Orbit of Mercury

January 27, 2011

Purpose: To use astronomical observations to deduce the path and period of Mercury's orbit.

Theory and Background

Tycho Brahe spent the greater part of his life systematically measuring and recording the locations of the planets. After his death in 1601 Brahe's data was passed on to Johannes Kepler. At the time Kepler believed that the planets moved in circular orbits around the Sun at a constant speed. Careful analysis of the observational data indicated that this was not quite correct. Eventually Kepler concluded that while all of the orbis are roughly circular none are exactly. He cleverly took the data for the "angle of greatest elongation from the Sun" for Mercury and used it to construct a figure which showed him that Mercury orbit is actually an elipse.

In this exercise we will take the same data that was available to Kepler and use his techniques to determine the shape and size of the orbit of Mercury. We will also be able to measure the time it takes Mercury to orbit the sun, (i.e. Mercury's orbital period).

Because the orbit of Mercury is closer to the Sun than the Earth's orbit Mercury (as seen from Earth) is never very far from the Sun; and so the angle between the direction of the Sun and the direction of Mercury is always small. This angle is called the angle of elongation. When Mercury passes between the Earth and the Sun the angle of elongation is 0^o. As time passes both planets move and the angle of elongation increases until it reaches a maximum or greatest value. The angle of greatest elongation occurs when Mercury is either west or east of the Sun as illustrated in Fig. 1.

In looking at Brahe's data Kepler noticed that the angles of greatest elongation were not always the same angles; they varied considerably. He apparently decided to construct a diagram to help him understand why these angles were not always the same. You will use the same technique to make sense of the angles as he did in this exercise. The angles of maximum elongation are shown in Table 1.

The excentricity, e of an orbit is a measure of how much the orbit differs from a circle. The value of e is smallest for a circle ($e = 0$). The largest possible

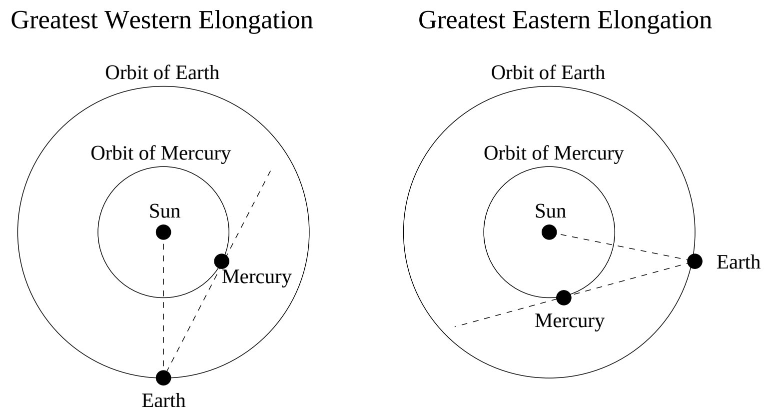

Figure 1: Orbital paths of Earth and Mercury around the Sun. Mercury is to the right of the Sun during western elongations and to the left of the Sun during eastern elongations.

value is $e = 1$ which corresponds to a long skinny orbit.

Procedure

Figure 4 shows the orbit of the Earth around the Sun with marks to represent the location of the Earth at various times of the year. For this exercise we assume (as did Kepler) that the Earth moved in a circle around the Sun. Use a very sharp pencil to carefully draw on Fig. 4 as follows:

1. For each of the dates in Table 1. locate the position of the Earth for the particular date on Fig. 4 and draw a very light pencil line from the Earth to the Sun.

2. Place the observation number next to the location of the Earth. (This will reduce clutter on your drawing and make it easier to understand.)

3. Center a protractor at the position of the Earth and draw a second line, the line of sight from Earth to Mercury at the point of greatest elongation (or simply line of sight) so that the angle between the Earth-Sun line and the line of sight is equal to the angle of greatest elongation for this particular date. **But before you draw the line of sight, be very careful about the direction (East or West) of the elongation.** For dates on which the elongation is western the line of sight from the earth to Mercury must be to the west of the Sun, likewise for those dates on which there is an eastern elongation the line of sight must be to the east of the Sun. To determine which is east and west orient your figure so the Earth is in front of you and you are looking from the Earth toward the Sun. West will be to the right of the Sun and east will be to the left. It

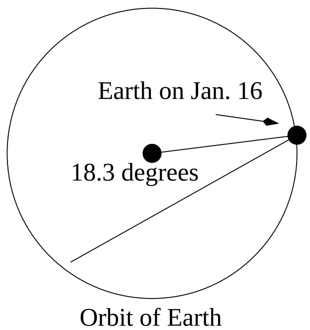

Figure 2: Example set of lines using the data in Table 1.

is very important to get these directions correct, so, after you have drawn the line for the first observation ask your instructor to check your work. Since we don't know right away where along the line of sight Mercury is, extend the line so it goes well past the Sun as shown in Fig. 2.

4. Repeat the steps above to plot the lines of sight for all the dates shown in Table 1. Be careful that you get the east and west correct. As you proceed you will start to see the shape of the orbit of Mercury emerge. Remember to make the line from the Earth to the Sun quite faint; otherwise they will overshadow the greatest elongation lines. Also be certain to extend the greatest elongation line well past the Sun.

