Physics 152 Practice exam #1

1) $$\vec{F} = \Sigma \vec{F} = \frac{kQq}{a^2}\hat{j} - \frac{kQq}{a^2}\hat{i} + \frac{2QkQ}{a^2+a^2}\left(\frac{a}{\sqrt{2a^2}}\hat{i} - \frac{a}{\sqrt{2a^2}}\hat{j}\right)$$

$$= \left(\frac{kQq}{a^2} - \frac{kQ^2}{\sqrt{2}\,a^2}\right)\hat{i} + \left(\frac{kQq}{a^2} + \frac{kQ^2}{\sqrt{2}\,a^2}\right)\hat{j} = 0$$

so $$\frac{\cancel{k}Qq}{\cancel{a^2}} = \frac{\cancel{k}Q^2}{\sqrt{2}\cancel{a^2}}$$

$$q = \frac{Q}{\sqrt{2}}$$

2) The electric field is useful because it can be used to find the force $\vec{F} = q\vec{E}$ that a charged object will experience if it is placed in the field.

3) $$\vec{E} = \int \frac{k\,dq}{r^2}\hat{r}$$

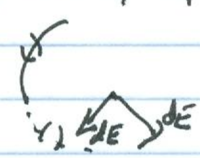
on top 1/4 of circle $\lambda = \frac{2q}{\pi}$

on bottom $\lambda = \frac{-2q}{\pi}$

$$\vec{E} = \int_{\pi/2}^{3\pi/2} \frac{k\lambda\,d\theta}{r^2}\hat{r}$$ and by inspection the net $\vec{E}$-field will point in the ~~+~~y direction (look at the force from ~~pairs~~ of pieces of charge

from

$$E_y = \frac{k\lambda}{r^2}\int_{\pi/2}^{\pi} -\sin\theta\,d\theta + \frac{k|\lambda|}{r^2}\int_{\pi}^{3\pi/2} \sin\theta$$

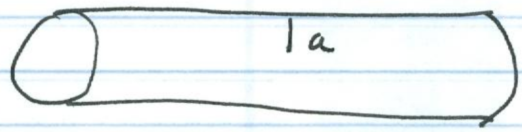


$$= -\left(\frac{k\lambda}{r^2}(-\cos\theta)\Big|_{\pi/2}^{\pi} + \frac{k|\lambda|}{r^2}(-\cos\theta)\Big|_{3\pi/2}^{\pi}\right)$$

$$= -\frac{k\lambda}{r^2}\left(1 - 0 + 1 - 0\right) = \frac{-2k\lambda}{r^2}$$

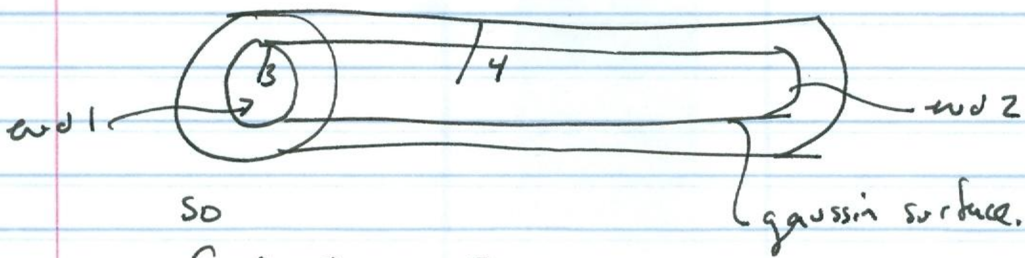
$$\vec{E} = -2\frac{k\lambda}{r^2}\hat{j}$$

4



$a = 4\,cm$

a at 3cm the gaussin surface is inside the cylinder



so

$$\oint \vec{E}\cdot d\vec{A} = \frac{Q_{enc}}{\epsilon_0}$$

becomes

$$\int_{side} \vec{E}\cdot d\vec{A} + \int_{end\ 1} \vec{E}\cdot d\vec{A} + \int_{end\ 2} \vec{E}\cdot d\vec{A} = \frac{\int \rho\, dV}{\epsilon_0}$$

$\vec{E}$ is strictly out from center of cylinder so the two end integrals have $\vec{E}\cdot d\vec{A} = 0$

$$\int_{side} \vec{E}\cdot d\vec{A} = \int_{side} \|\vec{E}\|\, dA = \|\vec{E}\| \int_{side} dA = \|\vec{E}\|\, 2\pi r l$$

$$\int \rho\, dV = \int_{\substack{\text{inside}\\ \text{gaussin surface}}} A r^2 (r\, dr\, d\theta\, dz) = l\, 2\pi A \int r^3 dr$$

$$= \frac{l\, 2\pi A}{4} r^4 \Big|_0^{3cm}$$

$$= \frac{l \pi A}{2} (0.03\,m)^4$$

or

$$\cancel{2\pi l}\, \|\vec{E}\| = \frac{\cancel{l \pi} A\, r^{\cancel{4}\,3}}{\cancel{2}\, 4\epsilon_0}$$

$$\|\vec{E}\| = \frac{A r^3}{4\epsilon_0} = \frac{2.5\,\frac{\mu C}{m^5}\,10^{-6}\,(0.03)^3 m^3}{4\times 8.85\times 10^{-12}}$$

$\|\vec{E}\| = 1.9 \frac{N}{C}$

b) @ 5cm the gaussian surface is outside the cylinder which charges

$$\frac{Q}{\epsilon_0} = \int_0^{4cm}\int_0^{2\pi}\int_0^{l} \rho r\, dr\, d\theta\, dz$$

$$= \frac{l\, 2\pi A}{4\epsilon_0} (0.04)^4$$

and

$$\|\vec{E}\|\, 2\pi\, (5cm)\, l = \frac{l\, 2\pi A\, (4cm)^4}{4\epsilon_0}$$

$$\|\vec{E}\| = \frac{A}{5\epsilon_0} (4cm)^3 = 3.6 \frac{N}{C}$$

5) Since the electric force is conservative we need only compare $V(\infty)$ to $V(0)$ to find $q\Delta V = -W$

$V(\infty) = 0$ by def.

$$V(0) = \frac{k4q}{2d} + \frac{k(-2q)}{d} = \frac{2kq}{d} - \frac{2kq}{d} = 0.$$

so $q\Delta V = 0 \Rightarrow W = 0.$