Former Exam #4

1) $\vec{\alpha} = \dfrac{\sum_i \vec{\tau}}{I}$ is the rotational version of Newton's 2^{nd} law

$\vec{\alpha}$ = angular acceleration $= \dfrac{d^2\vec{\theta}}{dt^2}$

I = moment of inertia $= \int_{\text{whole body}} r^2\,dm$

$\vec{\tau} = \vec{r} \times \vec{F}$ is the torque.

Conservation of momentum

$\vec{L}_i = \vec{L}_f$ where $\vec{L}$ is the angular momentum

$\vec{L} = \vec{r} \times \vec{p}$ or $I\vec{\omega}$

with $\vec{\omega} = \dfrac{d\vec{\theta}}{dt}$

Conservation of Energy

$\Delta KE = \sum_i W$ rotational $KE = \frac{1}{2} I\omega^2$

and rotational work is

$W = \int \vec{\tau} \cdot d\vec{\theta}$

2) $\alpha = 3\ \text{rad}/s^2$ $\omega_0 = 0.3\ \frac{\text{rad}}{s}$

a) for constant α we have

$\Delta\theta = \omega_0 t + \frac{1}{2}\alpha t^2$

so

$\Delta\theta = 0.3\frac{\text{rad}}{s}(10s) + \frac{1}{2}\, 3\frac{\text{rad}}{s^2}(10s)^2$

$= 3\text{rad} + 150\text{rad} = 153\text{ rad}.$

b) The torque causing this accel is

$\|\vec{\tau}\| = I\|\vec{\alpha}\|$

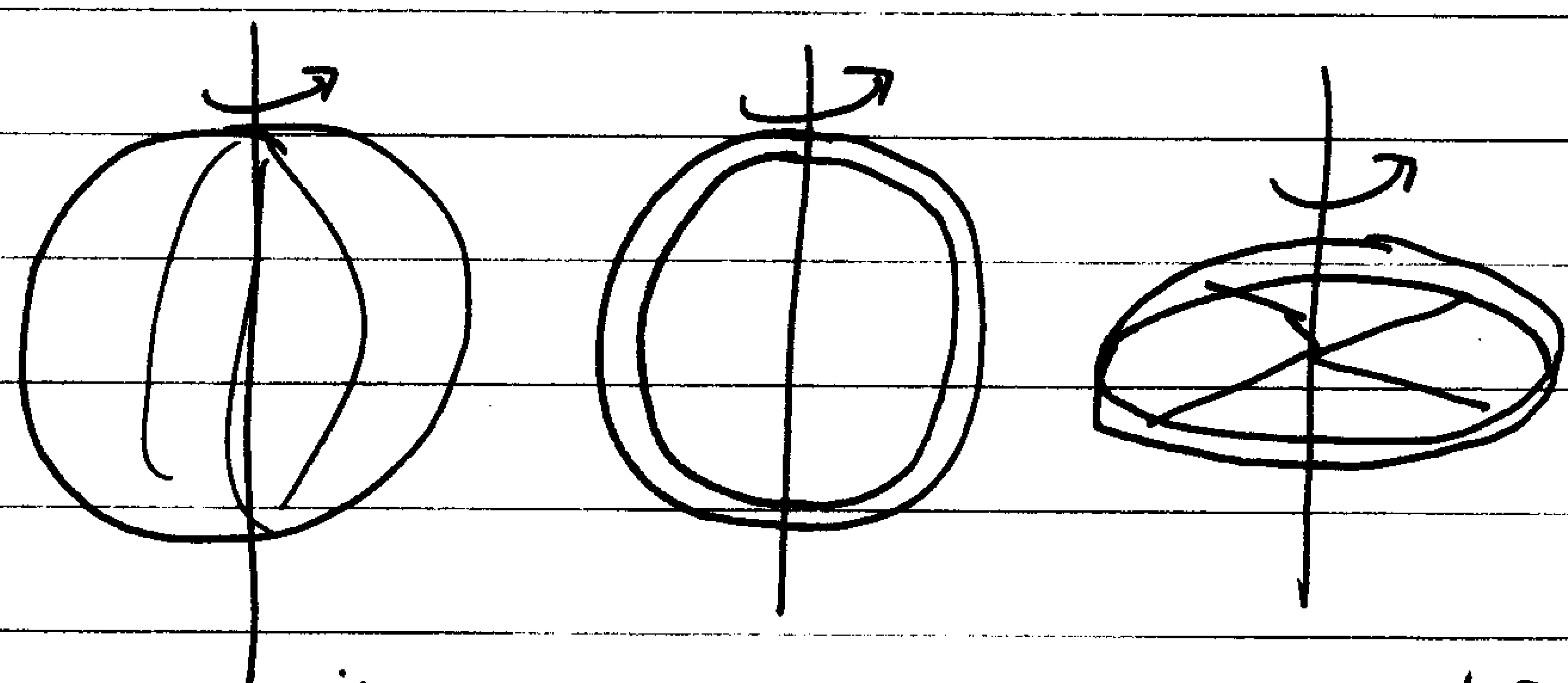
$= \left[1\text{kg}\,(0.5m)^2\right] 3.0\ \text{rad}/s^2$

