

MCS/PHY 186 Homework.

Pick two of the following four questions.

1. Population dynamics:

The number of creatures living in an enclosure can be described by the following differential equation:

$$\frac{dN}{dt} = a N - b N^2$$

Where  $N$  is the number of creatures alive,  $a$  measures how quickly they reproduce and  $b$  measures how much they compete with each other (rabbit on rabbit mortality rate). Start with two rabbits in a cage (large cage), use  $a = \frac{6}{yr}$  (female snowshoe rabbits have litters of 3-4 2-5 times a year),  $b = \frac{1}{500yr}$  and a time step of  $\Delta T = 3$  months to determine:

- i. The number of animal alive at the end of 10yrs
- ii. How long it takes the population to reach 3000 rabbits.

Be sure to be careful finding the result to both of these question as one does not immediately follow from the other.

Extra credit: Graph the population of rabbits over 10 yrs.

2. Loan interest:

Banks make money from you by, among other things, giving out loans. The following differential equation describes the principle,  $P$ , that you owe while you are paying back your loan given an interest rate,  $I$ , and a repayment amount,  $RP$

$$\frac{dP}{dt} = I P - RP$$

Using an initial loan amount of \$1000, and a credit card interest rate of 15%/year, a monthly repayment of \$20, and  $\Delta T = 1$  month to find:

- i. The time it takes to repay the entire loan
- ii. The amount of money the bank makes/you lose in this transaction.

3. Population dynamics extension:

The situation for the bunnies becomes more dire if they reproduce more quickly. Start with the same two rabbits in a cage (large cage), and increase the reproduction rate to  $a = \frac{9}{yr}$ . Keep  $b = \frac{1}{500yr}$  and a time step of  $\Delta T = 3$  months and find the number of rabbits: Examine the number of living animal between 7 and 10yrs. What do you notice and how would you interpret your results?

4. Integration:

Find the area under  $f(x)$ :

$$f(x) = 1/x$$

With the limits of integration  $10 \leq x \leq 100$  in two different ways:

- i. Use numerical integration with a step size  $\Delta x = 0.01$ .
- ii. Use standard calculus techniques to evaluate the integral. (If you are unsure how see me as calculus is not a prerequisite for this course.)

Report the size of the difference between these two results.