5. Upon completion it may be that one or more of your lines seem to be out of place. If so check to make sure that you have plotted them correctly. Adjust as necessary. After you have correctly plotted all the lines *carefully* sketch the orbit of Mercury on your figure. The orbit should not cross any of the greatest elongation lines (they should each be tangent to the orbit).

Analysis

Analysis of Orbit Shape

Use your figure to answer the following question:
1. Is the orbit of Mercury a circle? Is it posible that it is a circle? Is it definitely not a circle? Explain how you arrive at your answers.
2. Kepler recognized that the orbit of Mercury was an ellipse with one focus

Observation No.	Date	Elongation (degrees)	Direction	Elapsed Time (days)
1	Jan. 16, 1580	18.3	East	0
2	Feb. 27, 1580	27.6	West	42
3	May 9, 1580	22.8	East	114
4	June 27, 1580	20.8	West	163
5	Sep. 6, 1580	26.3	East	234
6	Oct. 17, 1580	18.7	West	275
7	Dec. 30, 1580	18.9	East	349
8	Feb. 9, 1581	26.8	West	390
9	Apr. 21, 1581	21.3	East	461
10	June 9, 1581	22.3	West	510
11	Aug. 20, 1581	27.1	East	582
12	Sep. 30, 1581	18.2	West	623
13	Dec. 13, 1581	19.6	East	697
14	Jan. 22, 1582	25.7	West	737
15	Apr. 4, 1582	20.2	East	809
16	May 22, 1582	24.0	West	857
17	Aug. 2, 1582	27.4	East	929
18	Sep. 14, 1582	17.9	West	972
19	Dec. 6, 1582	20.7	East	1055

Table 1: Greatest elongation angles for Mercury.

at the Sun. The longest direction of an ellipse is the major axis with a length of $2a$ and a shortest direction (perpendicular to the major axis) called the minor axis with a length of $2b$. These two axes cross at the center of the ellipse as shown in Fig. 3.

Carefully locate the major and minor axes of your ellipse. They must be perpendicular to each other. Measure and record the length of the major axis ($2a$) and minor axis ($2b$) in mm.

A convienent unit of measurement for distances in the solar system is the Astronomical Unit, $A.U.$, which is defined to be the distance from the Sun to the Earth. Now measure the size of $1.0 A.U.$ on the diagram you have made in Fig. 4 by measuring the distance from the sun to the Earth in mm.

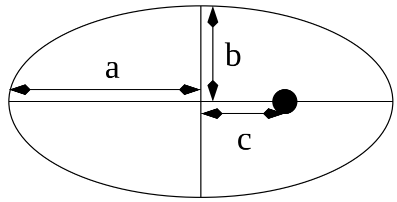

Figure 3: The semi-major axis a, semi-minor axis b, and the distance from the center of the ellipse to a focus c.

Based on your measurement of $1.0 A.U.$ you can find the scale for Fig. 4:

$$Scale = \left(\frac{1.0 A.U.}{Length\ of\ 1 A.U.\ in\ mm}\right) \tag{1}$$

Using this scale convert your distance measurements, namely the semi-major axis a, the semi-minor axis b, from mm to $A.U.$.

The accepted value for Mercury's a is $0.387 A.U.$ how does your measurement compare with this value?

Measure and record the length of c (see Fig. 3) in mm. Use the scale to convert c to $A.U.$.

Now calculate the $e = c/a$ the excentricity of the orbit. The references quote

the excentricity of Mercury's orbit as 0.206, how does your result compare?

Advanced/Extra Credit: Correction to N_{rev}
Notice that location 18 is a little past location 1. We can take this into account by including the fraction of a rotation that this represents. To do this draw a line from the Sun to location 1 and from the Sun to location 18. Measure and record the angle between these lines in degrees. Now convert this into the fractional number of revolutions that 18 is beyond 1 and add this to your whole number count of the revolutions to find a more precise value for N_{rev} the total number of revolutions durring the 972 days of observations.

Analysis of Orbital Period

When you drew the lines connecting Earth and Mercury you did not know where Mercury was located on the line. Now that you have sketched in the orbit it is possible to determine where Mercury was on each of the dates listed in Table 1. Locate the line that is associated with the data from January 16, 1580 (the very first data point). On that line make a dot where Mercury was on that date and label this dot with the observation number (i.e., 1). Repeat this process for each of the following dates on your drawing.

On your drawing Mercury's first location is labeled 1. You should notice that data point 18 is right beside 1. That means that the time between location 1 and 18 (which is 972 days) Mercury almost returned to its original location. By finding the number of orbits Mercury completed in this time we can calculate the planet's orbital period.

Mercury moves around the sun in the same direction that the Earth moves around the Sun. Starting with location 1, carefully count the number of revolutions, N_{rev}, that Mercury makes around the Sun before arriving at location 18; this is done by noting where each numbered location is and counting how many times Mercury passes by position 1 before arriving at position 18.

Use the length of the observation period (972 days) and N_{rev} to find the orbital period of Mercury in Earth days. The accepted value for the period of Mercury's orbit is 87.969 days. How close is your measurement to the quoted value? By how many days, hours does it differ?

Quantity	Value	Units
Major Axis		(mm)
Minor Axis		(mm)
Distance from Sun to center		(mm)
Length of 1 A.U.		(mm)
Semi major Axis		(A.U.)
Semi minor Axis		(A.U.)
Distance from Sun to center		(A.U.)
eccentricity		
N_{rev}		
Orbital period		(days)
Difference from accepted value (87.969 days)		

Table 2: Data Table.