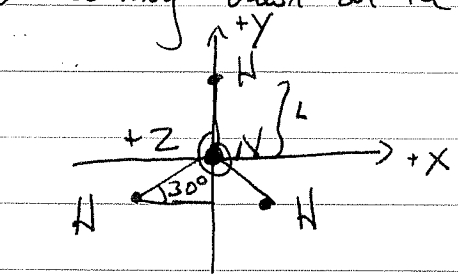
Former Exam 3.

1) looking down on the molecule



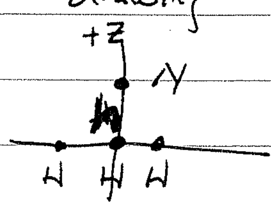
By ~~[illegible]~~ symmetry the x-coord. of the center of mass is @ $x=0$

The y-coord. is given by

$$y_{com} = \frac{m_H L + m_H(-L\sin 30°) + m_H(-L\sin 30°) + 0m_N}{m_H + m_H + m_H + m_N}$$

$$= \frac{m_H(L - 2L\sin 30°)}{3m_H + m_N} = \frac{m_H(0)}{3m_H + m_N} = 0$$

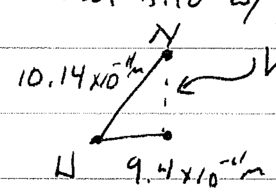
The z-component can be found with the aid of another drawing



$$Z_{com} = \frac{m_N h + 0m_H + 0m_H + 0m_H}{m_N + 3m_H}$$

$$= \frac{m_N h}{m_N + 3m_H} = \frac{\frac{m_N}{m_H} h}{\frac{m_N}{m_H} + 3}$$

So we need h which we can find w/ the pythagorean theorem

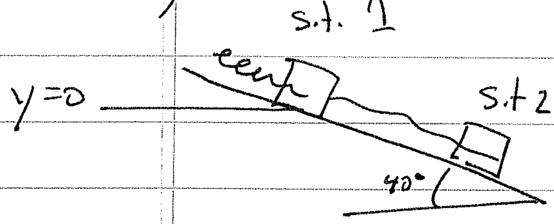


$$h = \sqrt{(10.14)^2 - (9.4)^2} \times 10^{-11}\,m$$

$$= 3.8 \times 10^{-11}\,m$$

$$Z_{com} = \frac{13.9 \quad 3.8\times10^{-11}\,m}{13.9 + 3} = 3.13\times10^{-11}\,m$$

2)

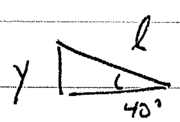


$$E = KE + PE = KE + GPE + SPE$$

$$E_1 = KE_1 + \cancelto{0 \text{ by choice of } y \text{ axis.}}{GPE_1} + \cancelto{0 \text{ (not stretched)}}{SPE_1}$$

$$= \tfrac{1}{2} m v_1^2$$

$$E_2 = \cancelto{0 \text{ (turning point)}}{KE_2} + GPE_2 + SPE_2$$



$y = l \sin 40°$

$$= mg\Delta y + \tfrac{1}{2} k l^2$$

$$= mg(-l \sin 40°) + \tfrac{1}{2} k l^2$$

so by cons. of Energy

$$E_1 = E_2$$

$$\tfrac{1}{2} m v_1^2 = -mg l \sin 40° + \tfrac{1}{2} k l^2$$

$$\tfrac{1}{2} k l^2 - mg \sin 40° \, l - \tfrac{1}{2} m v_1^2 = 0$$

$$l = \frac{mg \sin 40° \pm \sqrt{m^2 g^2 \sin^2 40° + k m v_1^2}}{k}$$

$$= \frac{mg \sin 40°}{k}\left(1 \pm \sqrt{1 + \frac{k v^2}{m g^2 \sin^2 40}}\right)$$

$$= 0.94\,m\,(1 \pm 1.17)$$

$$= 2.05\,m$$

+ root since assume $l > 0$.

b $\Delta E = W_{ext} = W_f$

$\frac{1}{2} k l^2 - mg l \sin 40° - \frac{1}{2} m v_1^2 = W_{ext}$

Now $l = 1.5\,m \Rightarrow$

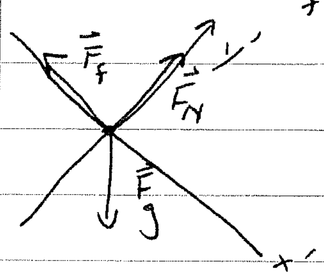
$\frac{1}{2}\, 20\frac{N}{m}\,(1.5m)^2 - 3kg\; 9.8\,m/s^2\; 1.5m\, \sin 40° - \frac{1}{2}\, 3kg\,(1.5\,m/s)^2 = W_{ext}$

$22.5\,J - 28.3\,J - 3.38\,J = -9.18\,J$

c $W = \vec{F} \cdot \Delta\vec{r} = -F_f\, l$

so

$F_f = \dfrac{9.18\,J}{1.5m} = 6.12\,N$



$\sum F_{y'} = 0 \quad = \vec{F}_{N_{y'}} - mg\cos 40° = 0$

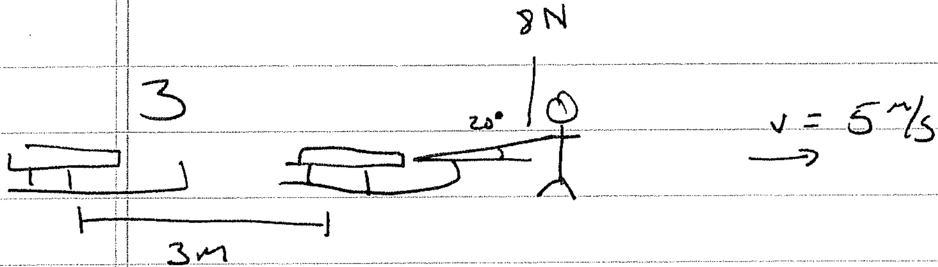
$F_{N_{y'}} = mg\cos 40° = \|\vec{F}_N\|$

so

$\mu_k \|\vec{F}_N\| = 6.12\,N$

$\mu_k = \dfrac{6.12\,N}{mg\cos 40°} = \dfrac{6.12\,N}{3kg\; 9.8\,m/s^2\,\cos 40°} = 0.27$

