

~~Summary~~

Last time

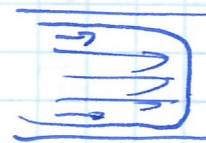
fluid flow dichotomies

- internal vs external flow
- passive vs active convection
(demo in fish tank)
- turbulent vs laminar flow

$$\text{Reynolds \# } R = \frac{\rho v L}{\mu}$$

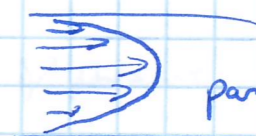
Specifies for pipe flow

turbulent
vel. profile



relatively
flat
vel. profile

laminar
vel. profile



parabolic $v(r)$ profile

integrated over r, θ yields

$$\dot{V} = \frac{\pi D^4 \Delta p}{128 \mu l} \text{ pressure gradient}$$

deceptive calculation from last time

find transition velocity for turbulent flow in pipe

$$R = 2000, L = D = \frac{1}{2} \text{''}, \rho = 1000 \text{ kg/m}^3, \mu = 1.12 \times 10^{-3} \frac{\text{Ns}}{\text{m}^2}$$

$$\rightarrow v = 0.2 \text{ m/s}$$

reason deceptive: find \dot{V}

Assume simplest (overly simple w/ error over estimating flow model for \dot{V} i.e. slug of water \rightarrow w/ $D = \frac{1}{2} \text{''}$, $v = 0.2 \text{ m/s}$

then

$$\dot{V} = \text{area of pipe} \times V \text{ of center}$$

$$= \pi \left(\frac{D}{2}\right)^2 \times V$$

$$= \pi \left(\frac{1.27 \text{ cm}}{2}\right)^2 20 \text{ cm/s} = \frac{25 \text{ cm}^3}{\text{s}}$$

How fast is this?

~~Attempts~~

Sec = 1 tsp

a typical person turns on a faucet to 1 - 1.5 gal

$$\frac{63 \text{ cm}^3}{\text{s}} - \frac{95 \text{ cm}^3}{\text{s}}$$

typical shower head is $\frac{2 \text{ gal}}{\text{min}} \rightarrow \frac{125 \text{ cm}^3}{\text{s}}$

• Full reasonable flow will be turbulent

To deal w/ the loss of pressure due to turbulence

in engineering texts this is called a head loss
as in pressure head

recall Bernoulli's eq.

for
laminar
flow $\rightarrow P_1 + \frac{\rho V_1^2}{2} + \rho g h_1 = P_2 + \frac{\rho V_2^2}{2} + \rho g h_2$

for
turbulent
flow $\frac{P_1 + \frac{\rho V_1^2}{2} + \rho g h_1}{\text{up stream location}} = \frac{P_2 + \frac{\rho V_2^2}{2} + \rho g h_2}{\text{down stream location}} + \rho g h_L$
↑ head loss

head loss has 2 sources

- the pipe itself (major head loss) h_{major}
- fittings between pipe segments (minor head loss) h_{minor}
fittings include elbows, Ts, valves, unions etc.

$$h_L = h_{major} + h_{minor}$$

↑
we will only consider this

$$h_{major} = f \frac{L v^2}{D 2g}$$

(so that $\rho g h_{major} =$
 $f \frac{\rho L v^2}{2D}$ or $f \frac{\rho L v \mu}{2D^2}$)

L = length of pipe

v = speed of fluid flow

D = diameter of pipe

g = accel. of gravity

f = friction factor ← set by empirical observations

f is found to depend on Reynolds # Re
and the roughness of the pipe.

for smooth pipe (glass, plastic tubing)

$$f = \frac{0.316}{Re^{1/4}}$$

for other pipes consult table for roughness f ,
calc. relative roughness e/D and need Re from
Moody Chart

Example pump water down & up a 300ft ground source heat pump well through 3/4" galvanized pipe at a rate of 12 gal/min. Find pressure needed to push water through this.

Conversion

$$1 \text{ gal} \approx 3785 \text{ cm}^3$$

$$\text{so } \frac{12 \text{ gal}}{\text{min}} = \frac{760 \text{ cm}^3}{\text{s}}$$

$$3/4" = 1.905 \text{ cm}$$

$$\text{area} \cdot v = \dot{V}$$

$$v = \frac{\dot{V}}{\text{area}} = \frac{760 \text{ cm}^3/\text{s}}{2.85 \text{ cm}^2} = 267 \frac{\text{cm}}{\text{s}} = 2.67 \frac{\text{m}}{\text{s}}$$

$$Re = \frac{\rho v D}{\mu} = \frac{1000 \frac{\text{kg}}{\text{m}^3} \cdot 2.67 \frac{\text{m}}{\text{s}} \cdot 0.019 \text{ m}}{1.12 \times 10^{-3} \frac{\text{Ns}}{\text{m}^2}} = 45,300$$

$\frac{E}{D}$ for galvanized pipe is

$$\frac{E}{D} = \frac{0.015 \text{ cm}}{1.905 \text{ cm}} = 0.008$$

$$\Rightarrow f = 0.036$$

$$L = 600 \text{ ft} \rightarrow 185 \text{ m}$$

$$h_f = \frac{0.036 \cdot 185 \text{ m} \cdot (2.67 \frac{\text{m}}{\text{s}})^2}{1.9 \times 10^{-2} \text{ m} \cdot 2 \cdot 9.8 \text{ m/s}^2} = 127 \text{ m}$$

$$P_t = P_c + \rho g h \Rightarrow P_t = \rho g h = 1000 \frac{\text{kg}}{\text{m}^3} \cdot 9.8 \frac{\text{m}}{\text{s}^2} \cdot 127 \text{ m}$$

$$\begin{aligned} P_i &= 1246 \text{ kPa} \\ &= 12.3 \text{ atmospheres} \\ &= 1815 \text{ psi} \end{aligned}$$