Computational Physics – HW #1

1. In a water purification process, one-nth of the impurity is removed in the first stage. In each succeeding stage, the amount of impurity removed is one-nth of that removed in the preceding stage. Show that if $n = 2$ the water can be made as pure as you like, but that if n=3, at least one-half of the impurity will remain no matter how many stages are used.

2. Find the limit of the sequence specified by

   $$\frac{(n+1)^2}{\sqrt{3+5n^2+4n^4}} \tag{1}$$

   in the limit $n \to \infty$.

3. Find the first non-zero term in the Taylor series of

   $$\frac{1}{\sqrt{1+x^4}} - \cos(x^2) \tag{2}$$

4. Find the limit

   $$lim_{x\to 0} \frac{1-cos(x)}{x^2} \tag{3}$$

5. Problem 27 from section 1.15.

6. It was realized long ago that the universe cannot be static, infinitely large, and have a uniform distribution of stars throughout space. This is because such a situiation would lead to a night sky that is bright because it looks competely covered with stars.

   To see how this follows, assume that the universe is static and the stars are uniformly distributed throughout space. Divide all space into thin spherical shells of constant thickness centered on the earth. Since all of the stars in a given shell are the same distace from the earth, the combine stars in any single shell by subtend a solid angle of ω_o. *Allowing for the blocking out of distant stars by nearer stars*, show that the total net solid angle subtended by all stars, shells extending out to infinity, is exactly 4π, the solid angle of a complete sphere (Which would imply that the night sky should be ablaze with light under these circumstances).