1. Consider any two continuous functions of the generalized coordinates and momenta $g(q_k, p_k)$ and $h(q_k, p_k)$. The Poisson brackets are defined by

$$[g, h] \equiv \Sigma_k (\frac{\partial g}{\partial q_k}\frac{\partial h}{\partial p_k} - \frac{\partial g}{\partial p_k}\frac{\partial h}{\partial q_k}) \tag{1}$$

   Verify the following properties of the Poisson brackets:

   (a) $\frac{dg}{dt} = [g, H] + \frac{\partial g}{\partial t}$

   (b) $\dot{q}_j + [q_j, H]$, $\dot{q}_j = [p_j, H]$

   (c) $[p_k, p_j] = 0$, $[q_k, q_j] = 0$

   (d) $[q_k, p_j] = \delta_{kj}$

   where H is the Hamiltonian. If the Poisson bracket of two quantities vanishes, the quantities are said to commute. If the Poisson bracket of two quantities equales unity, the quantities are said to be cannonically conjugate. Show that any quantitiy that does not depend explicitly on the time and that commutes with the Hamiltonian is a constant of the motion of the system. Poisson-bracket formalism is of considerable importance in quantum mechanics.

2. Discuss the implications of Liouville's theorem on the focusing of beams of charged particles by considering the following simple case. An electron beam of circular cross section (radius R_o) is directed along the z-axis. The density of the electrons across the beam is constant, but the momentum components transverse to the beam (p_x and p_y) are distributed uniformly over a circle of radius p_o in momentum space. If some focusing system reduces the beam radius from R_o to R_1, find the resulting distribution of the transverse momentum components. What is the physical meaning of this result? (consider the angular divergence of the beam.)