s^2 kg)$. Since the earth is so big, the distance from the center of the earth to the center of our cart will stay essentially the same throughout the experiment and is the radius of the earth which has the value $6x10^6 m$.

If we combine Eqs. 1 and 2 to find the acceleration of a cart because of a gravity pulling down on it we get

$$a = \frac{Gm_{earth}}{r^2} \tag{3}$$

. Notice that the acceleration of the cart is independent of its mass (it canceled out) just as Galileo observed. Rearranging Eq. 3 to get the mass of the earth on its own gives

$$m_{earth} = \frac{ar^2}{G} \tag{4}$$

and shows that if we can measure this acceleration then we can weigh the earth.

The one difficulty with the above is it requires us to drop the cart straight down on to the ground which makes measuring how quickly the cart speeds up hard to do because everything happens so quickly. One way around this is to send the cart down the track instead of dropping it. Then the track will partially hold the cart up by exerting a force on the cart and redirect the motion of the cart along the track's surface. A careful accounting of the effect of the track on the cart shows that the acceleration of the cart along the track, a_{track}, and the free fall acceleration, $a_{freeFall}$ are related to each other in the following way

$$\frac{a_{track}}{a_{freeFall}} = \frac{H}{L} \tag{5}$$

where L is the distance along the track the cart moves and H is the change of the height of the cart as it goes thorough the distance L. For each tilt of the track the ratio of H to L is constant. Equivalently we can rearrange Eq. 5 and write it in terms of $S = H/L$

$$a_{track} = Sa_{freeFall}. \tag{6}$$

2 Procedure

You will be measuring the acceleration of the cart for various levels of tilt, S. To find the acceleration at a given S value you will need to find how long it takes the cart to go from rest at the top of the track to its maximum speed at the bottom of the track. Detailed instructions are as follows:

1. Measure and record the distance L between the legs of your track.

2. Check the levelness of your track. To do this place the cart in the middle of the track. If the cart rolls then adjust the legs so that it no longer does so.

3. Select a piece of wood to prop up the track with. Measure its thickness H and calculate the value of S for this level of tilt.

4. Place your cart at the top of the track and use your stopwatch to time how long it takes for the cart to reach the bottom of the track. Do this a couple of times recording the values you find to be sure you are doing this correctly and getting a fairly consistent result.

5. To find the speed of your cart at the bottom of the track measure how long it takes the cart to move the last $10cm = 0.1m$ before hitting the end of the track. Again do this several times, recording each value you measure. Use this time and the distance traveled in this time to find the velocity of the cart at the bottom of the track in units of m/s.

6. The cart started at rest and finished with the speed you found above. Determine the acceleration of the cart along the track, $a_{track}=$ (change in velocity)/(time to make the change) in units of m/s^2.

7. Repeat the above measurements for a total of 4 tilt levels.

3 Analysis

You will now find the free fall acceleration of an object near the surface of the earth using the data you just collected.

1. Make a graph of a_{track} versus S. Your graph should have a_{track} on the vertical axis and S on the horizontal axis. Be sure to label your axis and give the units of the quantity.

2. Equation 6 tells us that the graph you just made should be a straight line. The standard form for an equation is $y = mx + b$. Explain what is playing the role of y, m, x, and b in Eq. 6.

3. Draw the line that best goes through your data points on the graph and calculate the slope of the line.

4. You are now ready to find the mass of the earth. Do the calculation and also find the accepted value for the mass of the earth. Are the two values similar?

Question: Both time measurements had some variability (or uncertainty) in them. Which time was the uncertainty more important in? In answering this question it may be helpful to calculate the percent uncertainty.

Your lab group should turn in a joint report that includes your data table, graph, answers to any questions, and a summary of what you did and/or learned.