Derivation of relative size of phase-phase voltage vs phase-neutral voltage for 3-phase WYE power.

In class I stated w/o proof that $V_{LL_{RMS}} = \sqrt{3}\, V_{LN_{RMS}}$

this is a proof.

Consider

$$\phi_A = V_0 \cos \omega t$$

$$\phi_B = V_0 \cos(\omega t - 120°)$$

so that

$$V_{LL} = \phi_A - \phi_B = V_0 (\cos \omega t - \cos(\omega t - 120°)).$$

using the trig identity $\cos(A-B) = \cos A \cos B + \sin A \sin B$

$$V_{LL} = V_0 (\cos \omega t - [\cos \omega t \cos 120°) + \sin \omega t \sin 120°])$$

$$= V_0 (\cos \omega t + \tfrac{1}{2}\cos \omega t - \tfrac{\sqrt{3}}{2} \sin \omega t)$$

$$= V_0 \left(\tfrac{3}{2}\cos \omega t - \tfrac{\sqrt{3}}{2} \sin \omega t\right)$$

$$(V_{LL})_{RMS} = \frac{V_0}{2}\sqrt{\frac{1}{T}\int_0^T (3\cos \omega t - \sqrt{3}\sin \omega t)^2\, dt}$$

$$= \frac{V_0}{2}\sqrt{\frac{1}{T}\int_0^T 9\cos^2 \omega t - 2\sqrt{3}\cdot 3 \cos \omega t \sin \omega t + 3\sin^2 \omega t\, dt}$$

using $\frac{1}{T}\int_0^T \cos^2 \omega t = \frac{1}{2} = \frac{1}{T}\int_0^T \sin^2 \omega t$ and $\frac{1}{T}\int_0^T \cos \omega t \sin \omega t\, dt = 0$

$$(V_{LL})_{RMS} = \frac{V_0}{2}\sqrt{\frac{9}{2} + \frac{3}{2}} = \frac{V_0}{2}\sqrt{6} = \frac{\sqrt{3}}{\sqrt{2}} V_0$$

and since

$\phi_{A_{RMS}} = \frac{V_0}{\sqrt{2}} = V_{LN_{RMS}}$ we have $V_{LL_{RMS}} = \sqrt{3}\, V_{LN_{RMS}}$