$= \frac{3}{4}\ Nm$

c) $W = \tau \Delta\theta = \frac{3}{4} \text{Nm} (153 \text{ rad}) = 114.75 \text{ J}$

$\sim 115 \text{ J}$

3)



for a solid sphere much matter is close to the axis of rotation

least I

less matter is close to the axis compared to the ball

middle

all of the matter is at the maximal distance from the axis

most I

4) $\vec{L}$ is conserved and

$$\vec{L}_i = I\vec{\omega}_{1i} + I\vec{\omega}_{2i}$$

$$\vec{L}_f = I\vec{\omega}_{1f} + I\vec{\omega}_{2f}$$

$$\vec{L}_i = \vec{L}_f$$

$$\cancel{I}\vec{\omega}_{1i} + \cancel{I}\vec{\omega}_{2i} = \cancel{I}\vec{\omega}_{1f} + \cancel{I}\vec{\omega}_{2f}$$

$$\vec{\omega}_{1i} + \vec{\omega}_{2i} = \vec{\omega}_{1f} + \vec{\omega}_{2f}$$

$$+35\pi \tfrac{\text{rad}}{s} - 25\pi \tfrac{\text{rad}}{s} = 30\pi \tfrac{\text{rad}}{s} + \omega_{2f}$$

$$10\pi \tfrac{\text{rad}}{s} = 30\pi \tfrac{\text{rad}}{s} + \omega_{2f}$$

$$\boxed{\omega_{2f} = -20\pi \text{ rad/s}}$$

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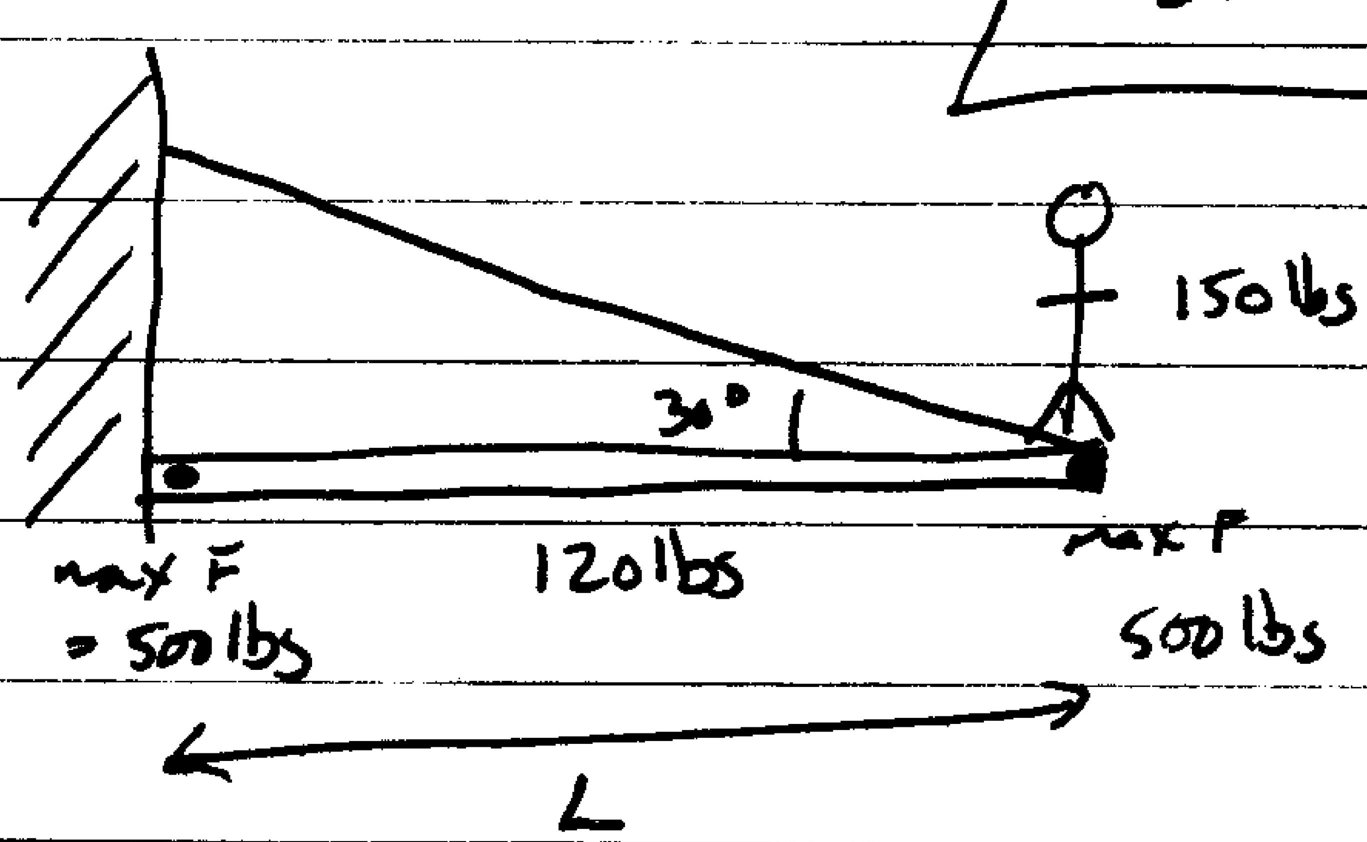
