

MCS/PHY 186 Homework for chapter 7.

1. Downey 7.1

Pick three of the following five questions.

2. Speeding up on a bicycle:

The following equation relates the speed, v , of a person on a bicycle

$$\frac{dv}{dt} = \frac{P}{m v}$$

With a combine mass of m , pedaling with power P , at time t . Find how long it takes a person to reach 6m/s if $m=70\text{kg}$, $P=100\text{W}$ if they were initially moving at 1m/s.

3. Population dynamics:

The number of creatures living in an enclosure can be described by the following differential equation:

$$\frac{dN}{dt} = a N - b N^2$$

Where N is the number of creatures alive, a measures how quickly they reproduce and b measures how much they compete with each other (rabbit on rabbit mortality rate). Start with two rabbits in a cage (large cage), use $a = \frac{6}{\text{yr}}$ (female snowshoe rabbits have litters of 3-4 2-5 times a year), $b = \frac{1}{500\text{yr}}$ and a time step of $\Delta t=3$ months to determine:

- i. The number of animal alive at the end of 10yrs
- ii. How long it takes the population to reach 3000 rabbits.

If you are surprised by the results of ii in light of i you might consider looking at the population at each time-step of part i.

4. Loan interest:

Banks make money from you by, among other things, giving out loans. The following differential equation describes the principle, P , that you owe while you are paying back your loan given an interest rate, I , and a repayment amount, RP

$$\frac{dP}{dt} = I P - RP$$

Using an initial loan amount of \$1000, and interest rate of 15%/year, a monthly repayment of \$20, and $\Delta t=1$ month to find:

- i. The time it takes to repay the entire loan
- ii. The amount of money the bank makes/you lose in this transaction.

5. Bicycle extension:

Question 2 ignored the effect of the force of drag from the air. Since the wind will slow the biker down we should subtract F_{air}/m from the right hand side

$$\frac{dv}{dt} = \frac{P}{m v} - \frac{F_{\text{air}}}{m}$$

where

$$F_{air} = \frac{\rho Area v^2}{2}$$

and $\rho = 1.25kg/m^3$ is the density of air and $Area = 0.33m^2$ is the cross-section of the person and bike. Keeping the same parameters and initial conditions (starting values for variables) as in question 2 to find how long it takes the biker to reach a speed of 6m/s.

6. Population dynamics extended:

The situation for the bunnies becomes more dire if they reproduce more quickly. Start with the same two rabbits in a cage (large cage), and increase the reproduction rate to $a = \frac{9}{yr}$. Keep $b = \frac{1}{500yr}$ and a time step of $\Delta T = 3$ months and find the number of rabbits: Examine the number of living animal between 7 and 10yrs. What do you notice and how would you interpret your results?