3



a) $W_{c\to s}$ $\quad \|\vec{T}\| = 8N$, $\|\vec{\Delta x}\| = 3m$ $\qquad W = 8N \; 3m \; \cos 20°$

$= 22.6 \text{ J}$

b) $W_{g\to s} = \underbrace{\vec{F}_g \cdot \vec{\Delta x}}_{\perp \text{ to each other}} = 0 \text{J}$

c) $W_{f\to s}$

$\Delta KE = \sum W$

$\cancel{\Delta KE}^{0} = W_{c\to s} + \cancel{W_{g\to s}}^{0} + \cancel{W_{N\to s}}^{0} + W_{f\to s}$

$W_{f\to s} = -W_{c\to s} = -22.6 \text{ J}$

d) $W_{N\to s} = 0 \quad \vec{F}_N$ & $\vec{\Delta x}$ are $\perp$.

e) $\Delta KE = \sum W = 0$, same KE at start as end.

4) The explosion of the gun powder adds a known amount of energy to the system, namely

$$\Delta E = \tfrac{1}{2}(40 \text{ lbs})(400 \text{ m/s})^2$$

$$= \tfrac{1}{2}(18.2 \text{ kg})(400 \text{ m/s})^2 = 1.45 \times 10^6 \text{ J}$$

In the field there will be no external force so momentum will be conserved and

$$\vec{P}_i = 0 = \vec{P}_f = m_c V_c + m_b V_b$$

$$\Rightarrow V_c = -\frac{m_b}{m_c} V_b$$

also

$$\Delta E = \tfrac{1}{2} m_c V_c^2 + \tfrac{1}{2} m_b V_b^2 = \tfrac{1}{2}(40 \text{ lbs})(400 \text{ m/s})^2$$

$$\frac{70}{4} V_c^2 + V_b^2 = (400 \text{ m/s})^2$$

Combining Energy & momentum

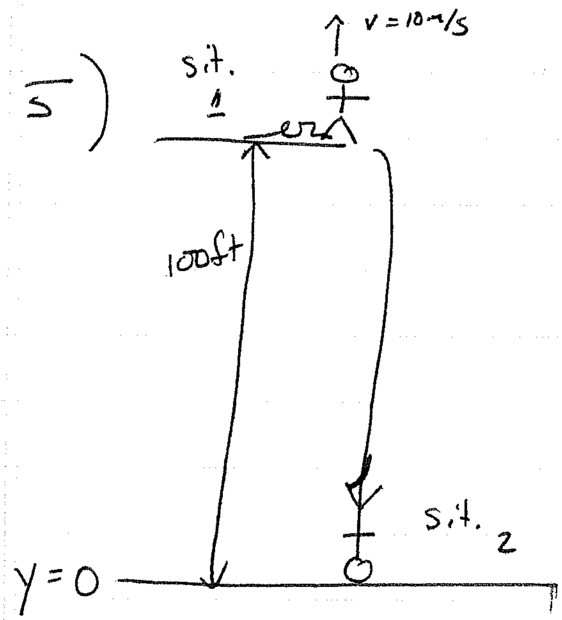
$$\frac{70}{4}\left(\frac{m_b}{m_c} V_b\right)^2 + V_b^2 = (400\,m/s)^2$$

$$\left(\frac{70}{4}\left(\frac{40}{700}\right)^2 + 1\right) V_b^2 = (400\,m/s)^2$$

$$V_b = \frac{400\,m/s}{\sqrt{1 + \frac{70}{4}\left(\frac{4}{70}\right)^2}} = \frac{400\,m/s}{\sqrt{1+\frac{4}{70}}}$$

$$= 389\,m/s$$

5)



$$E_1 = KE_1 + PE_1$$
$$= \tfrac{1}{2}mv_1^2 + mgy_1 + \cancel{\tfrac{1}{2}kx_1^2} \to 0$$
$$y_1 = 100\,ft + 3\,ft$$

$$E_2 = \cancel{KE_2} \to 0 + PE_2$$
$$= 0 + mgy_2 + \tfrac{1}{2}kx_2^2$$
$$y_2 = 3\,ft$$

$$\tfrac{1}{2}mv_1^2 + mgy_1 = mgy_2 + \tfrac{1}{2}kx_2^2$$
$$\tfrac{1}{2}mv_1^2 + mg(y_1 - y_2) = \tfrac{1}{2}kx_2^2$$
$$k = \frac{mv_1^2 + 2mg(y_1-y_2)}{x_2^2}$$

$m = 77.3\,kg$

$y_1 - y_2 = 30.5\,m$

$x_2 = 100\,ft - 75\,ft - 6\,ft$

$= 19\,ft = 5.8\,m$

$V_1 = 10\,ft/s = 3.05\,m/s$

$$= \frac{(77.3\,kg)(9\,m^2/s^2) + 2(77.3)(9.8)(30.5)\,J}{(5.8\,m)^2}$$

$$= \frac{719\,J + 46210\,J}{33.64\,m^2} = 1395\,\frac{N}{m}$$

which, it is worth noting, provides a force at max stretch equal to a 1816 lb weight dangling from Dave's leg.