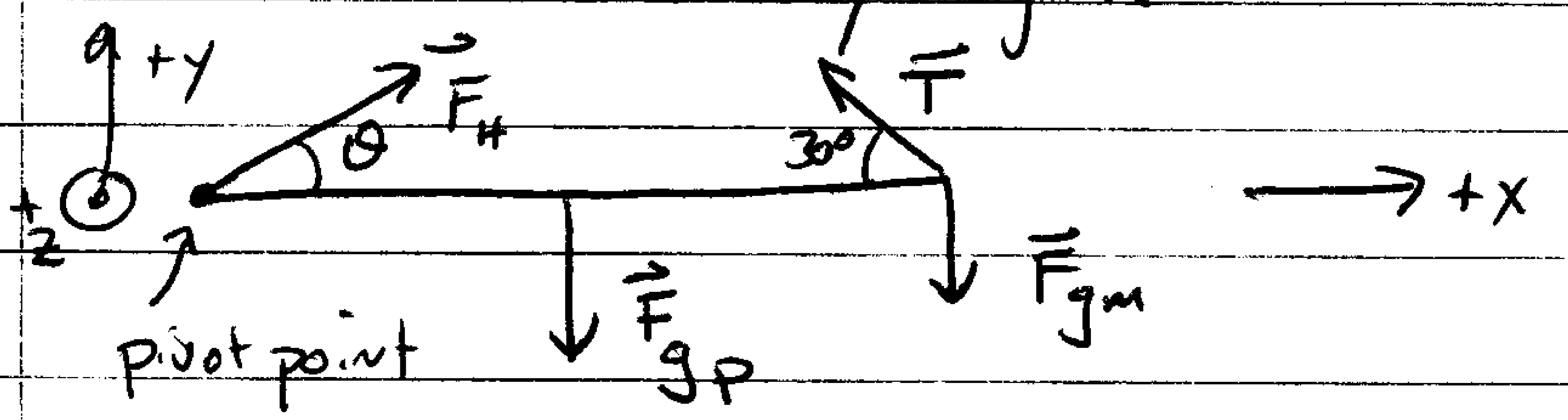


Assume the system is in static equilibrium then find the required forces and check them against the max values.

Extended free body diagram



$$\sum F_x = \|\vec F_H\|\cos\theta - \|\vec T\|\cos 30^\circ = 0$$

$$\sum F_y = \|\vec F_H\|\sin\theta + \|\vec T\|\sin 30^\circ - \|\vec F_{gp}\| - \|\vec F_{gm}\| = 0$$

$$\sum \tau_z = -\|\vec F_{gp}\|\frac{L}{2} - \|\vec F_{gm}\| L + \|\vec T\| L\sin 150^\circ = 0$$

(center of mass is at midpoint of pole By symmetry)

so from $\sum \tau_z = 0$ we have

$$\|\vec T\| \cancel{L\sin 150^\circ} = \frac{\|\vec F_{gp}\|\frac{\cancel{L}}{2} + \|\vec F_{gm}\|\cancel{L}}{\sin 150^\circ}$$

$$\|\vec T\| = \frac{\frac{120\text{ lbs}}{2} + 150\text{ lbs}}{\frac{1}{2}} = 420\text{ lbs}$$

plugging values into $\sum F_x = 0$ gives $\|\vec F_H\|\cos\theta = 364\text{ lbs}$ and into $\sum F_y = 0$ gives $\|\vec F_H\|\sin\theta = 88\text{ lbs}$

or $F_{H_x} = 364\text{ lbs}$ $F_{Hy} = 88\text{ lbs}$

so $F_H = \sqrt{F_{Hx}^2 + F_{Hy}^2} = 374.5\text{ lbs}$

So it is possible to do this maneuver