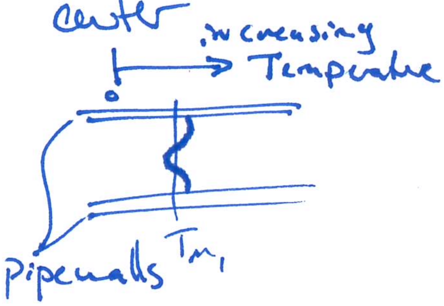
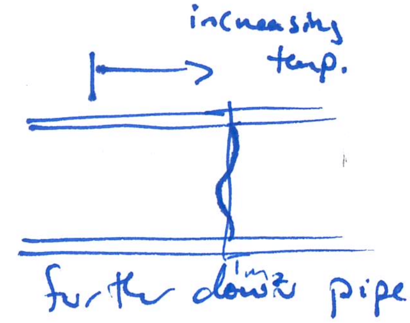
Heat flow into a pipe

when heat flows into a pipe it causes the fluid inside the pipe to warm leading to a temperature profile (on-average) as you move from the edge of the pipe to it's center



as the fluid moves along the temp profile tends to flatten



At each distance down the pipe you can define a mean temperature across the diameter of the pipe T_m

In general the relative strength of ~~conduction~~ convection vs conduction for a given Fluid by the Prandtl number, Pr

$$Pr \equiv \frac{C_p \mu}{K_T}$$

$C_p \mu$ ← relative tendency for convection (momentum diffusivity)

K_T ← relative tendency for conduction (thermal diffusivity)

for gases Pr is typically less than $\underline{1}$ (≈ 0.7 for air over a wide range of temps)

for liquids Pr is > 1 and often varies w/ temp. and by substance

water

$T = 0°C$	$Pr = 13$
$T = 27°C$	$Pr = 5.83$
$T = 100°C$	$Pr = 1.76$

$T = 27°C$

Engine oil	$Pr = 6400$
Ethylene glycol	$Pr = 151$
Water	$Pr = 5.83$

Modes of Heat flow into pipe

(1) Constant rate of Heat flux (units of W/m^2) $\dot{q}''_s$ through surface

this is typical of situations where a pipe is being "heated" w/ a fuel being burned, an electric coil, or a thermal solar collector.

with the surface heat flux constant

$$\dot{q} = \dot{q}'' \times \text{surface area}.$$

and the mean temperature of the liquid as it exits the pipe (of length L & circumference P (perimeter)) is

$$T_{m_o} = T_{m_i} + \frac{\dot{q}''_s \, PL}{\dot{m}\, C_p}$$

T_{m_o}: temp going out

T_{m_i}: temp going in

Note that w/ each additional meter of pipe the change in T_{m_o} is constant.

(2) Constant surface temperature T_s of pipe

this is typical of a pipe acting like a heat exchanger in a car radiator (air keeps pipe @ const. temp), a Hot water heater w/ pipe bathed in steam (think steam districts in NYC), or a well bore for a ground source heat pump (earth surrounding pipe stays at a constant ~50°F all year long).

T_s = constant Cont.

this senario is more complex to analyze. Before one can find q, one must determine T_{m_o}.

One can derive an equation for T_{m_o}

$$\frac{\Delta T_o}{\Delta T_i} = e^{-\frac{PL}{\dot{m} C_p}\bar{h}}$$

P = perimeter of pipe
L = length of pipe
$\dot{m}$ = mass flow rate (through pipe)
c_p = specific heat
$\bar{h}$ = average convection coefficient
$\Delta T_o = T_s - T_{m_o}$
$\Delta T_i = T_s - T_{m_i}$

The value of $\bar{h}$ is averaged in a temperature sense, along the length of the pipe. So solving for T_{m_o} is actually an iterative process where you must guess a value for T_{m_o} (since ~~h~~ h is temprature dependent), work out a value for $\bar{h}$ and then calculate T_{m_o}, with this better estimate $\bar{h}$ can be improved & a new, better estimate for T_{m_o} can be found.

Next: Calculating $\bar{h}$

Calculating h for heat transfer to / From a pipe

h is found by means of the Nusselt number Nu

$$Nu = \frac{hD}{k_T}$$

for laminar flow through a pipe w/a given cross-sectional shape and heat transfer method ($\dot{q}''$ = const or T_s = const) the Nusselt number is constant

Circular pipe cross-section

$Nu = 4.36$ when $\dot{q}''$ = const.

$Nu = 3.66$ when T_s = const.

However, for turbulent flow (most situations of actual interest) Nu depends on both the Reynolds # & Prandtl #.

Experimental observations yield the following fit

$$Nu = 0.023 \, Re^{4/5} Pr^{n} \quad \text{where } n = \begin{cases} 0.4 & \text{for heating} \\ 0.3 & \text{for cooling} \end{cases}$$

what ever value is found for Nu

$$h = \frac{Nu \, k_T}{D}$$

and h depends on T by way of Re (μ is temp dependent)
Pr
and
k_T

finding $\dot{q}$

you can either calculate $\dot{q}$ by comparing the thermal energy in the fluid at the beginning vs end on a per mass basis

$$u_o - u_i = c_p T_{m_o} - c_p T_{m_i}$$

and in units of W (or kW)

$$\dot{q} = \dot{m}(u_o - u_i) = \dot{m} c_p (T_{m_o} - T_{m_i})$$

an attempt to relate this heat flow back to the surface area the heat passes through is given by

$$\dot{q} = \bar{h} P L \Delta T_{lm}$$

where

$$\Delta T_{lm} = \frac{\Delta T_o - \Delta T_i}{\ln(\Delta T_o / \Delta T_i)}$$

Note that unlike the $\dot{q}'' =$ const. case there are diminishing returns for longer & longer pipes. Varying L gives