Total Harmonic Distortion (THD)

Built on the idea of fourier Series

$$F(t) = a_0 + a_1 \cos \omega t + a_2 \cos 2\omega t + \cdots$$
$$+ b_1 \sin \omega t + b_2 \sin 2\omega t + \cdots$$
$$= a_0 + \sum_{n=1}^{\infty} a_n \cos n\omega t + \sum_{n=1}^{\infty} b \sin n\omega t$$

w/ $a_n = \frac{2}{T} \int_0^T F(t) \cos n\omega t \, dt$

and

$b_n = \frac{2}{T} \int_0^T F(t) \sin n\omega t \, dt$

Total harmonic distortion compares the part of the f^n that is in the Fundamental to the rest of the f^n

$$F_1^2 = a_1^2 + b_1^2$$
$$F^2 = F_0^2 + \sum_{n=1}^{\infty} F_n^2$$

so the part that is not in the fundamental is

$$F^2 - F_1^2 = F_0^2 + F_2^2 + F_3^2 + \cdots$$

and the ratio of non-fundamental to Fundamental

$$\frac{F^2 - F_1^2}{F_1^2}$$ which leads to

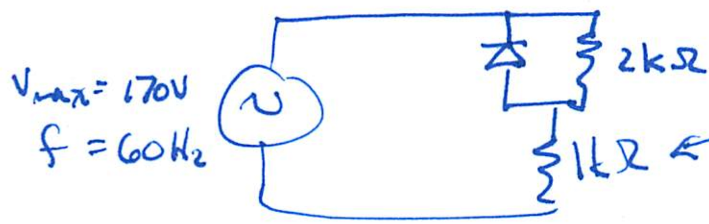
$$\boxed{THD = \sqrt{\frac{F^2 - F_1^2}{F_1^2}} \quad \text{or in terms of RMS values} \quad = \sqrt{\frac{F_{RMS}^2 - F_{1RMS}^2}{F_{1RMS}^2}}}$$

* Note THD typically is found for current as V(t) is often constructed to be sinusoidal

Calculating THD using LTSpice

Construct wave to analyze

in class example we found the current through this resistor



found on RMS basis

- control click waveform title
  → RMS value for $i(t)$

- To Find F_{1RMS}
  - select ~~graph~~ window w/ waveform
  - under view menu select FFT
  - change vertical & horizontal scales to linear scales
  - read peak of FFT @ fundamental (60 Hz)

    peak value is F_{1RMS}

from simulation

$i_{RMS} = 87.4\ mA$

$i_{1RMS} = 78.1\ mA$

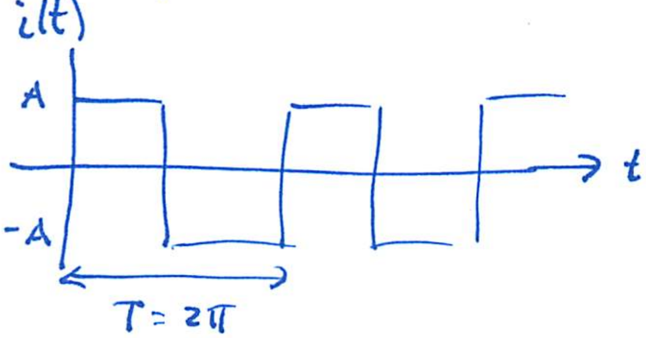
$$THD = \sqrt{\frac{87.4^2 - 78.1^2}{78.1^2}} = 0.50$$

or

50%

Calculating THD by hand

Example in class



$$i_{RMS} = \sqrt{\frac{1}{T}\int_0^T i^2\,dt}$$

($\sqrt{2}$ conversion only applies for sinusoidal f^ns).

break integral into 2 pieces

$$i_{RMS} = \sqrt{\frac{1}{2\pi}\int_0^{\pi}(A)^2\,dt + \frac{1}{2\pi}\int_{\pi}^{2\pi}(-A)^2\,dt}$$

$$= \sqrt{\frac{1}{2\pi}A^2\pi + \frac{1}{2\pi}A^2\pi} = \sqrt{\frac{A^2\pi + A^2\pi}{2\pi}} = \sqrt{A^2} = A.$$

to find $i_{1_{RMS}}$ we need a_1, b_1

$$a_1 = \frac{1}{\pi}\int_0^{2\pi} i(t)\cos t\,dt$$

again break into 2 pieces

$$= \frac{1}{\pi}\int_0^{\pi} A\cos t\,dt + \frac{1}{\pi}\int_{\pi}^{2\pi} -A\cos t\,dt$$

$$= \frac{A}{\pi}(+\sin t)\Big|_{t=0}^{t=\pi} - \frac{A}{\pi}(+\sin t)\Big|_{t=\pi}^{t=2\pi}$$

$$= \frac{A}{\pi}(0-0) - \frac{A}{\pi}(0-0) = 0.$$

$$a_1 = 0$$

$$b_1 = \frac{1}{\pi}\int_0^{2\pi} i(t)\sin t\,dt$$

break into 2 pieces

$$= \frac{1}{\pi}\int_0^{\pi} A\sin t\,dt + \frac{1}{\pi}\int_{\pi}^{2\pi} -A\sin t\,dt$$

$$= \frac{A}{\pi}(-\cos t)\Big|_0^{\pi} + \left(\frac{-A}{\pi}\right)(-\cos t)\Big|_{\pi}^{2\pi}$$

$$= \frac{A}{\pi} - -\frac{A}{\pi} + \frac{A}{\pi} - -\frac{A}{\pi}$$

$$= \frac{4A}{\pi}$$

and $b_{1_{RMS}} = \frac{4A}{\pi\sqrt{2}}$

$$I_{1_{RMS}}^2 = a_{1_{RMS}}^2 + b_{1_{RMS}}^2$$

$$= \frac{8A^2}{\pi^2}$$

So THD

$$= \sqrt{\frac{A^2 - \frac{8A^2}{\pi^2}}{8A^2/\pi^2}} = \sqrt{\frac{\pi^2}{8} - 1} = 0.48$$

or 48%